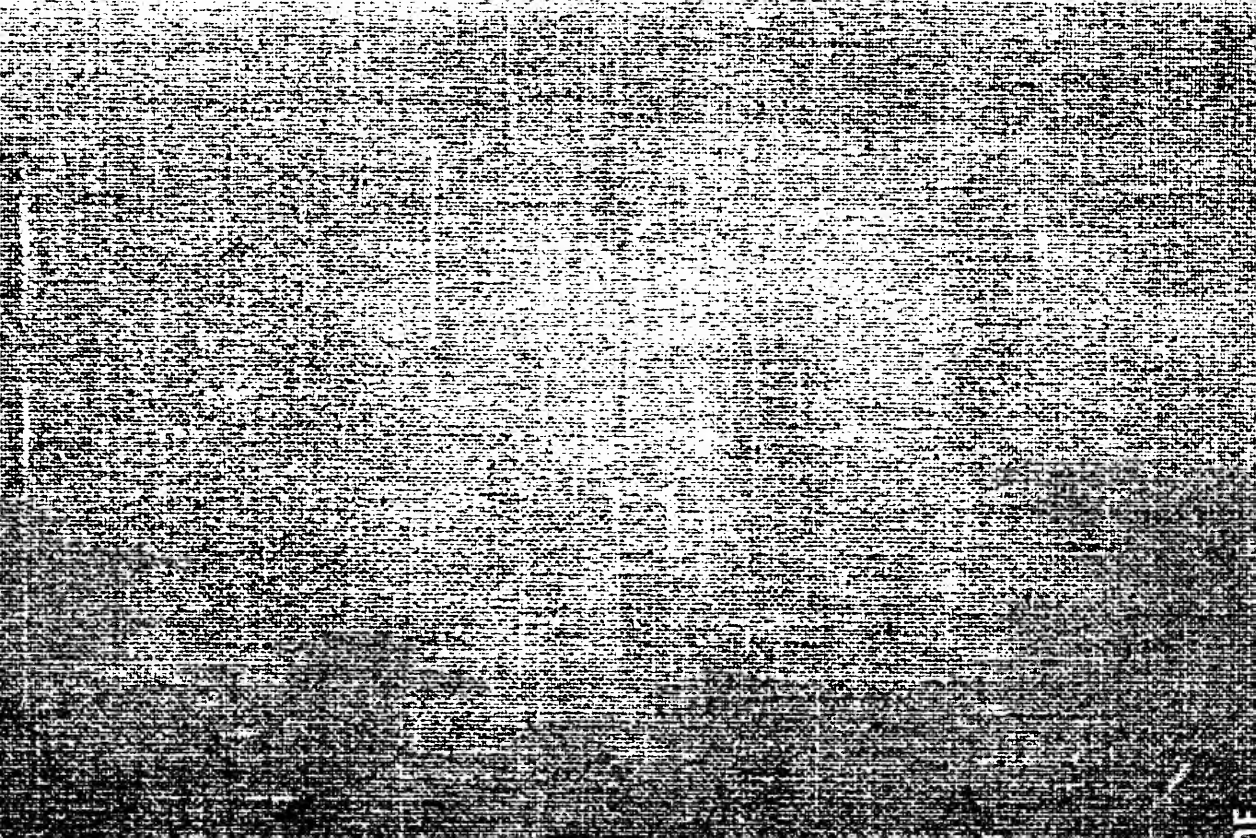

ACTUARIAL SOCIETY OF AMERICA.

"The work of science is to substitute facts for appearances and demonstrations for impressions."—RUSKIN.

Papers and Transactions.

1899-1900.

VOLUME VI.

Nos. 21, 22, 23 and 24.

With Index.

NEW YORK,
1900.

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NOTICE.

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these "Papers and Transactions."

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OCTOBER, 1900.

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Apr. 24, 1890	BLACKADAR, ALFRED KIMBALL, A.M., F.I.A., Actuary, Insurance Department, Ottawa, Canada.
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Oct. 8, 1896	DOVEY, WILLIAM ROADLEY, F.F.A., Actuary, Citizens Life Assur. Co., Sydney, Australia.
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Oct. 13, 1892	FRANKLAND, FREDERICK WILLIAM, F.I.A., Associate Actuary, New York Life Ins. Co., 346 Broadway, New York.
Apr. 30, 1896	GORE, JOHN KINSEY, A.M., Actuary, Prudential Ins. Co., Newark, New Jersey.
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†	HALL, CLAYTON COLMAN, LL.B., Actuary, Maryland Life Ins. Co., Baltimore, Maryland.
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†	HENDRY, WILLIAM, Consulting Actuary, Mutual Life Assur. Co. of Canada, Waterloo, Ontario.
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Oct. 19, 1893	HUBBELL, GEORGE WILLIAM, A.B., Assistant Actuary, United States Life Ins. Co., 261 Broadway, New York.
Oct. 6, 1898	HUGHES, WILLIAM, F.I.A., Joint Manager, Prudential Assur. Co., Holborn Bars, London.
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Apr. 28, 1892	KING, GEORGE, F.I.A., F.F.A., COR. MEM., I.A.FR. and A.A.BELG., Actuary, London Assurance Corporation, 7 Royal Exchange, London.
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Oct. 3, 1889	KYLE, CHRISTOPHER, New York Life Ins. Co., 346 Broadway, N. Y.
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Oct. 7, 1897	LEPREUX, OMER, COR. MEM. I.A.ENG., Director General of the "Caisse Générale d'Epargne et de Retraite," 48 Rue Fossé-aux-Loups, Brussels; President of the Permanent Committee, International Congress of Actuaries.
Apr. 28, 1893	LESLIE, GEORGE, Assistant Actuary, Govt. Life Ins. Dept., Wellington, New Zealand.
Apr. 24, 1890	LEWIS, CHARLTON THOMAS, Ph.D., 32 Nassau St., New York.
†	LOVELAND, CHARLES ALVIN, Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wisconsin.
†	LUNGER, JOHN BODINE, (TREASURER), Managing Actuary, New York Life Ins. Co., 346 Broadway, New York.
†	MCCABE, WILLIAM, LL.B., F.I.A., COR. MEM. I.A.FR., Managing Director and Actuary, North American Life Assur. Co., Toronto, Ontario.
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†	MACDONALD, WILLIAM CAMPBELL, Actuary, Confederation Life Assoc., Toronto, Ontario.
Oct. 10, 1895	MANLY, HENRY WILLIAM, F.I.A., COR. MEM. I.A.FR., Actuary, Equitable Life Assur. Soc., Mansion House St., London.
Apr. 28, 1893	MARIE, LÉON, A.B., COR. MEM. I.A.ENG., and A.A.BELG., Actuary, "Compagnie Le Phénix," 32, Rue Jouffroy, Paris.
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†	MARSHALL, WILLIAM ANDREW, Actuary, Home Life Ins. Co., 256 Broadway, New York.
Oct. 3, 1889	MASON, JAMES WEIR, A.M., 32 West 129th St., New York
Oct. 10, 1895	MEIKLE, JAMES, F.I.A., F.F.A., Actuary, Scottish Provident Institution, 6 St. Andrew Square, Edinburgh.
Apr. 24, 1890	MESSENGER, HIRAM JOHN, Ph.D., Actuary, Travelers Ins. Co., Hartford, Connecticut.
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Oct. 3, 1889	NICHOLS, WALTER SMITH, A.M., Editor *Insurance Monitor*, 100 William St., New York.
†	NITCHIE, JOSEPH HOWARD, A.B., Actuary, 161 La Salle St., Chicago, Illinois.
Oct. 13, 1892	OGDEN, SYDNEY NORRIS, Assistant Actuary, Mutual Benefit Life Ins. Co., Newark, New Jersey.
Apr. 24, 1890	PATERSON, JOHN SHERMAN, Actuary, Insurance Department, Albany, New York.
Apr. 28, 1893	PATERSON, WILLIAM BROCKIE, F.F.A., A.I.A., Assistant Actuary, Norwich Union Life Ins. Soc., Norwich, England.
Oct. 23, 1890	PEILER, MAXIMILIAN HEINRICH, Ætna Life Ins. Co., Hartford, Connecticut.
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Apr. 28, 1893	PLUMLEY, GARDNER LADD, Assistant Actuary, Home Life Ins. Co., 256 Broadway, New York.
Apr. 26, 1894	REITER, CHARLES GRANT, Assistant Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York.
Apr. 24, 1890	RICHARDSON, JOSEPHUS HARGREAVES, F.F.A., A.I.A., Commissioner, Govt. Life Ins. Dept., Wellington, New Zealand.

Elected.	
Apr. 24, 1890	RICHTER, JOHN GEORGE, Actuary, London Life Ins. Co., London, Ontario.
*June 7, 1900	ROCHE, JOHN FRANCIS, New York Life Ins. Co., 346 Broadway, New York.
Oct. 11, 1894	RYAN, GERALD HEMMINGTON, F.I.A., Actuary and Secretary, British Empire Mutual Life Assur. Co., 4 King William St., London.
†	ST. JOHN, HOWELL WILLIAMS, Ph.B., (EX-PRESIDENT), COR. MEM. I.A.FR., Actuary, Ætna Life Ins. Co., Hartford, Connecticut.
Oct. 19, 1893	SALTER, GEORGE FERRY, M.S., Prudential Ins. Co., Newark, New Jersey.
†	SANDERS, GEORGE WHITE, A.B., Actuary, Michigan Mutual Life Ins. Co., Detroit, Michigan.
Apr. 25, 1895	SANDERSON, FRANK, A.M., A.I.A., F.S.S., Actuary, Canada Life Assur. Co., Toronto, Ontario.
Apr. 28, 1892	SARTELLE, EDWARD JAMES, Assistant Actuary, State Mutual Life Assur. Co., Worcester, Massachusetts.
Apr. 28, 1893	SCOTT, ERNST WILLEM, A.I.A., COR. MEM. I.A.FR., Managing Director, "Algemeene Mattschappij van Levensverzekering en Lijfrente," Amsterdam.
Apr. 24, 1890	SMITH, WILLIAM SIMEON, A.M., Actuary, John Hancock Mutual Life Ins. Co., Boston, Massachusetts.
Apr. 24, 1890	STABLER, EDWARD LINCOLN, Ph.D., 135 Willow St., Brooklyn, New York.
†	STANDEN, WILLIAM THOMAS, Actuary, United States Life Ins. Co., 261 Broadway, New York.
†	STARR, WILLIAM ELI, Actuary, State Mutual Life Assur. Co., Worcester, Massachusetts.
Apr. 24, 1890	TATLOCK, JOHN, JR., A.M., F.R.A.S., (SECRETARY), Associate Actuary, Mutual Life Ins. Co., 32 Nassau St., New York.
Apr. 29, 1891	TEECE, RICHARD, F.I.A., F.F.A., Actuary, Australian Mutual Provident Soc., Sydney, Australia.
Oct. 13, 1892	THISELTON, HERBET CECIL, F.I.A., F.F.A., Actuary, Hand in Hand Ins. Soc., 26 New Bridge Street, London.
Oct. 19, 1893	TORREY, MORRIS WHITTEMORE, Actuary, Manhattan Life Ins. Co., 66 Broadway, New York.
Oct. 23, 1890	VAN CISE, JOEL GARRETSON, Actuary, Equitable Life Assur. Soc., 120 Broadway, New York.
†	WEEKS, RUFUS WELLS, A.I.A., COR. MEM. I.A.FR., Actuary, New York Life Ins. Co., 346 Broadway, New York.

Elected.	
Apr. 24, 1890	WEGENAST, GEORGE, Manager and Actuary, Mutual Life Assur. Co. of Canada, Waterloo, Ontario.
Apr. 24, 1890	WELCH, ARCHIBALD ASHLEY, A.B., Actuary, Phœnix Mutual Life Ins. Co., Hartford, Connecticut.
†	WELLS, DANIEL HALSEY, Ph.B., Actuary, Connecticut Mutual Life Ins. Co., Hartford, Connecticut.
Oct. 6, 1898	WHITTALL, WILLIAM JOSEPH HUTCHINGS, F.I.A., Actuary and Secretary, Clerical, Medical and General Life Assur Soc., 15 St. James's Square, London.
†	WING, ASA SHOVE, COR. MEM. I.A.FR., Vice-President, Provident Life and Trust Co., 409 Chestnut St., Philadelphia, Pennsylvania.
Apr. 30, 1896	WOLTERBEEK, LEONARD JOAN, LL.D., COR. MEM. A.A.BELG., Keizersgracht 311, Amsterdam.
Oct. 6, 1898	WOODS, ERNEST, F.I.A., Actuary, Westminister and General Life Assur. Assoc., 28 King St., Covent Garden, London.
†	WOODWARD, GEORGE BADGER, Secretary, Metropolitan Life Ins Co., 1 Madison Ave., New York.
Apr. 30, 1896	WRIGHT, PETER TROTH, A.M., Assistant Actuary, Penn Mutual Life Ins. Co., 921 Chestnut St., Philadelphia, Pennsylvania.
†	WRIGHT, WALTER CHANNING, Consulting Actuary, 45 Milk St. Boston, Massachusetts.
Oct. 6, 1898	WYATT, FRANK BERTRAND, F.I.A., Actuary, Clergy Mutual Assur. Soc., 3 The Sanctuary, Westminster, London.
Oct. 10, 1895	YANO, TSUNETA, COR. MEM. I.A.FR., Expert to the Imperial Japanese Ministry of Agriculture and Commerce, 12 Fukuromachi, Kanda, Tokio.
Apr. 29, 1897	YOUNG, THOMAS EMLEY, A.B., F.I.A., COR. MEM. I.A.FR. and A.A.BELG., Actuary, Commercial Union Assur. Co., 24 Cornhill, London.

ASSOCIATES.

Enrolled.	
Oct. 7, 1897	ANGELL, CHARLES HART, A.B., Mass. Mutual Life Ins. Co., Springfield, Massachusetts.
Sept. 12, 1900	BUTTOLPH, HENRY WRIGHT, A.M., Mathematician, Interstate Life Ins. Co., Indianapolis, Indiana.
Apr. 13, 1900	DAUGHTREY, WILLIAM LAMB, JR., C.E., Actuary, Life Ins. Co. of Va., Richmond, Virginia.
Sept. 12, 1900	DAVENPORT, JOHN SYDNEY, A.B., 35 Nassau St., New York.
Sept. 12, 1900	DOW, HERBERT BEEMAN, A.M., Assistant Actuary, New England Mutual Life Ins. Co., Boston, Massachusetts.
Apr. 13, 1900	EMERY, JOHN MAYNARD, M.D., Actuary, American Union Life Ins. Co., 11 Broadway, New York.
Sept. 12, 1900	FORSTER, ROBERT ELDER, Ph.B., Consulting Actuary, 1001 Chestnut St., Philadelphia. Pennsylvania.
Apr. 13, 1900	GAYLORD, WILLIAM STANDISH, A.B., 35 Nassau St., New York.
Apr. 13, 1900	GRAHAM, WILLIAM JOSEPH, Actuary, Sun Life Ins. Co., Louisville Kentucky.
Oct. 7, 1897	HALL, SAMUEL STICKNEY, A.B., Mutual Life Ins. Co., 32 Nassau St., New York.
Apr. 13, 1900	HARDCASTLE, EDWARD EDGINGTON, M.A., Union Central Life Ins. Co., Cincinnati, Ohio.
Apr. 13, 1900	HENDERSON, ROBERT, B.A., F.I.A., Equitable Life Assur. Soc., 120 Broadway, New York.
Apr. 28, 1898	HITCHINS, WILLIAM RICHMOND. A.B., Assistant Actuary, Manufacturers' Life Ins. Co., Toronto, Ontario.
Apr. 13, 1900	HUNTER, ARTHUR, F.F.A., A.I.A., New York Life Ins. Co., 346 Broadway, New York.
Sept. 12, 1900	HUNTER, ROBERTSON GILBERT, New York Life Ins. Co., 346 Broadway, New York.
May 17, 1900	HUTCHESON, WILLIAM ANDERSON, F.I.A., F.F.A., Associate Actuary, Mutual Life Ins. Co., 32 Nassau St., New York.
Apr. 13, 1900	JOHNSTON, FREDERICK HENRY, A.I.A., Assistant Actuary, Prudential Ins. Co., Newark, New Jersey.
Apr. 28, 1898	KAUFMAN, HENRY NICHOLAS, A.I.A., Phœnix Mutual Life Ins. Co., Hartford, Connecticut.

Enrolled.	
Apr. 13, 1900	LINZMEYER, LOUIS, Manhattan Life Ins. Co., 66 Broadway, New York.
May 17, 1900	SHEPPARD, HERBERT NORMAN, B.A., A.I.A., Union Central Life Ins. Co., Cincinnati, Ohio.
Oct. 7, 1897	ROSE, DOUGLAS HALL, Maryland Life Ins. Co., Baltimore, Maryland.
Apr. 13, 1900	WASHBURNE, ALVA COURTENAY, Metropolitan Life Ins. Co., 1 Madison Ave., New York.
Apr. 28, 1898	WOOD, ARTHUR BARTON, A.B., A.I.A., Sun Life Assur. Co. of Canada, Montreal, Quebec.

NOTATION.

ADOPTED OCTOBER 23, 1890.

RESOLVED, that in the presentation of papers the Symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that in any event the use of such symbols in other senses be avoided; that the letters denoting commutation columns, according to the system of Dr. Farr, customarily followed by American Actuaries, be printed in plain, "full-face" type; and that otherwise there be entire liberty as regards notation.

$$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}.$$

$$u_x = \frac{l_x}{l_{x+1}}(1+i) = \frac{D_x}{D_{x+1}}.$$

S=Sum insured.

K=Cost of insurance=q ($S-V$).

Π=Single premium (alternative for A).

CORRIGENDA.

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VOL. VI. NO. 21.

PAPERS AND TRANSACTIONS.

MAY 18TH AND 19TH, 1899.

NOTE ON INTEREST RATES.

HOWELL W. ST. JOHN.

A RECENT publication* containing the opinions of distinguished financiers relating to the probable future rates of interest upon securities of the higher class has already aroused public attention, and the belief that the question treated therein may be profitably discussed by the members of the Actuarial Society forms the *raison d'être* of the following comments :

Ambiguity of language is often the occasion of confusion of thought, and nowhere, perhaps, more frequently than within the field of economics ; the word "wealth," for example, has caused such conflict of opinions and fruitless discussion that Archbishop Whateley, among the earlier, and Prof. Perry, of the later economists, reject it altogether.

A pertinent illustration of the truth of this proposition may be found in some of the expert opinions already referred to, viz., the popular and easily besetting fallacy that the rates of interest under discussion are dependent upon the quantity of money (currency) existing in a community at a given epoch—that an abundance of currency is, in consequence, attended by low interest, and vice versa. This error commonly arises from ambiguity in the use of the term capital, a confusion of the *means or instruments of production*, with *money*, "the standard of value and the medium of exchange" ; interest is by universally accepted definition *the income that flows from capital*, its rate (or amount) is expressed or measured in terms of money, as a standard of value, which again, as a medium of exchange, is paid for the possession or control of capital.

There are, however, conditions under which interest is paid for the use of *money per se*—legal tender, the solvent of debts—*e. g.*, for the protection of credit during a panic, or for the reinforcement

* Letters from prominent financiers on interest rates, published by the Equitable Life Assurance Society.

of "margins" in speculation, but such transactions are temporary and spasmodic, and should not be classed among the phenomena relating to the future rates of interest.

Again, there is a degree of ambiguity in the term interest as it is commonly employed, which may be illustrated somewhat by a few definitions, and it may be remarked, *en passant*, that the income thus known includes a premium of insurance—against the risk of investment—as well as the cost of management; this fact points out the distinction between *net* and *gross* interest; the annual returns upon funds invested in the national or state debts are examples as near as may be of the former.

Included, in popular language, under the common name of interest may be recognized, first, the remuneration of professional lenders of money for short-time loans, called by bankers discount; an excellent term, distinguishing clearly this class of transactions, and worthy to be generally adopted.

The *rates* paid under this sort of loans depend largely upon the activity of trade—the flood and ebb of the great tide of bargain and sale—*and they follow with a remarkable degree of correspondence the rise and fall of the average price of commodities;* they are affected by the local conditions of banking privileges, and furnish an example of variable and frequently alternating quantities best valued through the method of averages. Second in order may be placed that form of interest which is the remuneration for loans of capital employed in productive operations extending over considerable periods. This may be termed conveniently loan interest.*

The chief factors which determine the *rates* (as given by economists) are the following: (a) the quantity of disposable (or free) capital; (b) the quantity and quality of labor available for combination with capital; (c) the advance in the industrial arts or the condition of technical knowledge; (d) the natural advantages offering opportunities for the profitable investment of capital; (e) commercial security, *i. e.*, administration of justice, inviolability of contracts, freedom from legislative interference, equity in taxation, etc. There is a third form, which may be termed *normal* interest. It pertains to an equation between the *effective* demand for capital and the total supply offered. Its rates, compared with those of loan interest, bear a relation analogous to that between normal and market prices, and it forms a standard to which the variations

* Loans to the State for political or military purposes are not included in this definition.

in the latter, in the long run, tend to conform. This is the interest dealt with by the economists when they state that "its rates tend to equality and to decline."

Assuming the substantial accuracy of these definitions, and referring to the opinions already mentioned, one may easily detect another fallacy arising from a confusion of the variable rates of discount and the more equable rates of loan interest, which are quite independent of the former; the error consists in deducing conclusions respecting the future of *interest* from perturbations in *discount.*

There is one clearly expressed opinion (in the publication already referred to) that is of greater value for what it possibly suggests than for what it so tersely expresses: "It seems to me, however, to be fully established by the last quarter of a century's experience that the cheapening of productive forces diminishes the earning capacity of capital, and I see no reason for expecting that this law will cease to operate within the period you suggest."

It is difficult to perceive how the cheapening of productive *forces* (labor and the forces of nature) operates to diminish the earning capacity of capital; but it is certain that the cheapening, in terms of money, of the *fruits* of production (commodities) has resulted in a reduction of the prices paid for loans of capital during the period mentioned, and the two concurrent phenomena have moved downward in nearly parallel lines; a fact perhaps not generally observed.

The concentration of industry (a modern economic phenomenon) which is progressing with remarkable acceleration, presents a most interesting problem in its relation to the probable future rates of interest, and it has occasioned the following noteworthy opinion, which may be found among the others previously mentioned: . . . . "and the tendency toward industrial consolidations here will make for permanently lower rates in this country."

A letter addressed a few years ago to a distinguished scholar and economist upon this topic elicited the reply that the present tendency to industrial concentration was pregnant with grave sociological changes; that he saw, however, no reason why it should materially affect the future rates of interest in any way.

Notwithstanding the confidence which ought to be given to the opinion from the bench, there are reasons for believing that the layman has more nearly valued the probabilities of the future, though he presents nothing in support of his conclusions.

It is certain that great industrial consolidations tend to establish monopolies, whose economic relations are not easily determined, but this much may be said with confidence, large combinations of capital *do* enable the "captains of industry" *to lower the limit at which the arts of production can be profitably pursued*, and this alone tends to lower the rates of interest.

Finally, it may be stated that in general the opinions expressed by the distinguished financiers in the publication mentioned appear to have been influenced chiefly by consideration of the business experiences of the past twenty years. It is well established historically that the rise and fall of interest rates are *alternating* events (during variable and considerable intervals), and all conclusions ought to be considered as merely provisional that are not based upon a study of the history and causes of these phenomena during at least the modern period within which the great industrial and commercial interests have been dominant.

Special Tables for the Estimation of Mortality among Annuitants.

EMORY M'CLINTOCK.

It has appeared to me that there is no existing mortality table which can be used with satisfaction in the computation of reserves upon life annuities.

The standard tables on this subject are those presented by Mr. A. J. Finlaison, on February 10, 1883, to the Comptroller-General of the National Debt Office of England, and published by the House of Commons in 1884. I shall refer to these as Finlaison's tables. They comprise observations on 30,788 annuitants, of whom 22,998 had died. About two-thirds of the annuitants were women. Separate life tables were developed for male and female life.

A committee, representing the four principal French insurance companies, has more recently made public a large volume of experience on the lives of annuitants. Concerning this experience Mr. G. F. Hardy's paper, in Volume XXXIII of the Journal of the Institute of Actuaries, states that it "agrees very closely with that of the English Government experience" as presented by Mr. Finlaison.

Thus corroborated, Mr. Finlaison's tables would appear to meet all requirements, were it not for a still more recent paper read by Mr. Weeks before this Society and published in Volume IV of its Papers and Transactions. In this paper Mr. Weeks gave the results of a careful examination into the experience on annuitants in fifteen American life insurance companies, of which, however, only about one-fourth was actually on American lives, the remaining three-fourths representing annuities granted in Europe by the American companies in question. The number of lives taken into consideration comprises 4,365 males and 4,821 females. Considered merely as a body of experience, this is quite small as compared with those embraced in the English and French tabulations. It is, however, large enough to deserve serious attention. It contains, indeed, a large proportion of recent business, and therefore exhibits distinctly the effect of self-selection in reducing the rate of mortality. Apart from this, however, it appears also to

indicate a somewhat greater vitality on the whole, which would seem to be the result of the improving physical conditions of modern life. The hygienic condition of every country is better than it has been, and people generally know better how to take care of themselves. Owing to the great progress made latterly in medicine and surgery, diseases and accidents are less fatal. It is therefore really improbable that the mortality among annuitants during the greater part of the nineteenth century will repeat itself during the twentieth century.

It has been questioned whether, for the valuation of annuities, select tables are not to be preferred, showing a rate of mortality quite small during the first policy year, somewhat larger the next year, and so on, until after say four years the effect of selection may be regarded as worn off. For my own part, I am inclined to prefer for this purpose the use of an average table showing a distinctly low mortality at all ages, rather than to take advantage of the somewhat higher mortality usually found to prevail after the self-selection among annuitants has worn away. The use of a single table is much more readily explained, and involves less trouble. Whatever error occurs is always on the side of safety, and since we cannot exactly measure the progressive increase in vitality which has already occurred and may occur by virtue of human progress, there is every reason for adopting a basis of valuation which we know to be abundantly safe.

Finlaison's tables show that male annuitants are actually worse risks during middle age than insured lives of the same age. The same result appears from the French experience, and, to some extent, also from the experience of the American companies. The reasons have been discussed by Mr. Macaulay before this Society and by Mr. Hardy before the Institute. Men still young do not usually buy annuities for themselves, and those for whom annuities are bought by others are not necessarily good risks. It is reasonable to suppose that the mortality below sixty among male annuitants will continue to be rather heavy. If so, there is no reason why rates should not be made accordingly, having due regard to Mr. Hardy's warning against combining, without due circumspection, annuities according to one mortality table with insurance contracts according to another.

In the absence of tables fulfilling the requirements which I have sketched, I have devised two special mortality tables for male and female annuitants respectively, which will be found annexed.

They are graduated on the Makeham system, and have a common value for the exponential constant, so as to permit of the joint use of the two tables, in accordance with Mr. Makeham's theory. The logarithms of the numbers living have been obtained by successive addition of the logarithms of the annual probabilities of life, and the latter have been computed according to the formula $-\log p_x = \log b_x + c^x \log h$ where $\log c = 0.04$, and for the male table $\log b = 0.0032$, $\log \log h = 5.55$; for the female $\log b = 0.0015$, $\log \log h = 5.43$.

The extent to which these tables correspond with Mr. Weeks's data is shown by columns giving the number of expected deaths by these tables alongside of the actual deaths which occurred in the annuitants' experience of the fifteen American companies. The tables differ somewhat from those which might be framed as a mere adjustment of Mr. Weeks's data, yet I have endeavored to admit only such divergencies as have seemed reasonably allowable. The female table, for example, shows a somewhat higher rate of mortality at ages under 70, because I have considered that the experience in question has at those ages been materially affected by the large proportion of recent entries, tending naturally to reduce the average mortality. I have thought it best that the male table should diverge from the experience, if at all, in the opposite direction, so as to be on the safe side.

TABLE A.

MALE ANNUITANTS' MORTALITY.

Age.	$log\ l_x$	$log\ p_x$	q_x	μ_x	Exposures, 15 Am. Cos.	
					Expected by Table A.	Actual.
10	6.0000000	9.9967109	.00754	.00757		
11	5.9967109	.9967023	.00756	.00758		
12	.9934132	.9966928	.00759	.00761		
13	.9901060	.9966825	.00761	.00763		
14	.9867885	.9966712	.00764	.00765		
15	.9834597	.9966587	.00766	.00768		
16	.9801184	.9966451	.00770	.00771		
17	.9767635	.9966302	.00773	.00774		
18	.9733937	.9966138	.00777	.00778		
19	.9700075	.9965958	.00781	.00782		
20	.9666033	.9965761	.00785	.00786		
21	.9631794	.9965545	.00790	.00791		
22	.9597339	.9965308	.00796	.00796		
23	.9562647	.9965049	.00802	.00802		
24	.9527696	.9964764	.00808	.00808		
25	.9492460	.9964452	.00815	.00815		
26	.9456912	.9964110	.00823	.00823		
27	.9421022	.9963734	.00832	.00831	.1	
28	.9384756	.9963323	.00841	.00840	.1	
29	.9348079	.9962871	.00851	.00850	.1	
30	.9310950	.9962377	.00863	.00861	.1	
31	.9273327	.9961834	.00875	.00873	.1	
32	.9235161	.9961239	.00889	.00886	.2	
33	.9196400	.9960587	.00903	.00900	.2	
34	.9156987	.9959872	.00920	.00916	.3	
35	.9116859	.9959087	.00938	.00933	.3	1.
36	.9075946	.9958228	.00957	.00952	.3	
37	.9034174	.9957285	.00979	.00973	.5	1.
38	.8991459	.9956251	.01002	.00995	.5	
39	.8947710	.9955118	.01028	.01020	.7	
40	.8902828	.9953875	.01056	.01047	.8	
41	.8856703	.9952512	.01087	.01077	1.0	1.
42	.8809215	.9951018	.01122	.01110	1.1	
43	.8760233	.9949379	.01159	.01146	1.4	1.
44	.8709612	.9947583	.01200	.01186	1.6	2.
45	.8657195	.9945613	.01244	.01229	2.0	5.
46	.8602808	.9943453	.01294	.01277	2.2	
47	.8546261	.9941085	.01347	.01329	2.8	5.
48	.8487346	.9938488	.01406	.01386	3.1	2.
49	.8425834	.9935641	.01471	.01448	3.7	7.
50	.8361475	.9932519	.01542	.01517	4.5	2.
51	.8293994	.9929095	.01619	.01592	5.5	10.
52	.8223089	.9925342	.01704	.01675	6.3	7.
53	.8148431	.9921226	.01797	.01765	7.2	5.
54	.8069657	.9916714	.01899	.01864	8.4	10.
55	.7986371	.9911766	.02011	.01973	10.1	9.
56	.7898137	.9906340	.02134	.02092	12.0	11.
57	.7804477	.9900392	.02267	.02223	13.9	20.
58	.7704869	.9893869	.02414	.02367	15.7	15.
59	.7598738	.9886717	.02575	.02524	17.4	18.

TABLE A—(Continued).

MALE ANNUITANTS' MORTALITY.

Age.	$log\ l_x$	$log\ p_x$	q_x	μ_x	EXPOSURES, 15 AM. COS.	
					Expected by Table A.	Actual.
60	.7485455	.9878875	.02750	.02696	21.0	16.
61	.7364330	.9870276	.02943	.02885	24.5	25.
62	.7234606	.9860848	.03153	.03092	27.4	23.
63	.7095454	.9850510	.03384	.03320	30.1	32.
64	.6945964	.9839175	.03635	.03569	34.1	38.
65	.6785139	.9826746	.03911	.03842	37.8	41.
66	.6611885	.9813118	.04212	.04142	41.3	40.
67	.6425003	.9798176	.04541	.04470	45.5	39.
68	.6223179	.9781791	.04900	.04830	49.2	55.
69	.6004970	.9763826	.05293	.05225	51.8	47.
70	.5768796	.9744128	.05721	.05658	58.1	61.
71	.5512924	.9722529	.06189	.06133	59.2	56.
72	.5235453	.9698846	.06699	.06653	61.4	63.
73	.4934299	.9672879	.07256	.07224	63.0	66.
74	.4607178	.9644406	.07862	.07850	63.4	72.
75	.4251584	.9613187	.08522	.08536	63.2	68.
76	.3864771	.9578955	.09240	.09289	60.6	66.
77	.3443726	.9541420	.10021	.10114	57.0	59.
78	.2985146	.9500265	.10869	.11019	56 0	59.
79	.2485411	.9455139	.11791	.12011	51.6	43.
80	.1940550	.9405659	.12790	.13098	50.8	45.
81	.1346209	.9351405	.13873	.14291	48.8	44.
82	.0697614	.9291917	.15044	.15598	43.2	49.
83	4.9989531	.9226690	.16311	.17032	37.4	44.
84	.9216221	.9155169	.17678	.18604	31.6	34.
85	.8371390	.9076749	.19151	.20328	25.5	30.
86	.7448139	.8990763	.20736	.22218	19 3	14.
87	.6438902	.8896481	.22438	.24291	16.8	13.
88	.5335383	.8793102	.24263	.26563	13.8	12.
89	.4128485	.8679751	.26214	.29055	11.0	12.
90	.2808236	.8555463	.28296	.31787	8.2	6.
91	.1363699	.8419183	.30511	.34783	5.8	3.
92	3.9782882	.8269756	.32861	.38067	4.3	4.
93	.8052638	.8105913	.35347	.41669	2.5	2.
94	.6158551	.7926262	.37967	.45618	2.3	2.
95	.4084813	.7729279	.40717	.49948	1.2	
96	.1814092	.7513291	.43594	.54696	1.3	
97	2.9327383	.7276465	.46587	.59901	1.4	
98	.6603848	.7016791	.49687	.65609	1.5	
99	.3620639	.6732063	.52880	.71868	1.6	
100	.0352702	.6419866				
101	1.6772568	.6077549				
102	.2850117	.5702205				
103	0.8552322	.5290648				
104	.3842970	.4839386				
105	9.8682356	.4344587				
106	.3026945	.3802050				

TABLE B.

FEMALE ANNUITANTS' MORTALITY.

Age.	$log\ l_x$	$log\ p_x$	q_x	μ_x	EXPOSURES, 15 AM. COS.	
					Expected by Table B.	Actual.
10	6.0000000	9.9984324	.00360	.00360		
11	5.9984324	.9984259	.00362	.00361		
12	.9968583	.9984187	.00363	.00363		
13	.9952770	.9984109	.00365	.00365		
14	.9936879	.9984023	.00367	.00366		
15	.9920902	.9983928	.00369	.00369		
16	.9904830	.9983825	.00372	.00371		
17	.9888655	.9983712	.00374	.00373		
18	.9872367	.9983587	.00377	.00376		
19	.9855954	.9983451	.00380	.00379		
20	.9839405	.9983302	.00384	.00382		
21	.9822707	.9983138	.00388	.00386		
22	.9805845	.9982958	.00392	.00390		
23	.9788803	.9982761	.00396	.00394		
24	.9771564	.9982545	.00401	.00399		
25	.9754109	.9982308	.00407	.00404		
26	.9736417	.9982049	.00412	.00410		
27	.9718466	.9981764	.00419	.00416		
28	.9700230	.9981452	.00426	.00423		
29	.9681682	.9981110	.00434	.00431	.1	
30	.9662792	.9980734	.00443	.00439	.1	
31	.9643526	.9980323	.00452	.00448	.1	
32	.9623849	.9979871	.00462	.00458	.1	
33	.9603720	.9979377	.00474	.00469	.2	
34	.9583097	.9978834	.00486	.00481	.2	
35	.9561931	.9978239	.00500	.00494	.3	
36	.9540170	.9977587	.00515	.00508	.3	
37	.9517757	.9976872	.00531	.00524	.4	2.
38	.9494629	.9976087	.00549	.00541	.4	1.
39	.9470716	.9975228	.00569	.00560	.4	
40	.9445944	.9974285	.00590	.00581	.7	
41	.9420229	.9973251	.00614	.00603	.8	2.
42	.9393480	.9972118	.00640	.00628	.8	
43	.9365598	.9970875	.00668	.00655	1.0	2.
44	.9336473	.9969512	.00700	.00685	1.2	5.
45	.9305985	.9968018	.00734	.00718	1.4	3.
46	.9274003	.9966379	.00771	.00754	1.8	1.
47	.9240382	.9964583	.00812	.00794	2.2	1.
48	.9204965	.9962613	.00857	.00837	2.6	1.
49	.9167578	.9960453	.00906	.00885	3.0	1.
50	.9128031	.9958085	.00960	.00937	3.9	7.
51	.9086116	.9955488	.01020	.00994	4.7	4.
52	.9041604	.9952641	.01085	.01056	5.5	5.
53	.8994245	.9949519	.01156	.01125	6.3	9.
54	.8943764	.9946095	.01234	.01200	7.2	8.
55	.8889859	.9942342	.01319	.01283	8.6	6.
56	.8832201	.9938226	.01412	.01373	9.6	7.
57	.8770427	.9933714	.01515	.01472	11.0	8.
58	.8704141	.9928766	.01627	.01581	12.8	13.
59	.8632907	.9923340	.01750	.01700	14.8	16.

TABLE B—(Continued).

FEMALE ANNUITANTS' MORTALITY.

Age.	$log\ l_x$	$log\ p_x$	q_x	μ_x	EXPOSURES, 15 AM. COS.	
					Expected by Table B.	Actual.
60	.8556247	.9917392	.01884	.01831	17.1	13.
61	.8473639	.9910869	.02031	.01975	20.0	26.
62	.8384508	.9903717	.02193	.02132	23.1	28.
63	.8288225	.9895875	.02369	.02304	25.5	19.
64	.8184100	.9887276	.02562	.02493	27.5	25.
65	.8071376	.9877848	.02773	.02700	31.0	30.
66	.7949224	.9867510	.03005	.02928	33.1	35.
67	.7816734	.9856175	.03257	.03177	36.1	33.
68	.7672909	.9843746	.03534	.03450	38.6	39.
69	.7516655	.9830118	.03836	.03750	39.9	38.
70	.7346773	.9815176	.04166	.04078	42.0	30.
71	.7161949	.9798791	.04527	.04438	44.6	43.
72	.6960740	.9780826	.04921	.04833	47.1	44.
73	.6741566	.9761128	.05352	.05266	47.8	58.
74	.6502694	.9739529	.05821	.05741	47.6	43.
75	.6242223	.9715846	.06334	.06261	47.4	49.
76	.5958069	.9689879	.06891	.06832	47.7	48.
77	.5647948	.9661406	.07500	.07458	46.5	46.
78	.5309354	.9630187	.08163	.08144	45.1	40.
79	.4939541	.9595955	.08884	.08897	42.0	41.
80	.4535496	.9558420	.09668	.09722	38.2	49.
81	.4093916	.9517265	.10520	.10627	34.2	31.
82	.3611181	.9472139	.11445	.11619	32.2	34.
83	.3083320	.9422659	.12448	.12706	29.0	24.
84	.2505979	.9368405	.13535	.13899	25.7	30.
85	.1874384	.9308917	.14711	.15206	22.5	22.
86	.1183301	.9243690	.15983	.16640	19.2	19.
87	.0426991	.9172169	.17355	.18212	15.6	18.
88	4.9599160	.9093749	.18834	.19936	12.8	14.
89	.8692909	.9007763	.20425	.21826	10.0	8.
90	.7700672	.8913481	.22134	.23899	8.2	6.
91	.6614153	.8810102	.23966	.26171	6.5	7.
92	.5424255	.8696751	.25924	.28663	4.7	3.
93	.4121006	.8572463	.28014	.31395	4.2	3.
94	.2693469	.8436183	.30238	.34391	3.0	3.
95	.1129652	.8286756	.32598	.37675	2.0	1.
96	3.9416408	.8122913	.35093	.41277	1.4	
97	.7539321	.7943262	.37723	.45226	1.1	2.
98	.5482583	.7746279	.40485	.49556	.4	
99	.3228862	.7530291	.43372	.54304	.4	
100	.0759153	.7293465				
101	2.8052618	.7033791				
102	.5086409	.6749063				
103	.1835472	.6436866				
104	1.8272338	.6094549				
105	.4366887	.5719205				
106	.0086092	.5307648				
107	0.5393740	.4856386				
108	.0250126	.4361587				
109	9.4611713	.3819050				

MALES.

Age.	Table A. q_x	Finlaison. (After 4 Yrs.) q_x	30 Am. Offs. (Male.) q_x	Age.	Table A. q_x	Finlaison. (After 4 Yrs.) q_x	30 Am. Offs. (Male.) q_x
10	.00755	.....	.0065	55	.02010	.0248	.0189
11	.00757	.....	.0065	56	.02134	.0255	.0202
12	.00759	.....	.0065	57	.02267	.0262	.0216
13	.00761	.....	.0065	58	.02413	.0270	.0231
14	.00764	.....	.0066	59	.02575	.0277	.0247
15	.00766	.....	.0066	60	.02750	.0285	.0265
16	.00768	.....	.0066	61	.02942	.0293	.0285
17	.00773	.....	.0066	62	.03154	.0301	.0307
18	.00778	.....	.0067	63	.03384	.0322	.0331
19	.00780	.....	.0067	64	.03635	.0356	.0357
20	.00784	.....	.0068	65	.03912	.0395	.0386
21	.00791	.....	.0068	66	.04212	.0439	.0418
22	.00796	.....	.0069	67	.04540	.0490	.0453
23	.00803	.....	.0069	68	.04900	.0546	.0490
24	.00807	.....	.0070	69	.05293	.0596	.0532
25	.00814	.....	.0070	70	.05722	.0646	.0578
26	.00823	.....	.0071	71	.06190	.0694	.0628
27	.00832	.....	.0072	72	.06700	.0734	.0682
28	.00841	.....	.0073	73	.07255	.0781	.0741
29	.00851	.....	.0074	74	.07862	.0846	.0807
30	.00862	.....	.0075	75	.08521	.0906	.0878
31	.00876	.....	.0076	76	.09239	.0985	.0955
32	.00889	.....	.0077	77	.10021	.1085	.1040
33	.00903	.....	.0079	78	.10869	.1180	.1132
34	.00919	.....	.0080	79	.11792	.1266	.1232
35	.00937	.....	.0082	80	.12789	.1386	.1341
36	.00958	.....	.0084	81	.13873	.1494	.1458
37	.00978	.....	.0086	82	.15045	.1592	.1587
38	.01001	.....	.0088	83	.16311	.1720	.1725
39	.01028	.....	.0091	84	.17677	.1863	.1875
40	.01056	.0164	.0094	85	.19152	.1984	.2036
41	.01088	.0169	.0096	86	.20735	.2133	.2208
42	.01122	.0173	.0100	87	.22438	.2312	.2399
43	.01158	.0178	.0103	88	.24263	.2466	.2595
44	.01199	.0183	.0108	89	.26213	.2628	.2926
45	.01245	.0188	.0112	90	.28295	.2773	.3281
46	.01293	.0194	.0117	91	.30510	.2941	.3585
47	.01347	.0199	.0122	92	.32860	.3128	.3897
48	.01406	.0205	.0128	93	.35347	.3358	.4250
49	.01472	.0210	.0135	94	.37966	.3647	.4624
50	.01542	.0216	.0142	95	.40717	.4011	.5000
51	.01619	.0222	.0150	96	.43593	.4466	.5588
52	.01705	.0229	.0158	97	.46587	.5028	.6000
53	.01798	.0235	.0167	98	.49687	.5712	.6667
54	.01900	.0242	.0178	99	.52879	.6536	1.0000

FEMALES.

Age.	Table B. q_x	Finlaison. (After 4 Yrs.) q_x	30 Am. Offs. (Female.) q_x	Age.	Table B. q_x	Finlaison. (After 4 Yrs.) q_x	30 Am. Offs. (Female.) q_x
10	.00361	.....	.0031	55	.01320	.0181	.0173
11	.00361	.....	.0042	56	.01413	.0185	.0185
12	.00363	.....	.0051	57	.01515	.0194	.0203
13	.00365	.....	.0059	58	.01626	.0204	.0220
14	.00368	.....	.0064	59	.01751	.0209	.0235
15	.00370	.....	.0066	60	.01884	.0219	.0253
16	.00372	.....	.0070	61	.02031	.0233	.0273
17	.00375	.....	.0077	62	.02193	.0248	.0302
18	.00377	.....	.0083	63	.02368	.0266	.0326
19	.00379	.....	.0087	64	.02562	.0292	.0353
20	.00384	.....	.0091	65	.02775	.0320	.0384
21	.00388	.....	.0091	66	.03005	.0348	.0417
22	.00391	.....	.0102	67	.03257	.0375	.0454
23	.00395	.....	.0109	68	.03535	.0403	.0495
24	.00402	.....	.0110	69	.03837	.0434	.0540
25	.00407	.....	.0113	70	.04166	.0467	.0590
26	.00414	.....	.0116	71	.04527	.0511	.0645
27	.00418	.....	.0114	72	.04922	.0565	.0705
28	.00425	.....	.0113	73	.05352	.0629	.0772
29	.00434	.....	.0114	74	.05822	.0697	.0845
30	.00443	.....	.0115	75	.06334	.0771	.0926
31	.00453	.....	.0116	76	.06891	.0843	.1015
32	.00462	.....	.0111	77	.07500	.0921	.1112
33	.00473	.....	.0107	78	.08163	.0996	.1218
34	.00487	.....	.0112	79	.08883	.1085	.1334
35	.00501	.....	.0111	80	.09668	.1176	.1462
36	.00514	.....	.0111	81	.10519	.1285	.1601
37	.00530	.....	.0113	82	.11446	.1391	.1753
38	.00549	.....	.0115	83	.12447	.1521	.1918
39	.00569	.....	.0113	84	.13535	.1652	.2097
40	.00590	.0108	.0118	85	.14712	.1810	.2291
41	.00613	.0110	.0120	86	.15982	.1951	.2501
42	.00640	.0113	.0112	87	.17354	.2142	.2725
43	.00668	.0115	.0109	88	.18835	.2319	.2965
44	.00700	.0117	.0112	89	.20424	.2515	.3227
45	.00734	.0120	.0112	90	.22134	.2671	.3505
46	.00771	.0122	.0113	91	.23966	.2920	.3794
47	.00812	.0125	.0109	92	.25924	.3074	.4112
48	.00857	.0127	.0111	93	.28014	.3280	.4427
49	.00905	.0130	.0118	94	.30238	.3435	.4794
50	.00960	.0132	.0122	95	.32597	.3798	.5132
51	.01019	.0141	.0134	96	.35093	.4006	.5405
52	.01085	.0151	.0146	97	.37723	.4456	.5882
53	.01156	.0162	.0151	98	.40485	.4902	.5714
54	.01233	.0175	.0159	99	.43373	.6538	1.0000

ANNUITIES AT 3½ PER CENT.

MALES.

Age.	Table A.	Finlaison. At Entrance.	Finlaison. After 4 Years.	Age.	Table A.	Finlaison. At Entrance.	Finlaison. After 4 Years.
10	21.143	.....	.....	55	11.956	11.746	11.617
11	21.049	.....	.....	56	11.628	11.451	11.330
12	20.952	.....	.....	57	11.298	11.146	11.034
13	20.851	.....	.....	58	10.965	10.832	10.728
14	20.746	.....	.....	59	10.629	10.506	10.411
15	20.637	.....	.....	60	10.292	10.178	10.083
16	20.524	.....	.....	61	9.954	9.857	9.742
17	20.407	.....	.....	62	9.615	9.543	9.388
18	20.286	.....	.....	63	9.276	9.228	9.018
19	20.160	.....	.....	64	8.937	8.913	8.644
20	20.030	.....	.....	65	8.598	8.597	8.276
21	19.895	.....	.....	66	8.262	8.289	7.918
22	19.756	.....	.....	67	7.927	7.987	7.572
23	19.611	.....	.....	68	7.594	7.689	7.240
24	19.462	.....	.....	69	7.265	7.388	6.926
25	19.307	.....	.....	70	6.940	7.087	6.622
26	19.147	.....	.....	71	6.619	6.790	6.328
27	18.981	.....	.....	72	6.302	6.495	6.037
28	18.810	.....	.....	73	5.991	6.208	5.743
29	18.634	.....	.....	74	5.686	5.934	5.448
30	18.452	.....	.....	75	5.387	5.669	5.159
31	18.264	.....	.....	76	5.095	5.410	4.872
32	18.070	.....	.....	77	4.810	5.163	4.594
33	17.871	.....	.....	78	4.533	4.925	4.333
34	17.665	.....	.....	79	4.264	4.689	4.084
35	17 453	.....	.....	80	4.003	4.463	3.840
36	17.235	.....	.....	81	3.751	.....	3.615
37	17.011	.....	.....	82	3.508	.....	3.398
38	16.780	.....	.....	83	3.273	.....	3.183
39	16.543	.....	.....	84	3.048	.....	2.979
40	16.300	15.365	15.182	85	2.832	.....	2.789
41	16.051	15.158	14.976	86	2.626	.....	2.601
42	15.795	14.947	14.766	87	2.429	.....	2.423
43	15.533	14.733	14.552	88	2.241	.....	2.262
44	15.265	14.514	14.335	89	2.062	.....	2.107
45	14.991	14.292	14.114	90	1.892	.....	1.958
46	14.711	14.065	13.888	91	1.731	.....	1.804
47	14.425	13.833	13.658	92	1.579	.....	1.646
48	14.134	13.595	13.424	93	1.433	.....	1.478
49	13.837	13.351	13.184	94	1.295	.....	1.304
50	13.535	13.103	12.939	95	1.160	.....	1.124
51	13.228	12.845	12.688	96	1.025	.....	.943
52	12.916	12.582	12.430	97	.881	....	.764
53	12.600	12.311	12.167	98	.707	.....	.590
54	12.280	12.032	11.896	99	.455	.....	.425

Annuities at 3½ per Cent.

FEMALES.

Age.	Table B.	Finlaison. At Entrance.	Finlaison. After 4 Years.	Age.	Table B.	Finlaison. At Entrance.	Finlaison. After 4 Years.
10	23.139	.....	.....	55	13.464	12.942	12.699
11	23.036	.....	.....	56	13.122	12.620	12.386
12	22.929	.....	.....	57	12.776	12.292	12.061
13	22.818	.....	.....	58	12.427	11.960	11.731
14	22.703	.....	.....	59	12.075	11.625	11.395
15	22.584	.....	.....	60	11.720	11.287	11.045
16	22.461	.....	.....	61	11.363	10.943	10 688
17	22.334	.....	.....	62	11.004	10.601	10.326
18	22.202	.....	.....	63	10.645	10.257	9.959
19	22.066	.....	.....	64	10.285	9.907	9.589
20	21.925	.....	.....	65	9.9245	9.546	9.224
21	21.780	.....	.....	66	9.5647	9.183	8.862
22	21.630	.....	.....	67	9.2060	8.818	8.502
23	21.475	.....	.....	68	8.8489	8.456	8.143
24	21.315	.....	.....	69	8.4943	8.100	7.782
25	21.150	.....	.....	70	8.1424	7.754	7.420
26	20.980	.....	.....	71	7.7936	7.426	7.056
27	20.804	.....	.....	72	7.4489	7.110	6.696
28	20.622	.....	.....	73	7.1087	6.801	6.346
29	20.435	.....	.....	74	6.7736	6.500	6.009
30	20.242	.....	.....	75	6.4441	6.204	5.685
31	20.044	.....	.....	76	6.1207	5.915	5.375
32	19.840	.....	.....	77	5.8038	5.633	5.076
33	19.630	.....	.....	78	5.4940	5.359	4.787
34	19.414	.....	.....	79	5.1917	5.092	4.502
35	19.192	.....	.....	80	4.8973	4.834	4.227
36	18.964	.....	.....	81	4.6112	.....	3.958
37	18.729	.....	.....	82	4.3336	.....	3.701
38	18.488	.....	.....	83	4.0650	.....	3.449
39	18.241	.....	.....	84	3.8054	.....	3.210
40	17.988	16.980	16.787	85	3.5551	.....	2.980
41	17.728	16.758	16.565	86	3.3142	.....	2.766
42	17.461	16.529	16.336	87	3.0827	.....	2.557
43	17.189	16.294	16.101	88	2.8606	.....	2.368
44	16.910	16.051	15.858	89	2.6478	.....	2.190
45	16 625	15.801	15.608	90	2.4438	.....	2 029
46	16.334	15.543	15.350	91	2.2483	.....	1.865
47	16.037	15.276	15.083	92	2.0604	.....	1.727
48	15.734	15.000	14.808	93	1.8788	.....	1.580
49	15.425	14.719	14.524	94	1.7012	.....	1.434
50	15.111	14.434	14.230	95	1.5239	.....	1.261
51	14.791	14.149	13.925	96	1.3401	.....	1.104
52	14.466	13.859	13.618	97	1.1370	.....	.907
53	14.137	13.558	13.311	98	.8896	.....	.693
54	13.803	13.252	13.004	99	.5471	.....	.406

The Equities of the Policy-holder under Term Extension where Liens Exist on the Policy.

JESSE J. BARKER.

IN this day and generation, when the term extension idea has taken so firm a hold on the insuring public, the system of term extension is practiced in various ways by different companies, showing a diversity of opinion among them as to what constitutes its correct application to retiring policy-holders. It was originally introduced to the public by the Massachusetts Extension Law of 1861, with the workings of which all present are, of course, familiar, including its inconsistencies. Later the State of Maine adopted a modification of the Massachusetts non-forfeiture extension law, which in a measure corrected its faults. The faults of the Massachusetts law were mainly its inapplication to endowment forms of policies, which were then scarcely known, and the advantage it gave to note policies over those paid all cash.

That it is a most benevolent system, and one which works no hardship on the companies, is shown by the fact that it has not increased the mortality experienced even where it is automatic. Care, however, must be taken that in its application, which should be *general* where a medical examination is not required, the chance of selection against a company should either be entirely eliminated or reduced to its minimum, for it must be conceded that if all healthy lives upon retiring select paid-up insurance, while the unhealthy ones are permitted to take extension, then the selection will surely work against the company by increasing the average amount of its policies expiring by death, as compared with the average amount of its policies in force, modified, perhaps, by the cases where the date of death occurs beyond or without the period of extension.

Where no liens exist and the extension is granted to all alike, no such selection against the company, of course, can take place.

Where options are given to the retiring policy-holder of taking either paid-up or extension, as he may prefer, it is more likely that a selection against the company will be shown in the death losses.

Where loans exist, the problem of equitable extension is more complicated, whereas the question of a paid-up policy for a smaller

amount presents no difficulties of selection. If we cancel the liens from the reserve and grant extension for the full amount of the policy free of any further charge, using the balance of the reserve as a single premium to purchase such extension, it is a necessity that the insured shall be in good health to secure the proper average and prevent a selection against the company being made, for the contract by the cancellation of the indebtedness changes its character by reason of the insured being relieved of his obligation, and to that extent the obligation of the company is increased, and no condition should be permitted to arise or exist whereby that contract could ever be made to cost the company more than the original amount of the policy. Cases might in this way occur where a company whose obligation—originally, we will say, was for $100,000—might by the cancellation of a loan of, say, $20,000, and the subsequent maturing of the policy as a death loss under the extension be made to lose $120,000 under a contract the outside limit of which should never exceed $100,000. To prevent such a selection being made against the company and to obviate the necessity of ascertaining the good health of the insured before changing the contract to a term policy carried by the reserve, it was formerly the practice with companies granting such extensions, to carry as a charge against the term insurance not only the liens but also the foreborne premiums, at least for a time sufficient to prove the risk not below the average in health when the change to term insurance was made. But the competition between the companies has outweighed their caution in some instances, and in looking around for new features of liberality to offer to the public the above precautionary measures have been done away with. In such cases the companies are exposed to the additional risks above alluded to. The question then arises in what way may the equities of the policy-holders be fully protected and the advantages of the automatic term extension, which are many, extended to them without the company incurring the hazard of paying the contract in full after a large proportion of the reserve has been withdrawn in the shape of a loan or premium note. It occurs to my mind that if the policy was considered as a sum of its parts, and not as an entirety, where a lien as above exists, were we to cancel as much of the original contract before starting it on its journey as a term policy, as may be necessary to furnish a cash value sufficient to pay off the liens and carry only the balance of the policy under the extension system, these difficulties would be largely

avoided, while all the equities of the policy-holder under such a contract would be preserved, and the original time of extension as named in or provided for by the policy would in no wise be disturbed. The company would by this plan be only cancelling that portion of the policy previously mortgaged to its full cash value, and the balance of the policy thus extended would be the only remaining portion in which the insured has any rights or equities.

A few familiar figures will serve to illustrate these points (using the Actuaries' Table with four per cent. interest):

An applicant, aged 40, takes a policy for $100,000 on the ten payment life plan and pays eight premiums thereon. He then borrows all the company will lend him thereon, the reserve held being, say, $36,782, the company lends him, say, $24,500. He pays no further premium and the company forfeits the policy, and if it follows the method laid down by the old Massachusetts non-forfeiture law, cancels the indebtedness out of the reserve and fining the balance of reserve 20 per cent. has then a remainder of $9,826 with which to extend the $100,000 of insurance, and at the net rate at the then age of 48 this would extend the face of the policy for about seven years and forty days. Now, should the policy-holder die within that period his heirs will receive the full $100,000 (no foreborne premiums being charged), and having had his loan of $24,500 cancelled, that policy-holder has cost the company $124,500 on a $100,000 contract, which result should never be made possible. In the case of endowment policies the loans may be much larger and the results much more disastrous than the example given.

Another plan which possesses some advantages because the insured when he borrows on his policy many times considers himself insured for the face of the policy, less the amount of the loan, would be to grant extended insurance for that net amount for such time as the balance of the cash value after paying off the liens will purchase, and in this way the company can never be called upon to pay under the contract more in all than the face of the policy. Thus, if A has a $10,000 policy, and has borrowed upon it $3,000, the amount to be extended would be $7,000, and not $10,000. This might avoid complaints among laymen who are unacquainted with the fact that it requires considerably more insurance to produce value enough to cancel the loan than is represented by the face of the loan.

These propositions may be represented by the following formulæ:

Let S = Sum insured,
S' = Amount to which policy is to be reduced,
L = Liens on policy,
g = % of Reserve for Cash Value.

Then to find the new amount of the policy to be extended free of all liens for the time (m) for which the original policy would have been extended had no liens existed:

$$Sg\,{}_nV_x : Sg\,{}_nV_x - L :: S : S'$$
$$\text{and } S'\,(Sg\,{}_nV_x) = S\,(Sg\,{}_nV_x - L)$$
$$\text{and } S' = \frac{S\,(Sg\,{}_nV_x - L)}{Sg\,{}_nV_x}$$

dividing by S, we have

$$S' = \frac{Sg\,{}_nV_x - L}{g\,{}_nV_x}$$

But if it is desired to extend some other amount of insurance for such time as the cash value of the policy, after deducting the liens, will purchase, then

Let $\Pi'_{\frac{1}{x+n}\,\overline{t}|}$ represent the new term single premium to extend the new amount for the unknown term, t years.

$$\text{Then } Sg\,{}_nV_x : Sg\,{}_nV_x - L :: S\,\Pi_{\frac{1}{x+n}\,\overline{m}|} : \Pi'_{\frac{1}{x+n}\,\overline{t}|}$$
$$\text{and } \Pi'_{\frac{1}{x+n}\,\overline{t}|}\,(Sg\,{}_nV_x) = S\,\Pi_{\frac{1}{x+n}\,\overline{m}|}\,(Sg\,{}_nV_x - L)$$
$$\text{and } \Pi'_{\frac{1}{x+n}\,\overline{t}|} = \frac{S\,\Pi_{\frac{1}{x+n}\,\overline{m}|}\,(Sg\,{}_nV_x - L)}{Sg\,{}_nV_x}$$

dividing by S this becomes

$$\Pi'_{\frac{1}{x+n}\,\overline{t}|} = \frac{\Pi_{\frac{1}{x+n}\,\overline{m}|}\,(Sg\,{}_nV_x - L)}{g\,{}_nV_x}$$

and if the amount to be extended is to be $S - L$, or S'', (which in effect is extending the full amount of the policy subject to the liens existing at lapse)

$$\text{then } \Pi'_{\frac{1}{x+n}\,\overline{t}|} = \frac{\Pi_{\frac{1}{x+n}\,\overline{m}|}\,(Sg\,{}_nV_x - L)}{g\,{}_nV_x} \text{ is to be divided by } S''$$
$$\text{and becomes } \Pi_{\frac{1}{x+n}\,\overline{t}|} = \frac{\Pi_{\frac{1}{x+n}\,\overline{m}|}\,(Sg\,{}_nV_x - L)}{S''\,(g\,{}_nV_x)}$$

giving the new term single premium for an insurance of 1 for the term for which the insurance of the amount $S - L$ is to be extended.

Notes on the Rate of Interest for Premiums and Reserves of Life Companies.*

WILLIAM MCCABE.

IN many of the United States, the premiums and reserves on contracts issued by life insurance companies, are based upon the Combined Experience Table of Mortality and four per cent. interest, and in several others, upon the American Experience Table and either 4 or 4½ per cent. interest.

About seventeen years ago, one of the oldest United States life insurance companies based its reserves upon the latter Table and three per cent. interest, and within the last few years several of the oldest and most successful companies have made up their premiums and reserves upon the same Table and rate of interest. In Canada, the table prescribed is the H^M Table and 4½ per cent. interest, but the Government is considering a change of this rate to probably 3½ per cent.

In an article, in the *English Bankers' Magazine* for November, 1898, it is stated on page 657 that during the first five years (1871 to 1875) for which returns were made to Parliament by the British life companies, that "not one office in those days (1871 to 1875) assumed less than three per cent. as a valuation basis, whereas seven existing offices now do so, a majority of these assuming 2½ per cent." (page 667), and that "in the not remote future it seems probable that three per cent. will be the maximum rate which a well-managed company will be justified in assuming it will earn" (page 668).

Among other reasons that may be given for that opinion the following are submitted:

(1) When the present standards were introduced there was a margin of from 1 and 1½ to 2 per cent. over the prescribed rate, upon the earning power of such investments as life companies were authorized or required to hold.

(2) An examination of the official reports of both the American and British companies will show that there has always been a con-

* Substantially as sent the Superintendent of Insurance of Canada in December, 1898, in answer to a request from him respecting the lowering of the present rate of 4½ per cent. interest, with the suggestion that any change made should not apply to existing business until after the lapse of five years, following the precedent laid down in Sec. 23, 40 Vic., Cap. 42, Canada.

siderable excess (practically the same as that which existed on this continent) in the rate which the British companies, as a rule, earned on their investments over the rate of interest upon which their premiums and reserves were calculated.

(3) This margin is probably important for the ultimate success, or even safety of the companies, to meet existing and future contracts (extending, as many of them do, over half a century or more), and such margin has been steadily declining, not only in Britain, but on this continent also. This decline has been very marked for the last few years, and especially so during the last year, in which the rate at which municipal and railway securities could be obtained was such as to yield only from three to four per cent. Several companies have also made during the same period large loans on real estate at a rate considerably under that upon which their premiums are calculated.

(4) The establishment of the conservative rate of three per cent. should have the effect of leading to economy in expenses—a result highly desirable.

(5) The opinions of the most experienced financiers and business men confirm the belief that the reduced earning power of money is permanent.

(6) It seems quite probable that several State Legislatures will likely do as the Parliament of Canada is about to do, reduce very materially the rate of interest at present prescribed by their respective insurance laws. Whatever rate may be then adopted, it is essential that it be a *conservative rate*, sufficiently under the earning power of the companies (restricted as the investing powers of many of them are) to secure the payment of their contracts, having regard to the fact that many of them may not fall in for a long term of years.

A reasonable excess of interest earnings over the rate fixed for reserves secures not only the payment of their contracts, but also provides for losses on investments, possible even in the most carefully managed companies, and also forms a part of the savings of such companies, returnable at intervals which return, under existing conditions, is probably necessary to satisfy policy-holders.

It should be borne in mind also, that although the present investments of many of the companies may be earning somewhat over the rate prescribed by the present laws for reserves, yet the margin of excess is very slight, and such margin has been rapidly disappearing. As investments mature, the moneys will then

have to be reinvested at much lower rates than those which are now being received.

(7) Possibly the large accumulation of capital at all the leading money centres of the commercial world available for investments may be deemed by some to be a temporary condition ; but the great increase in the amount of gold being produced in the gold-yielding parts of the world is an important factor tending to the reduction of the earning power of money and also the rate of interest on the class of securities the companies desire to hold.

(8) The increase in the facilities for transferring money from one financial centre to another is such that within a few hours vast amounts can be transferred to any point at which money is in demand, and this thereby tends to equalize the current rates obtainable over the whole business world, with the results (a) that the same amount of capital is much more effective than it was some years ago; (b) this consequent addition of available capital increases the competition for investments and reduces the rate of interest.

(9) The experience of both the governments of Canada and the United States illustrates the decline in the rate of interest, the obligations of each having on every recent offering been taken up at a lower rate than ever before.

Nearly all of the leading British companies have realized that this decline is permanent by basing their premiums and reserves upon three per cent. interest and the H^M Table. Some use the low rate of 2½ per cent., and others the severe test of the $H^{M(5)}$ Table and three per cent. interest.

(10) Correct mortality tables, and especially a safe rate of interest for the calculation of premiums and reserves, invested in sound and productive securities out of which to pay obligations as they mature, *are indispensable foundations* upon which sound life insurance must rest; lacking any, a company must sooner or later fail to meet the expectations of all interested.

A Notable Annuity Experience:

Massachusetts Hospital Life Insurance Company.

WALTER C. WRIGHT.

I AM permitted by the kindness of Charles F. Choate, Actuary of the Massachusetts Hospital Life Insurance Company, to exhibit in the accompanying table the single life annuity experience of that company, as determined by me in 1897. This is a limited experience as regards the number of lives involved, which was only 641. But it may be seen that nearly 60 per cent. of those lives was under observation from entrance till death. The number of yearly entrances was pretty evenly distributed throughout the whole period from 1823 to 1897. The annuities were all purchased upon wholly unsolicited applications coming exclusively from persons belonging to or connected with well circumstanced families, resident in Boston or its vicinity. A number were given as rewards of merit to valued servants. There is probably as little doubt as to the ages at entrance of these annuitants as in the case of any collection of such experience ever tabulated, whence the statistical value of this evidence, as showing the possible measure of selection exercised by such applicants, is unsurpassed.

The standard tables which I have used for comparison are all tables of high vitality. Finlaison's female lives of 1829 is the table on which the premiums of the Massachusetts Hospital Company itself were, until recently, based. It shows on the whole $5,221.00 more decrement than under the company's actual experience, and for females alone $9,204.00, or 10.4 per cent., more. But perhaps the most striking way to show the longevity of the company's annuitants is to compare a few aggregates of exposure and decrement with those of a table familiar to all, like the Combined Experience. I give such a comparison in the following summary:

Table.	Ages.	Exposure.	Decrement.	Rates of Decrement.	Ratio of Rates.
Mass. Hos..	65–74	$534,913	$18,008	.033665	1.83
Com. Ex....	65–74	368,256	22,654	.061517	
Mass. Hos..	75–84	488,508	40,836	.083593	1.52
Com. Ex....	75–84	147,296	18,683	.126840	
Mass. Hos..	85–94	122,132	22,713	.185971	1.38
Com. Ex....	85–94	20,776	5,328	.256450	

At ages 65–74 there were more numerous entrances than at the higher groups of ages, and the effect of selection by the applicant may be plainly seen in the falling ratios of percentages, at the same time that the vitality greatly exceeds the Combined Experience to the last.

Massachusetts Hospital Life Insurance Company.

Single Life Annuity Experience. Males and Females. From the beginning of Business in 1823, to December, 1897.

Male Annuitants, 142, holding 182 Annuities. Outstanding at close of Observation, 46, holding 58 Annuities.

Female Annuitants, 499, holding 647 Annuities. Outstanding at close of Observation, 214, holding 320 Annuities.

Ages of Exposure.	Male Exposure.		Male Decrements.		Female Exposure.		Female Decrements.		Expected Decrements according to Standard Tables. Amounts only.					
									Finlaison's Females, 1829.		Dr. Farr's English Life, Healthy Districts.		Weeks' Joint Experience, American and Foreign.	
	Number	Amount.	Number	Amount.	Number	Amount.	Number	Amount.	Males.	Females.	Males.	Females.	Males.	Females.
23	1	$1,000	....	....	....	.. .	....	....	$9	....	$7	....	$10	....
24	1	1,000	....	....	....	....	....	....	9	....	8	....	10	....
25	1	1,000	....	....	....	....	....	....	9	....	8	....	10	....
26	1	1,000	....	....	....	....	....	....	9	....	8	....	10	....
27	1	1,000	....	....	1	$150	....	....	9	$1	8	$1	10	$1
28	1	1,000	....	.. .	1	150	....	....	9	1	8	1	10	1
29	1	1,000	....	....	1	150	....	....	9	1	8	1	10	1
30	....	....	....	....	1	150	....	....	....	1	....	1	....	1
31	....	....	....	....	1	150	. .	....	. .	1	....	1	....	1
32	....	....	....	....	2	250	....	....	.. .	2	....	2	....	1
33	....	...	....	...	2	350	....	....	....	3	....	3	...	2
34	2	280	....	....	1	200	....	....	3	2	2	2	3	1
35	1	30	....	....	2	300	....	....	....	3	....	3	....	1
36	2	130	....	....	2	300	....	...	1	3	1	3	1	2
37	2	130	....	....	4	800	....	....	1	9	1	8	1	4
38	3	160	....	....	5	900	....	....	2	10	1	9	2	4
39	2	60	....	....	6	1,000	....	....	1	11	1	10	1	5

Ages of Exposure.	Male Exposure.		Male Decrements.		Female Exposure.		Female Decrements.		Expected Decrements according to Standard Tables. Amounts only.					
									Finlaison's Females, 1829.		Dr. Farr's English Life, Healthy Districts.		Weeks' Joint Experience, American and Foreign.	
	Number	Amount.	Number	Amount.	Number	Amount.	Number	Amount.	Males.	Females	Males.	Females.	Males.	Females.
40	2	$60	....	....	6	$1,000	....	....	$1	$11	$1	$10	$1	$5
41	3	247	....	....	6	1,100	....	....	3	12	2	11	3	7
42	2	60	....	....	6	1,100	....	....	1	13	1	11	1	7
43	3	160	....	....	8	2,330	....	....	2	27	2	25	2	14
44	2	130	....	....	9	2,930	....	....	2	34	1	31	2	18
45	3	330	....	....	10	4,030	....	....	4	47	4	44	4	25
46	2	130	....	....	14	6,850	....	....	2	80	1	76	2	45
47	2	130	....	....	15	7,150	....	....	2	84	1	81	2	49
48	2	250	1	$100	18	8,450	....	....	3	101	3	98	4	60
49	1	150	....	....	19	7,375	....	....	2	89	2	88	2	55
50	1	150	....	....	27	8,869	....	....	2	106	2	108	2	69
51	2	400	....	....	28	8,894	2	$443	5	108	5	112	7	73
52	2	634	....	....	40	13,771	....	....	8	172	9	177	11	120
53	4	2,034	....	....	44	16,821	1	100	26	217	30	223	38	157
54	6	2,634	....	....	47	17,012	....	....	35	229	41	233	51	170
55	7	3,834	....	....	51	17,651	....	....	55	252	63	249	78	190
56	8	4,434	....	....	55	16,661	....	....	67	252	77	243	96	194
57	12	8,934	....	....	64	18,760	2	600	143	300	164	303	203	237
58	12	8,934	....	....	65	18,754	1	50	152	318	173	341	215	257
59	12	8,934	1	150	67	20,113	1	200	160	361	191	408	228	301
60	21	12,934	....	....	86	24,339	....	....	240	452	306	542	351	397
61	21	12,504	....	....	88	25,969	2	600	246	511	326	634	361	463
62	21	12,404	....	....	89	26,158	....	....	262	552	353	694	382	511
63	24	12,854	1	3,000	105	30,249	1	400	292	688	399	872	436	647

Ages of Exposure.	Male Exposure.		Male Decrements.		Female Exposure.		Female Decrements.		Expected Decrements according to Standard Tables. Amounts only.					
									Finlaison's Females, 1829.		Dr. Farr's English Life, Healthy Districts.		Weeks' Joint Experience American and Foreign.	
	Number	Amount.	Number	Amount.	Number	Amount.	Number	Amount.	Males.	Females.	Males.	Females.	Males.	Females.
64	26	$9,974	2	$500	113	$35,802	3	$2,300	$246	$884	$338	$1,119	$351	$838
65	27	13,382	1	50	129	39,305	3	800	357	1,049	493	1,331	505	1,007
66	30	15,889	1	100	137	41,911	5	1,523	463	1,222	636	1,537	644	1,176
67	35	17,761	3	2,000	144	47,060	2	1,300	561	1,487	771	1,869	773	1,446
68	42	20,236	....	....	158	51,950	1	800	688	1,766	955	2,240	948	1,749
69	46	19,996	1	800	174	54,955	3	445	740	2,033	1,025	2,572	1,010	2,027
70	49	20,367	1	750	184	56,789	7	3,939	844	2,353	1,134	2,887	1,102	2,294
71	53	21,717	4	1,350	189	58,203	3	300	984	2,637	1,312	3,214	1,280	2,576
72	50	20,332	2	332	203	61,784	4	2,713	1,007	3,061	1,335	3,708	1,296	2,996
73	55	23,585	6	3,350	207	62,845	12	3,473	1,292	3,441	1,681	4,100	1,627	3,340
74	54	22,885	3	2,100	208	60,112	9	2,715	1,373	3,607	1,772	4,266	1,711	3,500
75	53	23,842	6	3,400	207	60,250	10	3,561	1,523	3,848	2,002	4,651	1,934	3,845
76	50	21,230	1	160	187	57,081	9	1,455	1,507	4,050	1,933	4,790	1,870	3,990
77	50	21,490	7	1,830	195	58,236	12	5,519	1,705	4,620	2,119	5,307	2,057	4,462
78	47	20,130	4	1,920	187	55,623	9	2,330	1,763	4,872	2,152	5,506	2,095	4,672
79	44	15,782	5	2,650	182	54,383	17	5,898	1,509	5,202	1,823	5,837	1,787	5,006
80	42	14,859	6	1,590	170	49,880	17	5,179	1,549	5,200	1,856	5,798	1,832	5,033
81	38	10,819	5	2,550	150	45,356	24	6,063	1,179	4,940	1,457	5,707	1,453	5,016
82	37	12,169	8	3,320	131	39,626	15	3,970	1,357	4,418	1,767	5,387	1,781	4,804
83	32	9,929	4	1,000	118	35,787	14	2,591	1,130	4,073	1,553	5,257	1,586	4,757
84	24	8,279	1	50	103	32,286	11	4,270	963	3,754	1,390	5,106	1,441	4,705
85	26	9,379	4	873	91	27,563	15	3,834	1,140	3,351	1,690	4,696	1,783	4,401
86	26	9,129	4	1,002	74	22,140	16	4,101	1,188	2,881	1,765	4,050	1,895	3,877

Ages of Exposure.	Male Exposure.		Male Decrements.		Female Exposure.		Female Decrements.		Expected Decrements according to Standard Tables. Amounts only.					
									Finlaison's Females, 1829.		Dr. Farr's English Life, Healthy Districts.		Weeks' Joint Experience, American and Foreign.	
	Number	Amount.	Number	Amount.	Number	Amount.	Number	Amount.	Males.	Females.	Males.	Females.	Males.	Females.
87	18	$6,799	3	$2,134	61	$20,379	12	$6,697	$965	$2,893	$1,404	$4,004	$1,541	$3,910
88	18	7,525	3	460	49	13,543	11	1,873	1,208	2,175	1,662	2,844	1,864	2,849
89	15	7,365	2	2,025	37	10,410	8	1,810	1,369	1,935	1,736	2,337	1,994	2,401
90	11	4,760	....	...	28	8,095	3	428	1,048	1,782	1,193	1,945	1,409	2,047
91	11	4,760	....	....	25	6,901	6	1,130	1,254	1,818	1,264	1,758	1,541	1,913
92	11	4,760	2	700	18	5,707	2	740	1,465	1,757	1,342	1,553	1,685	1,734
93	9	4,060	3	1,750	14	4,422	3	1,400	1,414	1,540	1,223	1,272	1,573	1,473
94	5	1,710	1	160	10	2,972	2	700	668	1,160	538	914	725	1,086
95	4	1,550	1	100	8	2,272	2	1,082	657	963	519	730	719	910
96	3	1,450	...	....	6	1,190	1	200	716	588	506	411	736	523
97	3	1,450	....	....	5	890	2	600	826	507	535	318	806	428
98	3	1,450	1	1,000	3	290	....	....	942	188	575	109	883	153
99	2	450	2	450	3	290	....	....	337	217	183	116	301	168
100	....	....	....	....	3	290	1	50	...	290	112	118	109	184
101	....	....	....	....	2	240	1	200	....	....	70	105	32	167
102	....	....	....	....	1	40	1	40	....	....	85	40	8	40
Totals,	1,243	$532,305	100	$43,706	4,910	$1,498,044	286	$88,422	$39,723	$97,626	$48,124	$115,168	$51,247	$101,599

Note.—The number of Male and Female Decrements, as distinguished by ages, exceeds the difference between the total number of such Annuitants and the number living at the close of observation, because, in five cases, when more than one annuity was held by the same person, the dates of payment were such as to make the age of decrement as applicable to one annuity differ from the age as applicable to another.

The Actuary's Place in Science.

WALTER S. NICHOLS.

THE motive which four years ago prompted the organization of an International Congress of Actuaries was more than the mere prospective gain from the interchange of ideas. Nor do I believe that the men who ten years ago drafted the plan of our own Actuarial Society thought only of the benefits which would result from their social intercourse with each other. Societies like ours are object-lessons for the public. They are recognized fountains of authority. They place on their members that badge of distinction which membership in a learned organization always confers, and I regard the assertion before the world of the actuary's right to a place among members of the learned professions as one of the legitimate objects of this Society. Our decennial anniversary seems a fitting occasion to elaborate this thought.

Four hundred years ago Copernicus and Kepler and Galileo in quick succession struck death blows to the authority of Aristotle and Ptolemy which then dominated scientific thought. The chains which had bound the world to the philosophic systems of the schoolmen were loosed by those blows. It was gradually seen that, splendid as were the intellectual achievements of the ancients, the introspective and deductive methods of their philosophy, with its disputations about verbal distinctions, must always be barren of results in a search for objective truth. Experiment and induction must be the fertile instruments in any attempt to unlock the mysteries of the natural world.

It was a hard lesson to learn. Poor half-blind Kepler spent many a weary year going over those splendid observations of Tycho Brahe and striving through epicycles to fit the orbit of Mars and his companions to a perfect curve before he realized that the very foundations of scholastic astronomy were wrong, and by the mere force of persistent experiments lighted on the laws which control the motion of the planets. It was a hard lesson to learn even after Newton had extended gravity to the heavens and proved it the force which bound the universe together. The Vortices of Descartes continued for a generation to rival the theory of gravitation in the face of Newton's demonstrations.

But each new department of scientific research emphasized the fact. Chemistry, geology, medicine, even biology itself, including the science of the mind, yielded up their secrets only when placed in the crucible of experiment. To-day, in a thousand scattered places, observers of the winds and storms are doing just what Tycho did for astronomy at Uraniburgh four centuries ago, gathering the material for a coming generation to digest and create a science of meteorology.

When, however, the lesson had been finally learned the revulsion became complete. The leaders in scientific thought discarded all authority. Objective truth, regardless of its consequences, became the alleged goal of their inquiry. The work of the investigator was defined as the collation of facts and their systematic arrangement under general laws. Causality reduced itself to a sequence of events whose real connection must remain forever beyond the range of discovery. When all the phenomena which human ingenuity could reach had been disclosed and reduced to their lowest terms under these laws the work of science would be done. Their essence could be left for scholastic philosophers as an intellectual puzzle.

It was in this spirit that the structure of our modern science was reared. The facts, once established, were beyond debate. The laws which harmonized them, when shown to fit the facts, became the stones of a temple on which each successive workman built. But as time has gone on all departments of scientific inquiry have been seen to be correlated, and in their generalization to be leading up to a fundamental unity which has at last reduced the problem of the universe to one of pure physics, in which only the elements of matter and force are to be differentiated. Correlation of forces and conservation of energy to-day embody the finality of all physical laws. Such, in a crude way, is the outcome of modern scientific inquiry.

But its development has produced results that tend to weaken the stones of the temple and throw discredit or doubt on the whole system. Try as they might, the investigators have not been able to keep within their own rules. When Tyndall reached the border land of science he was constrained to invade the domain of speculation and line up with the philosophers of two thousand years before. When Huxley had demonstrated that man was a machine he unwittingly joined hands with Calvin and his school. Old Lucretius became again an authority for the moderns, with the revival

of the atom, and Helmholtz went as deep into speculation on the essence as any peripatetic Greek when he worked out the mathematical laws of an etherial vortex. Yet these and all similar speculations were the inevitable outcome of the search for general principles which would coördinate and harmonize the profoundest discoveries of science. The trouble with it all has been that, under the operation of necessary mental laws, the results of speculation and theory have been unconsciously amalgamated with thoroughly demonstrated truths, and science has been discredited by combining in her own generalizations those very metaphysical conceptions against which her votaries have protested.

The same result has worked out along a different line. Laws which were found sufficient to explain observed facts, especially if they responded to that touchstone of truth, the ability to predict new ones, were treated as themselves fundamental truths until some new fact was discovered which compelled their abandonment or modification.

Laplace expanded the suggestion of Kant, and built up a nebular hypothesis which until recent years has been accepted as the demonstrated genesis of world systems, but lo, the stability of Saturn's rings, the apparently strongest evidence of its truth has forced its modification, and within the last few years the problem of the tides has led up to a method of world genesis in some cases the very reverse of that hypothesis.

That same great geometer investigated the stability of the solar system and when he had shown by the convincing logic of his mathematical analysis that the perturbations of the planets were self-adjusting, the whole world accepted his conclusion that so long as gravitation lasted, the sun would continue to swing his satellites in their courses. But, that same problem of the tides has come in to upset the conclusion and show these planetary bodies working out their own destruction. And yet again, the ink is hardly dry which chronicles this latest revelation before we are confronted with liquid air, which suggests the query whether the very flexibility on which the tidal action depends may not in time disappear with the exhaustion of solar heat and relegate us back to the position of Laplace. The nebulous rings of that philosopher are to-day replaced by the picture of rotating spheres torn apart and driven asunder by a tidal force.

Turning to geology, within the memory of our older members, the Plutonic doctrines of heat and cataclysms have been pushed

aside in favor of sedimentation and existing forces calling for unlimited time for their operation, and these in turn have been recast to meet the mechanical theory of solar heat, with its time limitations. The chemist's explanation of the sun's fires is replaced by that of the physicist. The slowly disintegrating power of the rainfall is partially replaced by those same tidal disturbances, which at an earlier day must have swept the continents with their tremendous waves.

Turning to chemistry, the doctrines of combination and substitution must be remodeled to tally with the advance of organic chemistry. The kinetic theory of gases and heat and electricity have reduced the chemist to an explorer in a single department of physics. The atom, with its uniformity, its impenetrability, and its infinite elasticity, and the ether, with its continuity and its stresses and strains, become the final summation of the chemist and physicist alike, regardless of the fact that their adoption has simply removed old by raising new difficulties.

Turning to biology in its widest sense, we have seen the accepted principles of Cuvier and Agassiz replaced by those of Lamarck and Darwin and Huxley. We have heard the scientific world asserting that though life may be more than mere matter and force, the conservation of energy is as true of man as of the falling stone. Yet if this be so, every impulse to thought and action must itself be mechanical, for matter can only be moved by the expenditure of energy, or else life must be a creator of force, and the whole doctrine of the conservation of energy falls to the ground. There seems no escape from the dilemma.

Trace the recent history of medicine as a science, where in all the callings of life will be found a more constant replacement of old doctrines and methods with those that are new and antagonistic? Disease is now the ravages of a germ, and again the germ becomes the product of the disorder. The pharmacopia of fifty years ago is like an old almanac. The pathologist and etiologist have revolutionized their ideas, and warring schools of practice are continually shifting their own grounds while contending with each other.

The naturalist and the botanist have abandoned their distinctions of orders and genera and species, except as convenient groupings for tracing out the development of a principle of life, which, whether called plant or animal, has its starting point in the germ and the protoplasm.

The political economists, starting from Malthus and Adam Smith,

and ending with the champions of Modern Communism and Socialism, have created as many antagonistic theories of the laws of wealth and its distribution as there were original thinkers engaged in the study.

Such are the conflicts of science which have served to cast doubt on its assertions, and impair confidence in its conclusions. The explanation is a familiar one. We are told that the scientist distinguishes his established truths from the speculative assumptions which he tentatively uses to aid his investigations, and which he is prepared to abandon when their usefulness is gone, that neither atom nor ether are more than provisional aids in the advancement of chemistry and physics, that the symbolism of the chemist and the dynamic formulas of heat and light and electricity are mere algebraic symbols which correlate the facts, but whose real significance no scientist claims to understand, that the classification of the naturalist and the geologist and the rest are mere scaffolding to be changed or abandoned when the facts have been finally adjusted.

In theory this may be true, but as a practical fact, the average investigator, however good his intentions, does no such thing. The very laws of the human mind are in the way. Our concepts are verbal. We think in the imagery of words, and the language of every age is impregnated with the metaphysical as well as the physical ideas which are dominant. Unconsciously the investigator is constrained to read into his facts the ideas with which his language has been associated. He talked of centrifugal action as a force because he thought of it as a force, until closer analysis showed his error. He spoke of heat as caloric because he thought of it as a fluid until Davy and Joule and Fourier showed the contrary, and pointed out the way to the correlation of forces. He says to-day, as did Newton four hundred years ago, that action at a distance is unthinkable, unmindful, that the mere annihilation of distance does not remove his difficulties. He thinks of space as infinite and homologous until constrained by his physical theories to construct a geometry of four dimensions, and speculate on the relativity of motion and a reëntrant space.

The whole indictment may be summed up in the statement that the text-books of the past are dangerous guides, not because of new truths omitted, but because of supposed errors which they taught as truths, and our standard authorities to-day may, fifty years hence, be in the same category.

Now, where is the corrective for this weakness of science to be sought? The students of metaphysics and mental philosophy will tell us that it lies in their own discarded studies. We must understand the character and processes of the thinking mind before we can rightly comprehend its products. But I believe the corrective is of a different sort.

Carefully guarded at the great national capitols are kept standards of length and weight and volume, on whose accuracy every branch of physical research depends. So exhaustively have these been computed and their relations established, that an observer on a distant satellite, without a yardstick or pound or chronometer to aid him, could restore them all by a mere knowledge of their principles. When electricity became an important factor in the business community, the progress of the electrician was barred until his ohm and farrad and volt had been established and coördinated with the world's ordinary standards. A fixed standard is the first essential to accurate work and progress in every branch of physical research.

And here lies the weakness of science as a whole, it has never recognized an accepted standard by which to measure truth. All knowledge is relative, not only in the sense of Hamilton, but in the sense that in a last analysis, it reduces itself to a mere belief which, starting from the Cartesian doctrine of *conscio, ergo sum* as the most certain of all truth, ramifies into an endless network of probabilities more or less reliable. There is a risk of error in every judgment that I may form or belief that I may hold. Absolute certainty is an idea which the philosopher may dream of, but which the actual man can never reach.

The text-books of science are clouded by the mingling of alleged facts and principles, regardless of the measure of probability which sustains them. The unsuspecting student can never be sure when he is treading on dangerous and uncertain ground, because their authors have not been trained to discriminate between the relative weight of their assertions. This is the field where the actuary's studies have their practical application. The actuary holds the key to the standard by which all truth in its last analysis is measured, and his place in science is that of the conservator of this universal standard of truth.

To me it has always seemed strange that in the preparatory training of our universities so little thought has been given to the place which is filled by the doctrine of chances in all the affairs of

life, and it is so universally ignored in the mathematical curriculum. The trader in his speculations, the lawyer in his pleadings, and the clergyman in his homily, equally with the devotee of science, are dealing with problems whose correct solution is determined by the principles which belong to the actuary's pursuit.

I believe that for the purpose of science truth should be dealt with as a mathematical probability whose relative weight must determine the rank which every dictum and hypothesis should occupy and the measure of confidence it should enjoy. A science built up on such lines would rarely need to have its foundations disturbed. The demonstrated fallacy of some assumed hypothesis would not, as now, cast doubt on the structure into which it has been incorporated, and well-established principles would not suffer discredit because some accepted theory must be abandoned. The dogmatism of the scientist would no longer be hurled at him as a reproach if every recognized authority in his own department dealt with his facts as does the actuary with his observations.

That doctrine of chances which lies at the basis of our pursuit furnishes the only accurate rule by which error can be estimated. The law of error formulated by Gauss is the universal solvent for reducing physical observations to the category of truths. It is constructed from our doctrine of chances. I am not claiming that concepts and judgments must be brought to the touchstone of mathematics, nor that evidence should be sifted by an algebraic equation, but that the mind which weighs evidence and opinions and theories from the cold standpoint of their inherent probability, apart from all personal bias, and deals with the facts as does the actuary with his observations, is the truly scientific mind, and that the weakness of science lies in the extent to which it has ignored the principles of the doctrine of chances. Michael Faraday, in the fields of chemistry and electricity, was one of the most fertile explorers which England has produced. He could not even think in the language of the mathematician, but his mental make-up was mathematical to a high degree. It was because he treated every suspected law or principle as a mere hypothesis to be tested by facts, and dismissed or accepted according to its value as a probability, that his work was crowned with such brilliant success. It was the spirit of Newton abandoning his hypothesis of gravitation for years until Picard's corrected measurement of the earth eliminated the small discrepancy of an eighth from his original calculations, that was the secret of his God-like predictions.

Gentlemen of the Actuarial Society, when all the observations of science have been gathered, and every stone of her temple has been adjusted to its place, it must remain for minds that are disciplined in the logic of our profession to view the finished work and say how far the builders have realized their own boast that truth is the goal which they have won.

Within the past year there has been an awakening among our universities to the claims of insurance to a place in their curriculum. A distinguished member of this Society has been delivering courses of lectures on this topic before different bodies of students. May they prove the entering wedge to a fuller realization of the place which the actuary's studies fill in the field of mental training and physical research. The logic of the actuary, rather than of Aristotle and the schoolmen, should dominate the twentieth century.

What is the Proper Loading on Life Insurance Premiums?

JOEL G. VAN CISE.

WE must all agree that this is a vital question in the business of life insurance, even after proper care shall have been taken in the selection of a table of mortality and rate of interest to be used in the computation of net premiums. Commissions to agents and other necessary expenses must be provided for, and a margin should be left for unforeseen contingencies, such as an excessive rate of mortality or a sudden depreciation in the value of assets. This statement may seem almost like the affirmation of an axiom, or the repetition of an old, old story; but as some old stories, like those of the Bible, never lose their freshness nor their interest, so some truths, almost self-evident, must ever be kept in mind in the business of life insurance, and, in fact, in every other business, if it is to be carried on with wisdom and success.

I will then speak briefly on the question which I have taken as the subject of this paper, "What is the Proper Loading on Life Insurance Premiums?" and in this connection will limit myself to a consideration of the proper percentage to be added to the net premiums on ordinary life policies, taking it for granted that if this percentage shall be wisely determined the question of the proper loading in the case of other forms of policy can be readily and satisfactorily settled.

Before mentioning any figures, I wish to say right here, that, in my judgment, two important points ought always to be in the minds of actuaries when called upon by managers, or when taking it upon themselves, to construct tables of premiums: 1st. The absolute safety and security of their company's contracts. 2d. The accumulation of a surplus fund out of which to pay dividends to persistent policy-holders if the company is conducted upon the mutual plan. The first point as to safety will not admit of dispute, but unless I mistake the signs of the times, the second point as to the accumulation of surplus is fast losing its importance in the eyes of some of the gentlemen who have framed and introduced some of the new forms of policy which have come before the American public during the past few years.

Let us look at this question in the light of history, modern and not ancient, for I will only go back about thirty years. On February 1st, 1868, all the life insurance companies whose home offices were in New York City agreed upon a uniform table of premiums, based upon what is known as the American Experience Table of Mortality and four per cent. interest. The net ordinary life premiums were increased 40 per cent., thus making the loading equal to 28 4-7 per cent. of the gross premiums. While the companies in other States did not, as a rule, adopt this table, the rates of all who were transacting their business on the mutual plan were substantially the same, at least on life policies, as the rates of the New York companies.

Neither presidents nor actuaries then thought that too much was being charged, even when some companies began to defer not only the payment of the dividends but also the giving of surrender values, until the end of periods of ten, fifteen and twenty years. And yet what were the conditions thirty years ago as compared with those which exist in 1899? Then seven per cent. interest could be easily obtained in New York City on the same class of securities that now pay only four per cent., thus giving a profit margin of three per cent. as compared with only one per cent. now, even on the basis of a three per cent. reserve. The cost of new business in 1868 was only about one-half of what it is to-day, and the average ratio of expenses to premiums received is now also considerably increased by the lessening of the average time during which policies are kept in force by the payment of premiums. Extra care in the examination and selection of risks may have tended to decrease the death rate, but this has been offset by the more liberal provisions as to residence, occupation and incontestability, and the selection against a company owing to the withdrawal of healthy lives, who only take out policies because of extraordinary inducements on the part of agents, and who give them up largely because of similar attractions being held out by the representatives of rival institutions. Thirty years ago, in New York companies, the amount of the reserve on discontinued policies, less the amount paid in surrender values, was so large as to be sufficient in many cases to pay the greater part of the necessary expenses of conducting the business, while to-day, in the competition as to which company shall offer the most liberal terms to the man who stops paying premiums, the theory has been openly advocated that such a man ought to be allowed to withdraw without loss.

In the light of this simple statement of facts, which shows that the profits from nearly all sources are much less now than they were in 1868, has not the time arrived when the actuaries of America should stand like a stone wall against the reduction of rates of loading below what common prudence would seem to require? If 28 4-7 per cent. was not thought too much thirty years ago, should we not insist that at least 25 per cent. of the gross ordinary life premiums charged to-day should consist of loading, without going into the question of what rate of interest should be used in the calculation of net premiums and reserves? I, of course, refer to policies on which profits are promised, whether these profits are to be paid annually or at the end of a term of years; but even on policies without profits, should we not insist that at least 10 per cent. of the gross premiums should be loading? It is not likely that this low rate would cover expenses, but probably the deficiency could be made up from other sources.

In discussing this question of premium rates, we should never lose sight of the moral obligation under which we, as actuaries rest, not only to the trustees of the institutions by whom we are employed, but also to the policy-holders who are entrusting their money, hard earned in many cases, to the managers of these companies, as a provision for their families after their death. When I think of the competition now going on as to which company shall offer the largest surrender values, or the most liberal provisions as to residence and occupation, for the lowest premium, I am reminded of the old story of the gentleman who, in selecting a coachman, showed to each applicant for the position a precipice, and asked him how near he could drive without tumbling over it. It was wonderful how near the edge some thought they could drive without a smash, but at last another man came along who said he would keep out of danger by not driving near the edge of the precipice at all, and this was the coachman whom the wise employer selected. This is a homely story, but does it not point a moral, which it would be well for the presidents of American life insurance companies to heed to-day? Whatever others may do, let us as actuaries, who are naturally supposed to be careful and conservative, do all we can in the way of raising danger signals and putting on brakes, in the hope that in the race for new business, and amount of insurance in force, which is now going on, any possible catastrophe may be averted.

The Value of Actuarial Testimony in the Courts.

SYDNEY N. OGDEN.

I HAVE endeavored to put into readable shape a few thoughts upon a subject which is best expressed by the above heading, viz., "The Value of Actuarial Testimony in the Courts."

As the living and the dead are so closely identified with each other in all computations which the actuary is called upon to make, it is no more surprising that he should sometimes be called upon to fix a pecuniary value on a life which has suddenly ceased to exist, than it is for him to compute the value of the same life, assuming that it has an even chance of living out the natural lifetime.

It is not my intention to enter into a discussion of Chancery Court cases, where the "widow's dower," "life estates," etc., are at stake, for there is hardly a civilized country in the world where our mortality tables are not recognized as a valuable aid in determining these cases of long-drawn-out litigation. My purpose is rather to try and show in the suggestions which this paper may contain, that the testimony in courts of an expert actuary may oftentimes prove of great assistance to a jury in determining the amount of damages to be awarded to a plaintiff.

The English courts have for a long time past recognized the value of actuarial science in determining approximately the value of a life to the heirs of the deceased, but it is not until recently that the United States courts have permitted actuarial testimony to be publicly introduced in order to serve as a guide for the jury in fixing the amount of damages sustained.

In an opinion delivered in June, 1898, by Justice Collins, of the Supreme Court of New Jersey, in the case of the Camden and Atlantic Railroad vs. Margaret Williams, administratrix of John Williams, the learned Judge very ably discusses the "Carlisle Table of Mortality." He admits that these mortality tables are derived from statistics preserved through a number of years, and have become standard by the test of subsequent experience. He says that the courts have almost universally availed themselves of help from these tables when the expectancy of life became involved in a judicial proceeding, such as fixing the value of a life estate in

lands and other chancery cases. He declares that the "right to use such standard tables in evidence has never been expressly declared in this court, but the common practice of using them is recognized."

The opinion goes on to state that "the tables do not afford absolute rules for computation, but, they may help to form the judgment. Each case stands by itself, and has its own conditions, and the tables must be intelligently applied to that case." The Justice then declares that "it is proper, therefore, to admit this standard table in evidence without proof of its repute; that may be assumed." "It was legal evidence irrespective of the condition of the health of the deceased, for it is not a table compiled from statistics of selected lives only, but, of course, such condition had to be taken into account."

I shall endeavor to confine myself to cases in court where action is brought against the defendant to recover damages for the sudden taking of life by accident or otherwise.

The two all-important questions for the jury to decide are, first, whether the defendant was responsible for the death, and, second, if he was responsible, for what amount was the plaintiff damaged? It is in determining this latter point that the testimony of the actuary is called into play.

In some of the statutes creating a right of action for wrongful death, the recovery is confined to a sum not in excess of a fixed maximum limit. In Illinois, for instance, the limit is fixed at $5,000, while in Indiana, the Revised Statutes of 1881 fixed the maximum limit at $10,000.

In Missouri, the Revised Statutes of 1879 provide that a *fixed* penalty of $5,000 shall be recovered where the death results from certain causes, such as from injuries occasioned through the negligence of an officer or employee of a railroad or steamship company in operating a train or steamboat, etc.; and in Connecticut we find that the statute of 1853 fixed the *minimum* limit of recovery in an action against a person wrongfully causing the death of another at $1,000, and the *maximum* limit at $5,000.

It is difficult to understand by what process of reasoning our worthy legislators arrived at conclusions that enabled them to fix the amounts quoted above. Certain it is that the wisdom and propriety of such limitations have always been a subject of grave question, and they have been abrogated in many of the States.

It is true that even to-day there are a few of the Judges of our

Supreme Courts who decline to admit the testimony of the actuary, on the ground that life is too uncertain to be determined by even an actuary with any degree of mathematical precision.

Many will argue that the comparative value of any two lives suddenly terminated by an accident should be the same, even where the one was a millionaire and the other in moderate circumstances. The argument would undoubtedly hold true were we to compute the value of the two lives from a sentimental standpoint.

No one would dare to gainsay the fact that a poor man's life is fully as valuable to his family as is that of the millionaire.

If the jury were to be governed in their computation of the amount of damages sustained by the bereaved wife or children in proportion to what they believed the love for the deceased or the affection of the relatives had been injured, then indeed would we have such a display of grief in our courts as would try the very soul of every man on the jury, and make him absolutely incapable of rendering any verdict except that which the heart would dictate. But at this point the law steps in and says that all sentiment must be thrown aside, and nothing but the cold facts considered in the deliberations of the jury-room. First of all, is the defendant responsible for the death of the deceased, and, if so, how shall we arrive at the amount that will compensate the plaintiff for the loss of the life? The prudent lawyer will often foresee a verdict against himself, and it is then he calls in the actuary to help him out of his trouble. If the deceased were a poor man, and a small verdict expected, counsel for plaintiff will depend upon the actuary to help raise the award. If the deceased happens to be a rich man, and a large verdict is expected, then will the counsel for the defense turn to the actuary to help lower the award, showing an unconscious recognition of the law of average, from both sides. Both lawyers believe that the value of life calculated from a scientific basis, taking into consideration the mortality tables and the value of annuities, will bring about a fairer verdict than if the question is left entirely to the haphazard judgment of the twelve men who are to decide the case at issue.

The process of reasoning by which the counsel will arrive at his conclusions are, first, by proving the age of the deceased at time of death and the income he was earning at that time. After deducting a reasonable portion of said income, which would represent the actual living expense of the deceased, the remainder can fairly be said to represent the amount which the heirs

would annually lose during the remainder of his natural life-time. Of course, there are many questions that may be raised in making an equitable division of this income. It is difficult to say just how much a man does spend on himself and how much he gives to his family. He may have no immediate family, and he will naturally spend more on himself. He may have a large family, and every dollar earned will be required for their support and that of himself. His ambition, surroundings, future prospects and condition in life must all be carefully considered before we are in a position to consider the present value of an annuity for such a portion of the income as the heirs might reasonably expect to receive.

We will assume that the deceased was fifty years of age when he came to his untimely death; he earned annually, say, $2,000, out of which he had to support himself and family. Let us suppose that 10 per cent. of this income ($200) was put aside each year for unforeseen expenses, 40 per cent. ($800) was expended by the family for *actual* living expenses, and 50 per cent. ($1,000) was spent by the deceased himself for necessary *personal* expenses and such luxuries as his social position would entail upon him. Assuming that he will earn his maximum income at the age of fifty, and continue to earn the same amount during the balance of his natural life, we would then only have to determine the present value of a life annuity of $1,000 on a party now aged fifty to arrive at a fair estimate of the pecuniary value of the life to the family, which, according to the Carlisle Tables of Mortality and five per cent. interest, would be represented by $11,660.00.

While this amount would represent only an approximate value for the case under discussion, it would serve as a fair basis for determining the actual amount that should be awarded after giving due consideration to the many influences and arguments that may be brought forward by counsel.

A recent decision of one of our Judges of the Supreme Court of New Jersey set aside a verdict of $5,000 damages rendered in the case of a child who, I believe, had been killed in a railroad accident. His decision was based upon the assumption that there could be no pecuniary value to a child's life, for the reason that where the child had not reached the age where its money-earning capacity had been developed, it must necessarily be a source of expense rather than profit, and no claim for damages could be entertained.

From the general opinion of the press and the comments of the public at large, I am happy to say that the opinion of the learned Judge has been pretty generally condemned. The case has been retried, and a verdict for the same amount, $5,000, was again awarded.

The Statutes of New Jersey provide "That the father and grandfather, mother and grandmother and children and grandchildren, severally and respectively (of every poor, old, blind, lame and impotent person, or other poor person not able to work), being of sufficient ability, shall at his, her or their charges and expense, relieve and maintain every such poor person as aforesaid, in such manner as the court of quarter sessions shall order and direct," and a "penalty" is then provided in case any relative shall refuse to so maintain and relieve such poor person or persons as the Court shall have directed.

Oftentimes the poor man's children are his "stock-in-trade," and to say that they have no pecuniary value is as much as to say that his "staff," upon which he hopes to lean in old age, is but a fleeting fancy, and that the ties of blood are not recognized by mankind—except as the Courts may direct, when they compel the children to take care of the parents, where no other means of support are provided.

If we take the case above referred to of a party aged fifty as an average case, and assume that a man will earn his maximum income of $2,000 at that age, and continue to earn the same income during the balance of his natural life, we may with safety apply the law of average, and say that at age twenty he will earn his first income of, say, $500; at thirty, $1,000; at forty, $1,500, and at fifty the $2,000.

To determine the value of a life suddenly terminated at any age after reaching the age of twenty—or the time when a man begins to earn his own living—and recognizing the theory that fifty per cent. of his income belongs to his family, it is an easy matter to formulate a table which will give the present value of the sum of a series of Annuities, covering the periods during which he earns the graded income above referred to.

While it would be impossible to determine the exact value of a child's life, who at the time of death was not earning any income, it would not be a difficult problem with the aid of actuarial science to arrive at an approximate conclusion. The prospective earning capacity of the child can only be determined by a

careful consideration of all the influences that surround the child's life.

By the same process of reasoning as in the case of a man aged fifty, the table can be mathematically extended through the younger ages of life. For example, should death occur at age two, the present value would be represented by the sum of a series of Deferred Term Annuities for increasing amounts, and would be expressed by the following formula :

$$\left(\frac{N_{20}-N_{30}}{D_2}\times\$250\right)+\left(\frac{N_{30}-N_{40}}{D_2}\times\$500\right)+\left(\frac{N_{40}-N_{50}}{D_2}\times\$750\right)$$
$$+\left(\frac{N_{50}}{D_2}\times\$1,000\right)=\$2,467.77.$$

This can be but a crude estimate at best, but a jury would receive material aid from it when trying to arrive at a definite conclusion.

I have appended a table illustrating the values to which I have alluded.

CARLISLE TABLE OF MORTALITY WITH FIVE PER CENT. INTEREST.

Age.	Yearly Income.	50 per cent. to support self.	40 per cent. to family.	10 per cent. set aside.	Total value to family.	Present value of an Annuity of 50 per cent. of Income.
2	..	..	..	..	..	$2,467.77
5	..	..	..	..	..	3,269.57
10	..	..	..	..	..	4,390.48
15	..	..	..	..	..	5.745.85
20	$500	$250	$200	$50	$250	7,586.02
30	1,000	500	400	100	500	10,066.57
40	1,500	750	600	150	750	11,592.77
50	2,000	1,000	800	200	1,000	11,660.00
60	2,000	1,000	800	200	1,000	8,939.60
70	2,000	1,000	800	200	1,000	6,335.80

Mortality Experience Among Lives Accepted at Ages Over Sixty by the Equitable Life Assurance Society of the United States.

ROBERT GEORGE HANN.

HAVING made an investigation of the loss experience of the Society for the twenty-six years ended December 31st, 1895, the observations ceasing at that date, I suggested to our President, Mr. James W. Alexander, the desirability of bringing the results before the Actuarial Society on the occasion of its decennial celebration. He immediately recognized the propriety of this being done since it affords correct information, with a view to judge as to the extent to which it is wise to grant assurances on old lives.

In a matter of this kind reliable averages, suitable for all, cannot be expected from the limited experience of one Society. It must emanate from that of many companies, especially since each may be expected to possess its own peculiar features. In the mean time further light on the subject will do no harm. The objective point of the companies should be to obtain correct observations of sufficient volume extending from the average age at entry to the extreme limits of the duration of life. This would enable us to predict the experience of the future from that of the past with far greater confidence than is possible at present.

The total number of *policies* involved in this investigation was 2,107, of which 563, or 27 per cent., were terminated by death.

The total exposures by amounts were $54,411,276, distributed as follows:

Table I.

Years of Assurance.	Ages at Issue.	Amount Exposed.	Totals.
First 5 yrs.,	61—65	$30,034,507	
	66—70	5,921,722	
	71—75	165,026	$36,121,255
6 to 10 yrs.,	61—65	11,391,042	
	66—70	2,114,087	
	71—75	41,435	13,546,564
11 to 26 yrs.,	61—65	4,151,131	
	66—70	592,326	4,743,457
Grand Total,			$54,411,276

At first we dealt with amounts only, and compared the actual claims with the expected rate of loss by the American Experience Table. Within the past two weeks I have computed the percentage of actual claims to those expected when lives only are dealt with. These are given below :

TABLE II.

Ages at Issue.	Years of Assurance.	Percentage of Actual to Expectedon Amounts.	Percentage of Actual to Expected on Lives.	Ages at Issue.	Percentage of Actual to Expectedon Amounts.	Percentage of Actual to Expected on Lives.
61—65,	1— 5	90.71	95.28	66—75	91.14	104.29
	6—10	105.79	99.27	(mostly	74.84	96.85
	11—26	84.81	93.40	66 to 70)	49.35	93.65
	All years,	93.98	96.04		80.59	100.19

Combining all ages at issue and grouping by years of assurance, we have

Years.	Percentage of Actual to Expected on Amounts.	Percentage of Actual to Expected on Lives.
1— 5,	90.81	97.25
6—10,	98.69	98.75
11—26,	79.03	93.44
Total experience,	91.00	96.87

Grouping the observations by ages attained as follows, we have :

Ages Attained.	Percentage of Actual to Expected on Amounts.	Percentage of Actual to Expected on Lives.
61–65	90.16	93.78
66–70	102.33	98.82
71 and over.	79.97	97.12
Total Experience,	91.00	96.87

The following are the actual loss ratios by years of assurance :

Years of Assurance.	Ratio on Amounts.	Ratio on Lives.
1–5	.0378	.0402
6–10	.0622	.0618
11–26	.0873	.1036
All years,	.0482	.0528

Comparing the observations with those corresponding thereto by the Institute of Actuaries' H^M Experience, we have the following :

Ages at Issue.	Years of Assurance.	Loss Ratio.	H^M Ratio.	Ratio per cent. of H^M.
61 to 65	1–5	.03454	.03226	107.06
	6–10	.06108	.06125	99.72
66 to 70	1–5	.05447	.05024	108.42
	6–10	.06722	.09045	74.32

Combining we have :

Ages at Issue.	Years of Assurance.	Ratio of Actual Claims to H^M.
61–65	1–10	104.06 per cent.
66–70	1–10	95.08 "
61–70	1–10	101.97 "
61–70	1–5	107.39 "
61–70	6–10	94.25 "
61–70	1–10	101.97 "

In this comparison, due allowance was made for the H^M Table being arranged by calendar years, and the assurance having been accepted at age next birthday.

These observations include the whole business of the Society originally issued within the period above mentioned at the higher ages. Paid-up policies issued in lieu of those surrendered were not included when the original policy was issued on a life below 61 years of age. The business is chiefly on the whole-life plan. There were 161 endowment assurances, 14 term assurances under one year's duration, 3 joint life assurances, and 1 survivorship assurance. These leave 1,928 policies, or 91½ per cent. of the whole, in the life class. The total amount assured was $12,631,896, and the average length of exposure was 4.31 years.

TABLE III.

MORTALITY EXPERIENCE OF LIVES ADMITTED OVER 60 YEARS OF AGE.

	YEARS OF ASSURANCE. 1—5.			YEARS OF ASSURANCE. 6—10.			YEARS OF ASSURANCE. 11–26.			TOTAL.		
Age.	Number Exposed.	Expected.	Actual.	Number Exposed.	Expected.	Actual.	Number Exposed.	Expected.	Actual.	Number Exposed.	Expected.	Actual.
61	1525.9	50.72	50	656.0	33.29	32	339.5	32.49	28	2521.4	116.50	110
62	1373.8	49.68	43	612.5	33.76	32	223.6	22.42	23	2209.9	105.86	98
63	1014.5	39.77	47	426.7	25.82	28	162.7	18.17	17	1603.9	83.76	92
64	757.4	32.20	30	295.0	19.45	20	115.4	13.02	13	1167.8	64.67	63
65	690.7	32.30	25	275.6	19.64	19	127.1	16.68	15	1093.4	68.62	59
	5362.3	204.67	195	2265.8	131.96	131	968.3	102.78	96	8596.4	439.41	422
Actual to Expected,		95.28			99.27			93.40			96.04	
66	299.7	15.12	21	119.7	9.44	7	69.0	10.43	3	488.4	34.99	31
67	220.5	12.30	11	85.3	7.23	4	26.8	3.41	7	332.6	22.94	22
68	233.6	14.28	15	82.5	7.46	9	28.4	4.20	6	344.5	25.94	30
69	116.2	7.70	4	46.4	4.58	6	2.3	.31	1	164.9	12.59	11
70	59.5	4.25	6	23.3	2.52	3	5.5	.87	1	88.3	7.64	10
71	29.6	2.34	0	10.3 (4 yrs.only)	1.15	3	....	....	..	39.9	3.49	3
72	10.3	.84	1	5.0	.66	0	.. .	....	..	15.3	1.50	1
73	....	....	..	....	....	..	....	....	..	....	....	.
74	5.0	.51	1	....	....	..	....	....	..	5.0	.51	1
75	2.0 (2 yrs.only)	.19	1	.. .	....	..	...	....	..	2.0	.19	1
	976.4	57.53	60	372.5	33.04	32	132.0	19.22	18	1480.9	109.79	110
Actual to Expected,		104.29			96.85			93.65			100.79	

YEARS OF ASSURANCE.

All Ages.	Number Exposed.	Expected.	Actual.	Actual to Expected, %.
1 to 5	6338.7	262.2	255	97.25
6 to 10	2638.3	165.0	163	98.75
11 to 26	1100.3	122.0	114	93.44
Total,	10077.3	549.2	532	96.87

The calculations throughout were made for the actual time of exposures and policy years were adopted. The results fluctuate between wide limits where amounts are dealt with, which is to be expected from their irregularity and comparatively limited extent at certain ages. The following table shows the intensity of the general class of diseases :

TABLE IV.

INTENSITY OF DISEASES.

		Amount of Claims.	Percentage of Total Claims.
Diseases of Brain and Nerves		$610,784	23.30
Constitutional Diseases		570,201	21.75
The principal being :			
Pneumonia	$266,827		
Phthisis	53,659		
Sarcoma	98,930		
Bronchitis	41,504		
Dropsy	28,500		
Rheumatism	25,750		
Diseases of Urinary Organs		343,682	13.10
" " Digestive "		289,559	11.05
" " Heart and Blood Vessels		240,047	9.16
Epidemic and Contagious		221,203	8.44
Violent Deaths, Accident	$141,500		
Suicide	36,444	177,944	6.78
Natural Decay		130,243	4.97
Integumentary System		6,500	.25
Diseases of Joints		5,000	.19
Undetermined		26,382	1.01
Total		$2,621,545	100.00

TABLE V.

DEATHS ARRANGED BY CLASS OF OCCUPATION.

OCCUPATION.	Number of Lives.	Amount Assured.	Average Policy.
Merchants and Manufacturers	204	$1,210,108	$5,932
Professional.	80	307,696	3,846
Bankers and Capitalists.....	63	457,136	7,256
Farmers.....	73	225,583	3,091
Brewers, Distillers, etc.....	10	139,880	13,980
Clerks, Bookkeepers, etc.....	35	99,613	2,846
Mechanics.....	24	72,773	3,032
Army and Navy.....	10	17,919	1,792
Domestics and Laborers	33	90,837	2,753
Total.....	532	$2,621,545	$4,928

Not having the occupations of all the lives under observation, I was unable to give the percentage of deaths by occupation.

By the nature of the case the duplicate policies were few, and were taken at the time of entering the Society. The assurance on any life was not therefore increased later on by the issue of additional policies after reëxamination, as is often the case. Hence there was less suspended mortality here for the same duration than is usually to be found among mortality statistics. The deaths included $172,030 of assurance on people residing in the Southern States, and $92,587 of assurance on those residing in the tropics. The extra mortality due to residence in these places was not taken into consideration. If due weight had been given to this in the observations, or if they had been excluded, the results would have been more favorable.

I was much surprised to find the rate of loss come out appreciably less than the rate of mortality. That it is so may be seen from the following consideration: If we divide the amount exposed by the number of lives exposed we get $5,399 for the average amount exposed. Again, if we divide the amount of loss by the number of deaths we get $4,928 for the average claim, which shows that the loss fell, in the aggregate, on the smaller policies.

The rate of loss and rate of mortality both fall within the expected by the table upon which the premiums were computed. Taking amounts into account, it will be observed that 66½ per cent. of the exposures is in the first five years. Medical selection

is evidently at its worst when dealing with those in advanced life. The insecurity attending such risks accepted after the age of 60 is probably due largely to the fact that it is more difficult to anticipate the prevailing causes of death among those advanced in years than among younger applicants even when applying the most careful method of examination. Again, when disease takes hold, the power to resist death steadily declines as the attacks increase in frequency.

In dealing with proponents for assurance at advanced ages the most careful attention should be paid to their family history, habits, financial standing and general environment. Where there is any doubt on a material point, our motto should be "Decline."

At some future time I hope to test this experience by comparing the actual death strain with the real amount at risk which would give the financial mortality of the Society on such business.

I would state that Mr. George F. Salter, one of my former assistants, prepared the experience by amounts under my direction, and his work was carried out with great accuracy. One of my present assistants, Mr. Robert Henderson, has displayed great ability in aiding me in the preparation of the mortality experience, which had to be threshed out at short notice.

Some of the Effects Produced by A Change in the Interest Basis of Valuation.

THOMAS BRADSHAW.

The two most momentous questions at present engaging the attention of the Life Insurance Companies in Canada are (1) the basis of valuation and (2) the powers of investment of funds. That these questions are so prominent at this moment is due to the Bill introduced by the Honorable Minister of Finance, at the present session of the Dominion Parliament, entitled "An Act to further Amend the Insurance Act."

Inasmuch as the provisions of the proposed Act do not merely affect the native life insurance companies but concern all life companies operating in Canada, I have been led to make these notes in the expectation that they will prove of some interest to the members of the Society.

At this time, I will touch only upon the former question, namely, that dealing with the basis of valuation, although it appears to me that both questions are more or less associated with one another.

I think it a significant fact that the measure proposes on the one hand a radical lowering of the interest basis of valuation, and on the other hand an enlargement of the powers of investments of some of the companies. Two native companies now possess almost unlimited powers of investing their funds; and these powers the Bill does not propose to disturb. My own opinion is that if the Government had been less restrictive in granting originally powers of investments to the large majority of the companies, it would not now be necessary for it to propose this radical change in the basis of valuation. It is, however, to be earnestly hoped that such broad powers will now be granted to all companies as will afford them a wide range of securities from which to choose. This would do away with the necessity of their seeking in the future further extension of limits (which, in view of the proposed general Bill, it is altogether likely the Government will be most chary in granting); or of their being obliged to disturb, for at least many years to come. the interest basis of valuation.

The proposed change in the basis of valuation will be seen from comparing the following sections, the one taken from the existing

Insurance Act, and the other taken from the Bill introduced recently :

EXISTING PROVISION.	PROPOSED PROVISION.
Once in every five years, or oftener, at the discretion of the Minister, the Superintendent shall himself value, or procure to be valued under his supervision, the Canadian policies of life insurance of all companies licensed under this Act to transact the business of life insurance in Canada; and such valuation shall be based on the mortality tables of the Institute of Actuaries of Great Britain, and on a rate of interest at four and one-half per centum per annum, except in the case of bonus additions or profits accrued or declared before the 28th day of April, one thousand eight hundred and seventy-seven, and then valued on the basis of a rate of interest other than that above mentioned, which, in any such valuation, shall continue to be valued on such other basis.	Once in every five years, or oftener, at the discretion of the Minister, the Superintendent shall himself value, or procure to be valued under his supervision, all the policies of life insurance of Canadian companies and the Canadian policies of life insurance companies other than Canadian companies, licensed under this Act to transact the business of life insurance in Canada; and such valuation shall, until the first day of January, 1907, as to policies issued prior to the first day of January, 1900, and bonus additions or profits accrued or declared in respect thereof, be based on the mortality table of the Institute of Actuaries of Great Britain, and on a rate of interest of four and one-half per cent. per annum, and as to policies issued on or after the said last mentioned date and bonus additions or profits accrued or declared in respect thereof, shall be based upon the said mortality table and a rate of interest of three and one-half per cent. per annum; provided that, on and after the said first day of January, 1907, the basis of valuation last mentioned shall be applicable to all such policies and bonus additions or profits accrued or declared in respect thereof. In this and the next succeeding paragraph of this subsection the word "policies" includes annuity contracts, provided that, in the valuation of annuity contracts the table of mortality experience of (British) Government Life Annuitants may be used instead of the table of the Institute of Actuaries in paragraph clause mentioned.

It will be observed that the valuations of all policies and bonuses granted by Canadian, American and British companies on and after 1st January, 1900, are to be made on the mortality table of the Institute of Actuaries of Great Britain and interest at 3½ per cent.; of all policies and bonuses granted prior to 1st January, 1900, on the same table of mortality and interest at 4½ per cent. until 1st January, 1907; after which date the valuations of all policies and bonuses are to be made on the higher basis. In the valuation of annuities, the table of mortality experience of (British) Government Life Annuitants may be used.

Without discussing the advantages and disadvantages of Governmental supervision in the matter of policy valuation, let us proceed to examine what the proposed changes will result in.

Although the existing Insurance Act merely requires that the valuations of policies of life insurance shall be made on at least as stringent a basis as the Institute of Actuaries' table of mortality and a 4½ per cent. interest rate, most of the companies have voluntarily valued the whole or a part of their obligations on a more severe standard. The following table sets forth the basis of valuation of the fifteen Canadian life insurance companies :

Office.	Table of Mortality.	Rate of Interest.
1	American Ex	4%
2	Institute of Act. H^M	4½% for insurances effected prior to 1st January, 1896. 3½% for insurances effected since 1st January, 1896.
3	Institute of Act. H^M	4½% for insurances effected prior to 1st July, 1896. 4% for insurances effected since 1st July, 1896.
4	Institute of Act. H^M	4½% General insurance. 4% Industrial insurance.
5	Institute of Act. H^M	4½%
6	Com. Ex.....	4%
7	Institute of Act. H^M	3½%
8	Institute of Act. H^M Com. Ex.....	4½% General insurance. 4% Industrial insurance.
9	Institute of Act. H^M	4½% for insurances effected prior to 1st January, 1896. 4% for insurances effected since 1st January, 1896, and foreign insurance.
10	Institute of Act. H^M	4½% for insurances effected prior to 1st January, 1897. 4% for insurances effected since 1st January, 1897.
11	Com. Ex.....	4%
12	Com. Ex.....	4%
13	Institute of Act. H^M	4½%
14	Institute of Act. H^M	4%
15	Institute of Act. H^M	4½% for insurances effected prior to 1st January, 1897. 4% for insurances effected since 1st January, 1897.

It will be observed that one company values its insurances entirely on a 3½ per cent. basis; one its new insurances on a 3½ per cent. basis, and its old on a 4½ per cent. basis; five their entire insurances on a 4 per cent. basis; six part of their insurances on a 4 per cent. basis, and part on a 4½ per cent. basis, and two their entire insurances on a 4½ per cent. basis. Practically all the companies, by their voluntary adoption of a more stringent reserve basis than that called for by the existing act, indicate that in their opinion, 4½ per cent. is too high a rate of interest to employ.

A 3½ per cent. valuation in respect of insurances on the companies' books at 1st January, 1900, is not proposed for seven years, or until 1st January, 1907. Of course, a very considerable proportion of insurances in force on 1st January, 1900, will have terminated prior to 1907, but, notwithstanding this, a drop of one per cent. in the rate of interest employed in the valuation of the remaining insurances of a company which has been doing business for many years, and which has a comparatively large amount of relatively old policies, will require a large transfer of funds from surplus to reserve, and consequently, policy-holders must expect diminished profits in such case.

From the official returns it would be impossible to approximate, with any degree of accuracy, how much would be involved in the change from a 4½ per cent. standard to a 4 per cent. or to a 3½ per cent. in any one company's business. A comparison of values, computed on different interest bases, will nevertheless enable us to determine what effect a reduced rate of interest has upon individual insurances.

Table 1 *shows the comparison of values for each* $1,000 *of Whole Life Insurance, by annual payments, by the Institute of Actuaries'* H^M *4½ per cent. and 3½ per cent. tables.*

The following are some of the features disclosed:

(1) At the usual lowest insurable age, 20, the 3½ per cent. value exceeds the 4½ per cent. value at the end of the first year by 26 per cent.

(2) This percentage is decreased from 6 to 4 points for every advance of 10 years in the age at entry; the excess of the 3½ per cent. value over the 4½ per cent. value being 7 per cent. at age at entry, 60.

(3) For every advance of 5 years in the duration of the insurance, the percentages by which the 3½ per cent. values exceed the 4½ per cent. values, decrease from 3 to 1 points, for example:

Age at Entry, 20.		Age at Entry, 40.	
Duration.	Excess 3½ per cent. values over 4½ per cent. values.	Duration.	Excess 3½ per cent. values over 4½ per cent. values.
5 Years.	25 per cent.	5 Years.	14 per cent.
10 "	22 "	10 "	12 "
15 "	20 "	15 "	10 "
20 "	18 "	20 "	9 "
25 "	16 "	25 "	7 "
30 "	14 "	30 "	6 "
Etc.	Etc.	Etc.	Etc.

Tables 2 and 3 show the comparison of values for each $1,000 *of Whole Life Insurance by annual payments, limited to* 10 *and* 20 *years, by the Institute of Actuaries'* H^M *4½ and 3½ per cent. tables.*

(a) Whole Life Insurance by 10 Annual Payments.

(1) At age at entry, 20, the 3½ per cent. value exceeds the 4½ per cent. value at the end of the first year by 33 per cent.

(2) This percentage is decreased from 5 to 6 points for every advance of 10 years in the age at entry; the excess of the 3½ per cent. value over the 4½ per cent. value being 16 per cent. at age at entry, 50.

(3) The differences in the two values from the end of the first year until the end of the tenth year, when the insurance is paid up, decline

at age of entry 20, from 33 to 28 per cent.
" " 30, " 28 " 23 "
" " 40, " 22 " 17 "
" " 50, " 16 " 12 "

(b) Whole Life Insurance by 20 Annual Payments.

(1) At age at entry, 20, the 3½ per cent. value exceeds the 4½ per cent. value, at the end of the first year, by the same percentage as the 3½ per cent. value exceeds the 4½ per cent. value under the 10-payment life form of insurance, viz., 33 per cent.

(2) This percentage is decreased from 6 to 7 points for every advance of 10 years in the age at entry; the excess of the 3½ per cent. value over the 4½ per cent. value being 14 per cent. at age at entry, 50.

(3) The differences in the two values from the end of the first year until the end of the twentieth year, when the insurance is paid up, decline

at age of entry		20	from	33	to	23	per cent.
"	"	30	"	27	"	17	"
"	"	40	"	20	"	12	"
"	"	50	"	14	"	8	"

Tables 4 and 5 show the comparison of values for each $1,000 of Endowment Insurance by equal annual payments for periods of 10 and 20 years by the Institute of Actuaries' H^M 4½ per cent. and 3½ per cent. tables.

(a) Endowment Period, 10 Years.

(1) For all insurable ages, the 3½ per cent. values may be said to exceed the 4½ per cent. values at the end of the first year by 5 per cent.

(2) This percentage may be said to decrease, for all ages at entry, at the rate of about one-half of one point for each additional year's duration after the first.

(b) Endowment Period, 20 Years.

(1) For all insurable ages, the 3½ per cent. values may be said to exceed the 4½ per cent. values at the end of the first year by from 10 to 11 per cent.

(2) This percentage may be said to decrease in the same manner as in the case of 10-year Endowment Insurance.

Tables 6 and 7 show the comparison of values for each $1,000 of paid-up Whole Life Insurance and Endowment Insurance for 20 years by the Institute of Actuaries' H^M 4½ per cent. and 3½ per cent. tables.

(a) Whole Life Insurance.

(1) At age 20, the 3½ per cent. value exceeds the 4½ per cent. value by 32 per cent.

(2) This percentage is decreased from 2 to 3 points for every advance of 5 years in the age, the excess of the 3½ per cent. value over the 4½ per cent. value at age 75 being 6 per cent.

(b) Endowment Period, 20 Years.

(1) For ages 20 to 50, the 3½ per cent. values at the end of the first year exceed the 4½ per cent. values by from 14 to 18 per cent.

(2) These percentages decrease from one-half to one whole point for each additional year's duration after the first.

I think it will be found that the major portion of life insurance in Canada has been effected on the Whole Life system by limited payments, and on the Endowment system for periods of 20 years and over. It will also be found that the aggregate insurance in force in Canadian companies on the first of January, 1898, is somewhat less than the total insurance effected in the preceding 8 years, and consequently the average duration of the policies cannot be very great. Of course, when the change comes into effect, the old insurance to which it will refer will have been running for at least 7 years. Assuming that insurance has been mainly effected on the systems referred to, that in 1907, the date of the change, the average duration of the old policies will be about 12 years, and that the average age at entry was 32, I estimate that the difference between a valuation on the 4½ per cent. standard and the 3½ per cent. standard will be 15 per cent. of the 4½ per cent. reserve.

It appears to me that a drop of 1 per cent. in the interest rate at the time proposed is too great and too sudden. If a company continued to value its present business on a 4½ per cent. basis until 1907, had made no provision for the change, and was then required to set aside as a liability the equivalent of a 3½ per cent. reserve on all of its obligations, it is altogether likely that it could not do so.

The change, in my opinion, should be made more gradually; for example, a drop of one-half of one per cent. to apply 5 years from 1st January, 1900, and a further drop of one-half of one per cent. to apply 5 years later, or on 1st January, 1910. The effect of this would be to give the companies a longer time to prepare for the severe 3½ per cent. standard, and to break the effect of the radical change proposed.

Whatever measure is finally adopted, the companies will require immediately to set about preparing for it. This may be done either by establishing a special reserve fund and adding to it yearly until the difference in standards has been accumulated, or by valuing an increasing amount of the old business yearly, on the higher standard, until the whole has been brought up to it.

Those companies which have heretofore adopted a 4 per cent. standard may be said to have already prepared themselves half-way for the change, for it will be found that the difference between the 3½ per cent. and 4 per cent. values is approximately one-half the difference between the 3½ per cent. and 4½ per cent. values.

TABLE 1.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$				${}_nV_{60}$			
	3½%	4½%	Δ	%	3½%	4½%	Δ	%	3½%	4½%	Δ	%	3½%	4½%	Δ	%	3½%	4½%	Δ	%
1	$7.48	$5.93	$1.55	26	$10.67	$8.86	$1.81	20	$15.37	$13.32	$2.05	15	$22.36	$20.15	$2.21	11	$31.80	$29.59	$2.21	7
5	38.92	31.11	7.81	25	56.20	47.27	8.93	19	81.59	71.76	9.83	14	115.52	105.42	10.10	10	156.76	147.24	9.52	6
10	85.23	69.68	15.55	22	119.13	101.81	17.32	17	170.45	152.25	18.20	12	236.32	218.73	17.59	8	310.52	295.15	15.37	5
15	136.64	113.65	22.99	20	191.00	166.27	24.73	15	266.28	241.62	24.66	10	356.03	333.76	22.27	7	451.33	433.55	17.78	4
20	194.21	164.39	29.82	18	269.28	238.56	30.72	13	366.49	337.68	28.81	9	473.46	449.32	24.14	5	568.77	550.93	17.84	3
25	259.95	224.36	35.59	16	353.69	318.83	34.86	11	465.79	435.19	30.60	7	580.99	557.45	23.54	4	658.71	641.93	16.78	3
30	331.56	291.62	39.94	14	441.96	405.11	36.85	9	563.21	533.16	30.05	6	670.68	649.15	21.53	3	748.80	734.20	14.60	2
35	408.78	366.29	42.49	12	529.44	492.70	36.74	7	652.41	624.83	27.58	4	739.36	720.25	19.11	3	869.59	860.65	8.94	1
40	489.52	446.56	42.96	10	615.24	580.69	34.55	6	726.81	702.57	24.24	3	808.16	792.34	15.82	2	....	....	....	..
45	569.54	528.05	41.49	8	693.82	663.02	30.80	5	783.79	762.84	20.95	3	900.41	891.13	9.28	1	....	....	....	..
50	648.04	609.90	38.14	6	759.36	732.85	26.51	4	840.86	823.95	16.91	2	....	....	....	..	....	....	....	..
55	719.91	686.50	33.41	5	809.54	786.99	22.55	3	917.39	907.70	9.69	1	....	....	....	..	....	....	....	..
60	779.87	751.46	28.41	4	859.82	841.88	17.94	2	....	....	...	..	....	....	....	..	....	....	....	..
65	825.78	801.83	23.95	3	927.23	917.10	10.13	1	....	....	....	..	....	....	....	..	....	....	....	..
70	871.77	852.89	18.88	2	....	....	....	..	....	....	....	.	....	....	....	..	....	....	...	..
75	933.43	922.88	10.55	1	....	....	....	..	....	....	....	..	....	....	....	..	....	....	....	..

TABLE 2.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY 10 ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M $3\frac{1}{2}$ PER CENT. AND $4\frac{1}{2}$ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$			
	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%
1	$28.76	$21.56	$7.20	33	$35.30	$27.59	$7.71	28	$43.28	$35.43	$7.85	22	$52.44	$45.13	$7.31	16
2	58.33	43.84	14.49	33	71.94	56.45	15.49	28	88.37	72.68	15.69	22	106.95	92.39	14.56	16
3	89.04	67.18	21.86	33	110.00	86.68	23.32	27	135.31	111.79	23.52	21	163.57	141.85	21.72	15
4	121.12	91.81	29.31	32	149.56	118.37	31.19	26	184.11	152.76	31.35	21	222.38	193.59	28.79	15
5	154.65	117.83	36.82	31	190.66	151.58	39.08	26	234.84	195.72	39.12	20	283.58	247.82	35.76	14
6	189.59	145.21	44.38	31	233.36	186.36	47.00	25	287.51	240.64	46.87	19	347.34	304.70	42.64	14
7	225.95	173.97	51.98	30	277.71	222.76	54.95	25	342.20	287.62	54.58	19	413.89	364.50	49.39	14
8	263.68	204.05	59.63	29	323.78	260.89	62.89	24	399.08	336.85	62.23	18	483.54	427.53	56.01	13
9	302.81	235.50	67.31	29	371.72	300.88	70.84	24	458.36	388.56	69.80	18	556.66	494.17	62.49	13
10	343.43	268.40	75.03	28	421.65	342.89	78.76	23	520.23	442.93	77.30	17	633.61	564.78	68.83	12

TABLE 3.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY 20 ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$			
	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%
1	$14.82	$11.11	$3.71	33	$18.47	$14.57	$3.90	27	$22.98	$19.11	$3.87	20	$28.57	$25.10	$3.47	14
2	29.87	22.41	7.46	33	37.55	29.71	7.84	26	46.83	39.12	7.71	20	57.93	51.05	6.88	13
3	45.44	34.18	11.26	33	57.27	45.49	11.78	26	71.54	60.01	11.53	19	88.02	77.81	10.21	13
4	61.74	46.63	15.11	32	77.68	61.94	15.74	25	97.04	81.72	15.32	19	118.77	105.32	13.45	13
5	78.84	59.86	18.98	32	98.78	79.07	19.71	25	123.35	104.27	19.08	18	150.22	133.60	16.62	12
6	96.65	73.78	22 88	31	120.58	96.88	23.70	24	150.35	127.55	22.80	18	182.35	162.65	19.70	12
7	115.19	88.38	26.81	30	143.04	115.36	27.68	24	178.02	151.53	26.49	17	215.17	192.48	22.69	12
8	134.30	103.54	30.76	30	166.20	134.54	31.66	23	206.42	176.29	30.13	17	248.75	223.15	25.60	11
9	153.99	119.25	34.94	29	190.15	154.50	35.65	23	235.66	201.92	33.74	17	283.15	251.74	28.41	11
10	174.29	135.56	38.73	29	214.94	175.31	39.63	23	265.77	228.49	37.28	16	318.35	287.22	31.13	11
11	195.23	152.48	42.75	28	240.69	197.10	43.59	22	296.90	256.13	40.77	16	354.40	320.63	33.77	11
12	216.92	170.15	46.77	28	267.48	219.96	47.52	22	329.16	284.95	44.21	16	391.37	355.04	36.33	10
13	239.41	188.60	50.81	27	295.33	243.92	51.41	21	362.53	314.98	47.55	15	429.36	390.56	38.80	10
14	262.73	207.89	54.84	26	324.19	268.94	55.25	21	397.05	346.22	50.83	15	468.53	427.31	41.22	10
15	286.92	228.05	58.87	26	354.12	295.07	59.05	20	432.79	378.76	54.03	14	509.16	465 58	43.58	9
16	311.98	249.09	62.89	25	385.07	322.26	62.81	20	469.84	412.67	57.17	14	551.57	505.71	45.86	9
17	337.93	271.03	66.90	25	417.06	350.53	66.53	19	508.28	448.07	60.21	13	596.13	548.07	48.06	9
18	364.80	293.92	70.88	24	450.17	379.99	70.18	19	548.28	485.11	63.17	13	643.26	593.08	50.18	8
19	392.68	317.84	74.84	24	484.53	410.75	73.78	18	590.01	523.96	66.05	13	693.53	641.32	52.21	8
20	421.65	342.89	78.76	23	520.23	442.93	77.30	17	633.61	564.78	68.83	12	747.38	693.23	54.15	8

TABLE 4.

COMPARISON OF VALUES OF 10 YEARS' ENDOWMENT INSURANCE BY EQUAL ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{10}$			
	3½%	4½%	Δ	%	3½%	4½%	Δ	%
1	$82.85	$79.07	$3.78	5	$81.54	$77.82	$3.72	5
2	168.82	161.89	6.93	4	166.68	159.84	6.84	4
3	258.31	248.93	9.38	4	255.54	246.27	9.27	4
4	351.62	340.57	11.05	3	348.25	337.31	10.94	3
5	448.92	437.06	11.86	3	445.04	433.26	11.78	3
6	550.31	538.57	11.74	2	546.10	534.41	11.69	2
7	655.92	645.33	10.59	2	651.73	641.14	10.59	2
8	765.92	757.60	8.32	1	762.31	753.95	8.36	1
9	880.53	875.70	4.83	1	878.24	873.37	4.87	1

TABLE 5.

COMPARISON OF VALUES OF 20 YEARS' ENDOWMENT INSURANCE BY EQUAL ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{10}$			
	3½%	4½%	Δ	%	3½%	4½%	Δ	%
1	$33.48	$30.18	$3.30	11	$33.40	$30.23	$3.17	10
2	67.98	61.55	6.43	10	68.17	61.98	6.19	10
3	103.82	94.43	9.39	10	104.31	95.28	9.03	9
4	141.25	129.11	12.14	9	141.77	130.12	11.65	9
5	180.34	165.69	14.65	9	180.64	166.57	14.07	8
6	221.09	204.19	16.90	8	220.83	204.59	16.24	8
7	263.50	244.64	18.86	8	262.37	244.25	18.12	7
8	307.54	287.05	20.49	7	305.40	285.68	19.72	7
9	353.26	331.49	21.77	7	350 08	329.04	21.04	6
10	400.76	378.10	22.66	6	396.51	374.58	21.93	6
11	450.15	427.00	23.15	5	444.90	422.46	22.44	5
12	501.55	478.40	23.15	5	495.43	472.91	22.52	5
13	555.10	532.46	22.64	4	548.21	526.09	22.12	4
14	610.90	589.33	21.57	4	603.39	582.23	21.16	4
15	669.06	649.18	19.88	3	661.19	641.61	19.58	3
16	729.71	712.19	17.52	2	721.85	704.46	17.39	2
17	792.96	778.53	14.43	2	785.66	771.25	14.41	2
18	858.97	848.45	10.52	1	853.00	842.40	10.60	1
19	927.93	922.19	5.74	1	924.28	918.44	5.84	1

TABLE 6.

COMPARISON OF VALUES OF PAID-UP LIFE INSURANCE, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

Age	3½%	4½%	Δ	%	Age	3½%	4½%	Δ	%
20	$282.26	$213.61	$68.65	32	50	$520.23	$442.93	$77.30	17
25	310.19	238.08	72.11	30	55	575.66	501.66	74.00	15
30	343.43	268.40	75.03	28	60	633.61	564.78	68.83	12
35	380.33	302.99	77.34	26	65	691.04	628.86	62.18	10
40	421.65	342 89	78.76	23	70	747.38	693.23	54.15	8
45	468.84	390.04	78.80	20	75	798.97	753.47	45.50	6

TABLE 7.

COMPARISON OF VALUES OF PAID-UP ENDOWMENT INSURANCE, 20 YEARS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND 4½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$			
	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%	$3\frac{1}{2}\%$	$4\frac{1}{2}\%$	Δ	%
1	$546.52	$463.31	$83.21	18	$553.97	$471.75	$82.22	17	$568.30	$487.96	$80.34	16	$600.96	$524.97	$75.99	14
2	562.71	480.66	82.05	17	569.95	488.93	81.02	17	583.83	504.72	79.11	16	615.58	540.94	74.64	14
3	579.52	498.86	80.66	16	586.55	506.93	79.62	16	599.97	522.30	77.67	15	630.65	557.52	73.13	13
4	597.08	518.05	79.03	15	603.80	525.81	77.99	15	616.70	540.70	76.00	14	646.14	574.70	71.44	12
5	615.43	538.30	77.13	14	621.71	545.61	76.10	14	634.06	559.94	74.12	13	662.08	592.52	69.56	12
6	634.54	559.60	74.94	13	640.31	566.36	73.95	13	652.00	580.02	71.98	12	678.49	611.00	67.49	11
7	654.44	581.99	72.45	12	659.63	588.10	71.53	12	670.57	600.96	69.61	12	695.40	630.19	65.21	10
8	675.11	605.46	69.65	12	679.68	610.88	68.80	11	689.78	622.83	66.95	11	712.85	650.15	62.70	10
9	696.56	630.05	66.51	11	700.54	634.80	65.74	10	709.73	645.74	63.99	10	730.91	670.98	59.93	9
10	718.85	655.84	63.01	10	722.26	659.93	62.33	9	730.47	669.78	60.69	9	749.59	692.71	56.88	8
11	742.01	682.91	59.10	9	744.91	686.40	58.51	9	752.08	695.05	57.03	8	768.97	715.44	53.53	7
12	766.13	711.35	54.78	8	768.55	714.29	54.26	8	774.65	721.69	52.96	7	789.13	739.28	49.85	7
13	791.26	741.26	50.00	7	793.24	743.68	49.56	7	798.22	749.77	48.45	6	810.17	764.39	45.78	6
14	817.44	772.74	44.70	6	818.98	774.64	44.34	6	822.86	779.41	43.45	6	832.24	790.94	41.30	5
15	844.73	805.86	38.87	5	845.87	807.27	38.60	5	848.68	810.77	37.91	5	855.53	819.23	36.30	4
16	873.18	840.73	32.45	4	873.93	841.67	32.26	4	875 78	843.95	31.83	4	880.28	849.56	30.72	4
17	902.86	877.44	25.42	3	903.27	877.96	25.31	3	904.28	879.22	25.06	3	906.74	882.31	24.43	3
18	933.83	916.13	17.70	2	933.98	916.33	17.65	2	934.35	916.79	17.56	2	935.25	917.92	17.33	2
19	966.18	956.94	9.24	1	966.18	956.94	9.24	1	966.18	956.94	9.24	1	966.18	956.94	9.24	1

TABLE 8.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$				${}_nV_{60}$			
	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%
1	$6.19	$7.48	$1.29	17	$9.45	$10.67	$1.22	11	$14.68	$15.37	$0.69	0	$22.74	$22.36	—$0.38	—0	$33.00	$31.80	—$1.20	—4
5	33.23	38.92	5.69	15	50.58	56.20	5.62	10	77.92	81.59	3.67	4	117.31	115.52	— 1.79	—2	164.34	156.76	— 7.58	—5
10	72.78	85.23	12.45	15	110.14	119.13	8.99	8	166.89	170.45	3.56	2	240.96	236.32	— 4.64	—2	321.81	310.52	—11.29	—4
15	119.68	136.64	16.96	12	179.47	191.00	11.53	6	264.62	266.28	1.66	1	365.70	356.03	— 9.67	—3	465.19	451.33	—13.86	—3
20	174.90	194.21	19.31	10	258.64	269.28	10.64	4	367.63	366.49	— 1.14	—0	485.23	473.46	—11.77	—2	597.84	568.77	—29.07	—5
25	239.19	259.95	20.76	8	345.61	353.69	8.08	2	471.56	465.79	— 5.77	—1	594.06	580.99	—13.07	—2	720.65	658.71	—61.94	—9
30	312.60	331.56	18.96	6	437.28	441.96	4.68	1	571.14	563.21	— 7.93	—1	694.75	670.68	—24.07	—4	830.16	748.80	—81.36	—11
35	393.24	408.78	15.54	4	529.76	529.44	—0.32	—0	661.80	652.41	— 9.39	—1	787.96	739.36	—48.60	—7	909.36	869.59	—39.77	—5
40	478.23	489.52	11.29	3	618.37	615.24	—3.13	—1	745.69	726.81	—18.88	—3	871.08	808.16	—62.92	—8	..	..	..	..
45	563.98	569.54	5.56	1	699.05	693.82	—5.23	—1	823.35	783.79	—39.56	—5	..	..	..	..	..	..	..	..

TABLE 9.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY 10 ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_{n}V_{20}$				${}_{n}V_{30}$				${}_{n}V_{40}$				${}_{n}V_{50}$			
	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%
1	$27.84	$28.76	$0.92	3	$33.89	$35.30	$1.41	4	$42.14	$43.28	$1.14	3	$52.35	$52.44	$0.09	0
2	56.83	58.33	1.50	3	69.18	71.94	2.76	4	85.98	88.37	2.39	3	106.61	106.95	0 34	0
3	87.03	89.04	2.01	2	105.94	110.00	4.06	4	131.61	135.31	3.70	3	162.87	163.57	0.70	0
4	118.49	121.12	2.63	2	144.21	149.56	5.35	4	179.10	184.11	5.01	3	221.27	222.38	1.11	0
5	151.27	154.65	3 38	2	184.08	190.66	6.58	3	228.53	234.84	6.31	3	281.94	283.58	1.64	1
6	185.42	189.59	4.17	2	225.63	233.36	7.73	3	280.01	287.51	7.50	3	345.06	347.34	2.28	1
7	221.00	225.95	4.95	2	268.91	277.71	8.80	3	333.63	342.20	8.57	3	410.83	413.89	3.06	1
8	258.09	263.68	5.59	2	314.02	323.78	9.76	3	389.50	399.08	9.58	2	479.52	483.54	4.02	1
9	296.74	302.81	6.07	2	361.03	371.72	10.69	3	447.74	458.36	10.62	2	551.43	556.66	5.23	1
10	337.02	343.43	6.41	2	410.03	421.65	11.62	3	508.49	520.23	11.74	2	626.92	633.61	6.69	1

TABLE 10.

COMPARISON OF VALUES OF WHOLE LIFE INSURANCE BY 20 ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$			
	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%	A. Ex. $3\frac{1}{2}\%$	I. of A. $3\frac{1}{2}\%$	Δ	%
1	$13.75	$14.82	$1.07	7	$17.30	$18.47	$1.17	6	$22.25	$22.98	$0.63	3	$28.87	$28.57	—$.30	—1
2	28.05	29.87	1.82	6	35.27	37.55	2.28	6	45.30	46.83	1.53	3	58.43	57.93	— .50	—1
3	42.91	45.44	2.53	6	53.94	57.27	3.33	6	69.17	71.54	2 37	3	88.70	88.02	— .68	—1
4	58.36	61.74	3.38	5	73.32	77.68	4.36	6	93.88	97.04	3.16	3	119.68	118.77	— .91	—1
5	74.44	78.84	4.40	6	93.46	98.78	5.32	5	119.46	123 35	3.89	3	151.35	150.22	—1.13	—1
6	91.16	96.66	5.50	6	114.39	120.58	6.19	5	145.93	150.35	4.42	3	183.74	182.35	—1.39	—1
7	108.55	115.19	6.64	6	136.12	143.04	6.92	5	173.31	178.02	4.71	3	216.84	215.17	—1.67	—1
8	126.64	134.30	7.66	6	158.69	166.20	7.51	5	201.62	206.42	4.80	2	250.70	248.75	—1.95	—1
9	145.46	153.99	8.53	6	182.12	190.15	8.03	4	230 88	235.66	4.78	2	285.33	283.15	—2.18	—1
10	165.03	174 29	9.26	5	206.47	214.94	8.47	4	261.10	265.77	4.67	2	320.77	318.35	—2.42	—1
11	185.39	195.23	9.84	5	231.75	240.69	8.94	4	292.31	296.90	4.59	2	357.08	354.40	—2.68	—1
12	206.58	216.92	10.34	5	258.01	267.48	9.47	4	324.55	329.16	4.61	1	394.34	391.37	—2.97	—1
13	228.62	239.41	10.79	5	285.29	295.33	10.04	3	357.85	362.53	4.68	1	432.64	429.36	—3.28	—1
14	251.54	262.73	11.19	4	313.63	324.19	10.56	3	392.27	397.05	4.78	1	472.11	468.53	—3.58	—1
15	275.39	286.92	11.53	4	343.07	354.12	11.05	3	427.87	432.79	4.92	1	512.92	509.16	—3.76	—1
16	300.22	311.98	11.76	4	373.67	385.07	11.40	3	464.74	469.84	5.10	1	555.27	551.57	—3.70	—1
17	326.05	337.93	11.88	4	405.45	417.06	11.61	3	502.94	508.28	5.34	1	599.45	596.13	—3.32	—1
18	352.93	364.80	11.87	3	438.48	450.17	11.69	3	542.61	548.28	5.67	1	645.80	643.26	—2.54	—0
19	380.91	392.68	11.77	3	472.81	484.53	11.72	2	583.89	590.01	6.12	1	694 78	693.53	—1.25	—0
20	410.03	421.65	11.62	3	508.49	520.23	11.74	2	626.92	633.61	6.69	1	746.98	747.38	.40	0

TABLE 11.

COMPARISON OF VALUES OF 10 YEARS' ENDOWMENT INSURANCE BY EQUAL ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_nV_{20}$			${}_nV_{10}$		
	Am. Ex. 3½%	I. of A. 3½%	Δ	Am. Ex. 3½%	I. of A. 3½%	Δ
1	$82.16	$82.85	$0.69	$81.65	$81.54	—$0.11
2	167.81	168.82	1.01	166.82	166.68	— .14
3	257.13	258.31	1.18	255.67	255.54	— .13
4	350.28	351.62	1.34	348.41	348.25	— .16
5	447.43	448.92	1.49	445.24	445.04	— .20
6	548.78	550.31	1.53	546.41	546.10	— .31
7	654.49	655.92	1.43	652.16	651.73	— .43
8	764.79	765.92	1.13	762.78	762.31	— .47
9	879.89	880.53	.64	878.60	878.24	— .36

TABLE 12.

COMPARISON OF VALUES OF 20 YEARS' ENDOWMENT INSURANCE BY EQUAL ANNUAL PAYMENTS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_nV_{20}$			${}_nV_{10}$		
	Am. Ex. 3½%	I. of A. 3½%	Δ	Am. Ex. 3½%	I. of A. 3½%	Δ
1	$32.71	$33.48	$0.77	$33.15	$33.40	$0.25
2	66.79	67.98	1.19	67.59	68.17	.58
3	102.29	103.82	1.53	103.38	104.31	.93
4	139.29	141.25	1.96	140.58	141.77	1.19
5	177.84	180.34	2.50	179.23	180.64	1.41
6	218.02	221.09	3.07	219.41	220.83	1.42
7	259.90	263.50	3.60	261.16	262.37	1.21
8	303.55	307.54	3.99	304.57	305.40	.83
9	349.05	353.26	4.21	349.71	350.08	.37
10	396.50	400.76	4.26	396.66	396.51	— .15
11	445.97	450.15	4.18	445.52	444.90	— .62
12	497.56	501.55	3.99	496.40	495.43	— .97
13	551.38	555.10	3.72	549.46	548.21	— 1.25
14	607.52	610.90	3.38	604.86	603.39	— 1.47
15	666.10	669.06	2.96	662.78	661.19	— 1.59
16	727.23	729.71	2.48	723.45	721.85	— 1.60
17	791.05	792.96	1.91	787.16	785.66	— 1.50
18	857.69	858.97	1.28	854.21	853.00	— 1.21
19	927.28	927.93	.65	925.01	924.28	— .73

TABLE 13.

COMPARISON OF VALUES OF PAID-UP LIFE INSURANCE, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

Age	Am. Ex. 3½%	I. of A. 3½%	Δ	%	Age	Am. Ex. 3½%	I. of A. 3½%	Δ	%
20	$284.97	$282.26	—$2.71	—1	50	$508.49	$520.23	$11.74	2
25	308.73	310.19	1.46	0	55	566.15	575.66	9.51	2
30	337.02	343.43	6.41	2	60	626.92	633.61	6.69	1
35	370.55	380.33	9.78	3	65	688.24	691.04	2.80	0
40	410.03	421.65	11.62	3	70	746.98	747.38	.40	0
45	456.00	468 84	12.84	3	75	800.48	798.97	—1.51	0

TABLE 14.

COMPARISON OF VALUES OF PAID-UP ENDOWMENT INSURANCE, 20 YEARS, $1,000, BY INSTITUTE OF ACTUARIES' H^M 3½ PER CENT. AND AMERICAN EXPERIENCE 3½ PER CENT. TABLES.

n	${}_nV_{20}$				${}_nV_{30}$				${}_nV_{40}$				${}_nV_{50}$			
	A. Ex. 3½%	I. of A. 3½%	Δ	%	A. Ex. 3½%	I. of A. 3½%	Δ	%	A. Ex. 3½%	I. of A. 3½%	Δ	%	A. Ex. 3½%	I. of A. 3½%	Δ	%
1	$550.18	$546.52	—$3.66	—1	$553.92	$553.97	.05	0	$564.01	$568.30	$4.29	1	$593.40	$600.96	$7.56	1
2	566.02	562.71	—3.31	—1	569.65	569.95	.30	0	579.55	583.83	4.28	1	608.48	615.58	7.10	1
3	582.53	579.52	—3.01	—1	586.02	586.55	.53	0	595.68	599.97	4.29	1	623.99	630.65	6.66	1
4	599.74	597.08	—2.66	—0	603.07	603.80	.73	0	612.46	616.70	4.24	1	639.95	646.14	6.19	1
5	617.66	615.43	—2.23	—0	620.83	621.71	.88	0	629.89	634.06	4.17	1	656.37	662.08	5.71	1
6	636.35	634.54	—1.81	—0	639.34	640.31	.97	0	648.00	652.00	4.00	1	673.27	678.49	5.22	1
7	655.82	654.44	—1.38	—0	658.61	659 63	1.02	0	666.83	670.57	3.74	1	690.69	695.40	4.71	1
8	676.12	675.11	—1.01	—0	678.69	679.68	.99	0	686.41	689.78	3.37	0	708.64	712.85	4.21	1
9	697.29	696.56	— .73	—0	699.62	700.54	.92	0	706.76	709.73	2.97	0	727.19	730.91	3.72	1
10	719.35	718.85	— .50	—0	721.44	722.26	.82	0	727.93	730.47	2.54	0	746.37	749.59	3.22	0
11	742.36	742.01	— .35	—0	744.18	744.91	.73	0	749.96	752.08	2.12	0	766.26	768.97	2.71	0
12	766 35	766.13	— .22	—0	767.91	768.55	.64	0	772.91	774.65	1.74	0	786.92	789.13	2.21	0
13	791.38	791.26	— .12	—0	792.66	793.24	.58	0	796.84	798.22	1.38	0	808.47	810.17	1.70	0
14	817.48	817.44	— .04	—0	818.49	818 98	.49	0	821.82	822.86	1.04	0	831.03	832.24	1.21	0
15	844.72	844.73	.01	0	845.47	845.87	.40	0	847.93	848.68	.75	0	854.76	855.53	.77	0
16	873.15	873.18	.03	0	873.65	873.93	.28	0	875.30	875.78	.48	0	879.86	880.28	.42	0
17	902.83	902.86	.03	0	903.10	903.27	.17	0	904.02	904.28	.26	0	906.57	906.74	.17	0
18	933.82	933.83	.01	0	933.92	933.98	.06	0	934.26	934.35	.09	0	935.21	935.25	.04	0
19	966.18	966.18	.00	0	966.18	966.18	.00	0	966.18	966.18	.00	0	966.18	966.18	.00	0

In respect to the valuation of new insurances, that is, insurances effected after 1st January, 1900, the companies will have no difficulty in complying with the 3½ per cent. valuation, provided, of course, adequate premiums are charged.

I will now refer to some of the comparisons of the values by the Institute of Actuaries' H^M and the American Experience tables of mortality and interest at 3½ per cent. on the basis of $1,000 insurance.

Table 8. Whole Life Insurance by annual payments.

(1) The values by the Institute table for ages at entry 20 up to almost 50 are in excess of those by the American table at the end of the first policy year by from 17 per cent. downwards.

(2) At age at entry 20 the excess of the Institute values over the American values continues for about 45 years, after which the American is in the ascendency. At age at entry 30 the Institute values are the greater for about 30 years. At age at entry 40 the Institute values are greater than the American values for about 15 years. At age at entry 50 and over the American values are greater than the Institute's for all durations.

Tables 9 and 10. Whole Life Insurance by annual payments, limited to 10 and 20 years.

(a) By 10 annual payments. For all ages at entry between 20 and 50, inclusive, and for all durations during the premium-paying term, the Institute values are greater than the American values by from 4 per cent. downwards.

(b) By 20 annual payments. For all ages at entry between 20 and almost up to 50, and for all durations, during the premium-paying term, the Institute values are greater than the American values by from 7 per cent. downwards. For age at entry 50 the American values for all years during the premium-paying term, except the last, are slightly higher than the American values.

Tables 11 and 12. Endowment Insurance by equal annual payments.

(a) Endowment period, 10 years. For age at entry 20 the Institute values are slightly higher than the American values for all durations. For age at entry 40 the reverse is the case.

(b) Endowment period, 20 years. For age at entry 20 the Institute values are slightly higher than the American values for all

durations; but for age at entry 40 this condition only holds up to the ninth year, after which the American values are greater than the Institute's values.

Tables 13 and 14. Paid-up Insurance.

(a) Whole Life. For ages between 24 and 73 the Institute values are greater than the American values; after age 73 the American values are the greater.

(b) Endowment period, 20 years. For age at entry 20 the American values exceed the Institute values for the first 14 years, after which the Institute values are the greater. For ages at entry 30 and higher and for all durations, the Institute values are greater than the American values.

From a consideration of these comparisons, it may be concluded generally that the Institute table is a more severe standard of valuation for a company doing a fair amount of new business than the American table, assuming equal interest rates; although for endowment insurance, especially where the endowment period is not great, there is little difference in the values brought out by the two tables.

Effect upon Premiums.

Assuming that a 3½ per cent. reserve standard will require to be maintained in respect of new insurances effected after the 1st January, 1900, it will be essential for all Canadian companies to adopt higher rates of premiums.

For some time past there has been a feeling among some that uniform rates would be desirable, and efforts have not been lacking to secure them. Nothing practicable, however, has yet resulted. If all earnestly believe that uniform rates are desirable, I do not know that a more favorable condition for their adoption will ever again present itself than the present. That a revision in the premium rates of certain companies is essential, apart altogether from the proposed new order of things, must be apparent from an examination of the following table:

HIGHEST, LOWEST AND AVERAGE PARTICIPATING PREMIUMS CHARGED BY 15 CANADIAN COMPANIES.

AMOUNT OF INSURANCE, $1,000. AGE, 35.

System.	Highest.	Lowest.	Average.	Excess of Highest over Lowest.
Whole life.....	27.15	24.84	25.97	2.31
10-payment life.....	54.20	46.27	51.31	7.93
15 " "	41.20	35.62	38.74	5.58
20 " "	35.00	31.55	32.94	3.45
10-year endowment..	107.00	97.80	101.61	9.20
15 " " ..	68.70	62.00	65.13	6.70
20 " " ..	50.45	44.75	47.61	5.70

It is a notable fact that all of the above "highest" premiums and all of the above "lowest" premiums (with a single exception) are the premiums of two companies only; and it is most significant that the company whose rates are the "highest" obtained in the year 1898, in Canada, a larger amount of new general business than any other company operating there. I point out this fact simply to illustrate that this company's high premiums did not prove an obstacle to the securing of an influential volume of business.

The defect between the two extremes is considerable and is equivalent to an annual reduction from the "highest" rates of the following percentages for each $1,000 of insurance, which represent a very fair dividend to the assured by way of permanent reduction of the premium :

Whole life by annual premiums..... 9%

10-payment life..... ....15%	10-year endowment..... 9%
15 " " 14%	15 " " 10%
20 " " 10%	20 " " 11%

The present values of these defects on a 4 per cent. interest basis, and allowing for the life contingency are as follows :

Whole life by annual premiums.....$39.73

10-payment life.....$64.28	10-year endowment ... 74.58
15 " " 60.61	15 " " 72.78
20 " " 44.71	20 " " 73.87

These amounts represent the actual value to the assured of the difference between the "highest" and the "lowest" premiums for each $1,000 of insurance for age at entry, 35.

Inasmuch as both companies are practically organized on the same basis—both having a capital stock; both reserving at the same rate of interest 4 per cent. and on mortality tables (Institute of Actuaries' H^M and Combined Experience), which bring out practically the same values, it will be of interest to examine how the "highest" and the "lowest" gross premiums compare with the net H^M 4 per cent.

AMOUNT OF INSURANCE, $1,000. AGE, 35.					
System.	Net.	Highest.	Loading.	Lowest.	Loading.
Whole life	19.69	27.15	7.46	24.84	5.15
10-payment life....	41.77	54.20	12.43	46.27	4.50
15 " "	31.18	41.20	10.02	35.62	4.44
20 " "	26.13	35.00	8.87	31.55	5.42
10-year endowment	84.90	107.00	22.10	97.80	12.90
15 " "	53.61	68.70	15.09	62.00	8.39
20 " "	38.70	50.45	11.75	44.75	6.05

I think it must be abundantly clear that, while the "highest" premiums are loaded fairly stiffly, the "lowest" premiums are defective for participating insurance.

The new rates of Canadian life companies, at age 35, for each $1,000 of insurance, will probably approximate as follows:

System.	Premium.	Excess over present Average premiums.
Whole life.....	27.70	$1.73 or 7 per cent.
10-payment life.....	57.50	6.19 " 12 "
15 " "	42.50	3.76 " 10 "
20 " "	36.00	3.06 " 9 "
10-year endowment....	104.55	2.94 " 3 "
15 " "	68.10	2.97 " 5 "
20 " "	50.55	2.94 " 6 "

The rate of interest to be employed in the calculation of revised rates will undoubtedly be 3½ per cent., but what table of mortality? Will the Mixed H^M Table of the Institute of Actuaries

be employed, or the more scientific Select Life Table of Dr. Sprague? The latter is, without doubt, the best measure of the risk, is held in high esteem by actuaries, and has been freely used by the British life companies. At the younger ages, the premiums brought out by it are higher than the corresponding premiums according to the Mixed Table, which are well known to be too low. It is recommended that the Select Table be employed.

The following paper was presented by Mr. JOHN B. LUNGER.

NEW METHOD OF APPROXIMATION.

ARTHUR HUNTER.

NEARLY all Actuaries have at one time or another been in need of some means whereby they could pass from a table of mortality at a certain rate of interest to the same table of mortality with a different rate of interest. For instance, it often occurs that the value of a policy on the Actuaries' 3 per cent. basis is required where the commutation and other values on the Actuaries' 4 per cent. basis only are at hand. The method which I am about to describe gives a means whereby this can be approximately done. The circumstances under which the discovery arose are as follows :

The New York Life Insurance Company has a "Select" table of mortality based upon the Company's own experience, which shows the effect of selection. Commutation columns are theoretically required under this Select table for each age at entry, but for practical purposes each fifth age has been computed. These commutation columns are based upon 4 per cent. interest. A short time ago it was desired to obtain certain values based upon 3 per cent. and also upon 3½ per cent. interest. As these were required in a hurry, it was impossible to construct the necessary commutation columns. A suggestion was made that the approximate value at a certain age might be obtained by taking the value by the American 3 per cent. experience, and finding at what age the corresponding value appeared on the 4 per cent. table and then using that age under the Select 4 per cent. table. Thus at age 30 the annual premium for a whole life policy on the American 3 per cent. basis is $18.28 per $1,000, which is the premium for a similar policy on the American 4 per cent. at age 34 ; consequently age 34 on our 4 per cent. Select table was used as the approximation for age 30 on the 3 per cent. basis. This method worked satisfactorily for whole life policies, but under endowment assurance it was not so good. It had the drawbacks that fractional ages had to be employed to obtain accuracy, and that the age had to be changed for each kind of policy. For example : under a single premium policy the age corresponding to 30 on the 3 per cent. basis is 42 on the 4 per cent. basis, while on the whole life plan it is 34.

A number of experiments were then made by Mr. Frank B.

Lucas and myself, in order to find a method which would be more satisfactory, with the result that a very serviceable rule was evolved. The idea of this rule was suggested either by Mr. Lucas or myself, but, as it did not appear to be of much moment, neither of us was particular to notice who made the first suggestion in its crude shape. It occurred to me, however, that the idea was capable of development, and this led me to make the investigations which are herein presented.

As the rule is a very simple one, I will give it at once : Take the difference between the values at the desired rates of interest under any table where these values can be readily found and deduct or add it to the value on the table under which only the value at one rate of interest is known. An example will make this clear : Suppose we desire to obtain the single premium on the Actuaries' 3½ per cent. basis when we only have the value on the Actuaries' 4 per cent. basis. As we have the single premiums on American 3½ per cent. and 4 per cent., that table will suit our purpose. The single premium at age 40 on American 3½ per cent. is $410.03 per $1,000, and on the 4 per cent. basis, $367.58, a difference of $42.45. The single premium on the Actuaries' 4 per cent. basis at age 40 is $381.04. Adding the $42.45 to the single premium, $381.04, gives $423.49, whereas the actual single premium on the 3½ per cent. basis is $423.24. The approximation and the actual value are so very close that it looks as if the example had been specially chosen, which was not the case.

As our first experiments were made on the difference between 3½ per cent. and 4 per cent., I shall make that my first Exhibit. The differences between the single premiums, annual premiums, and annuities at 3½ per cent. and 4 per cent. are given under five different tables : The Actuaries' was selected because it is the standard in the United States ; H^M, because it is the standard in Britain ; American, because it is used as an office standard in the United States ; Sprague's Select, because it was desirable to test the work on a select table ; and the Carlisle, because it was constructed from population returns. Surely these five tables represent all the conditions which are apt to face an Actuary with the exception of abnormal mortality, which will be treated later. The Exhibit shows the values at ages 20, 40, 60 and 80. These ages give a sufficiently good indication of the approximate accuracy of the method. I may mention that tests were made for quinquennial ages, from 20 to 80, inclusive.

Even though my method may not be used by a number of Actuaries for actual computation of reserves, etc., it is most valuable in checking errors in work. A number of mistakes in calculation were found by using this method, including an error in Meech's tables.

The variations between the differences of the single premiums, annual premiums, and life annuities have been given in Exhibit I. in order that the extent of the variation may be judged.

The variation of the differences of the single premiums at age 20 is 62 cents. As the single premium at that age on the American 3½ per cent. basis is $284.97, the maximum error would not amount to more than ¼ of 1 per cent. At age 60 the variation is 83 cents, being barely 1-7 of 1 per cent. of the single premium at that age, $626.92. The results are nearly as accurate for the annual premiums. For instance, at age 20 the variation is only 5 cents, while the premium is $13.48, and at age 60 the variation is 15 cents, while the premium is $56.83. The variation at age 20 under the life annuities is 7 cents and the annuity at that age $20.14, while at age 60 the variation is only 3 cents on an annuity of $10.03.

In case it may be thought that there is not an appreciable difference between the single premiums under any of the above tables, I may state that at age 20 the largest single premium of the five tables at 3½ per cent. is $303.18 and the smallest $281.59, a difference of $21.59 against a variation in the differences of the single premiums of only 62 cents. At age 60 the highest single premium is $635.17 and the lowest $625.88, a difference of $9.29 against a variation in the differences of the single premiums of 83 cents. The difference between the largest and the smallest annual premiums at age 20 is $1.46, and at age 60 $2.30, on the 3½ per cent. basis, while the variations in the differences of the annual premiums are 5 cents and 15 cents, respectively. The differences between the largest and the smallest life annuity is 64 cents at age 20 and $1.24 at age 60, on the 3½ per cent. basis, while the variations in the differences on the life annuities are 7 cents and 3 cents at these ages, respectively. I made similar tables in a number of other cases in order to prove that the usefulness of the method of approximation is not in any way diminished by the fact that the various values according to one table of mortality are not greatly different from those according to a different mortality experience.

The first Exhibit convinced me that the approximation method would be used by many American Actuaries if it applied to tables at higher and lower rates of interest. I accordingly proceeded to make a similar table to Exhibit I., which showed the differences between single premiums, annual premiums, and life annuities at 3 per cent. and 3½ per cent. interest. It proved that the method was quite as accurate for 3 per cent. and 3½ per cent. as it was for 3½ per cent. and 4 per cent. Space does not permit me to give the table.

The question naturally arose—does this approximation method hold good for higher rates of interest? In order to prove this, the differences were obtained between 5 per cent. and 6 per cent. under single premiums, annual premiums, and life annuities. As the matter is not of great importance at these rates of interest, only three typical tables were selected—Actuaries, H^M, and Carlisle. These are given in Exhibit II.

The differences were so very close, although slightly more irregular than under Exhibit I., that it appeared advisable to determine whether or not there was a greater divergence as the rate of interest decreased. In other words, the less the interest affected the rate of mortality the greater would be the variation in the differences. There is no published table based upon an American experience which shows the values on a 2½ per cent. interest assumption, and therefore I was forced to use tables which are employed in Great Britain and France owing to the lower rates of interest prevailing in these countries. The next Exhibit (III.) accordingly shows the difference between single premiums, annual premiums, and annuities on 2½ per cent. and 3 per cent. basis under H^M, $H^{M(5)}$, and the French experience, commonly known as A. F. It will be generally conceded that the variations in the tables are unimportant.

It may be advisable to mention that tests were made in all the foregoing cases for quinquennial ages from 20 to 80, but only vicennial ages are given as they were sufficient to demonstrate the correctness of the method.

Before introducing the subject of joint life values, and also reserves on single lives, I show in Exhibit IV., the results of the investigation, under the same three heads—single premiums, annual premiums, and life annuities—for abnormal rates of mortality. For the investigation such tables were chosen as could be applied without a great deal of labor. The tables are:

(1) Actuaries; (2) English Life Table No. 3, which shows mortality slightly in excess of that according to the Actuaries'; (3) Experience of the New York Life Insurance Company within the Tropics, which shows an average mortality of about 150 per cent. of the Actuaries' q; (4) a table based on double the Actuaries' q; and (5) a table based on three times the q of the H^M table, as given in the Institute Text Book. In order to see the matter in a broad light, the differences for the single premiums, annual premiums, and life annuities, have been taken with 3 per cent. and 4 per cent. interest. At the younger ages the variations in the differences are very small indeed, but at the higher ages they are considerable. The differences are thoroughly consistent, and decrease, except at the young ages, with the increasing rate of mortality. For instance, at age 60 the differences of the single premiums are as follows, taking them in order of the tables: $74.71, $73.80, $67.41, $59.40 and $50.03. When we come to annual premiums the variations in the differences are very small indeed. At age 20 the variation is only 19 cents; at age 40, 13 cents; and at age 60 14 cents, showing that the approximative method will apply for annual premiums under almost any and every condition. The variation in the differences between the life annuities is quite considerable, but is in accordance with the variation of the differences in the single premiums.

The fact that there are considerable variations in the single premiums and life annuities does not mean that the method cannot be used for high mortalities. It can readily be applied by taking the difference between the annual premiums under the Actuaries' or American table and adding it to, or subtracting it from, the annual premium at the given rate of interest under the desired table. Having obtained the annual premium in this way, the approximate single premium and life annuity may be obtained from Rothery and Ryan's Conversion tables. Thus, at age 60 (which shows a large variation in the single premium) the difference between the Actuaries' 4 per cent. and the Actuaries' 3 per cent. annual premium is $2.70. Adding this to the annual premium under the Double Actuaries' Experience 4 per cent., $92.86, gives $95.56. If we put this value ($95.56) through the Conversion tables it gives a single premium for Double Actuaries' 3 per cent. of $766.41, as against the calculated single premium of $766.52, or a difference of only 11 cents.

If this method applies so well to single lives, the inquiry sug-

gested itself—does it also apply to joint lives? I again selected the single premium, annual premium, and life annuity on the H^M, Actuaries', and Carlisle tables at 3 per cent. and 4 per cent. interest for five sets of ages which represent all practical varieties. The tables are given under Exhibit V.

It will be noticed that the variation at no place is greater than ½ of 1 per cent. on the single premiums, and 1 per cent. on the annual premiums. At the ages which are most used in insurance practice, the variation is very small.

A further development would be the application of the method to last survivorship annuities and insurances. Tables under this form of insurance are very scarce. Only two could be found which answered my purpose, that is, which did not require a great deal of labor in calculating values. A great many cases of joint life and last survivor were tested and it was found that the method gave approximately accurate results for every combination. Values representing various diverse combinations only are given for illustration. (Exhibit VI.)

It did not seem necessary to investigate at great length the accuracy of the approximation for the reserves under the various policies. It appeared to me that they would be nearly correct, because the elements which enter into their composition, namely, annual premium, single premium, and life annuity, had already been investigated with satisfactory results. An Exhibit (VII.) has, however, been constructed showing the difference between reserves on American, H^M, and Carlisle tables at 3 per cent. and 4 per cent. on ordinary life, ten-payment life and ten-year endowment plans.

The variations are so small for all plans that the method must commend itself to Actuaries when we consider that reserves are usually taken to the nearest dollar. Under endowment insurances the variations are remarkably small. This is due largely to the fact that the reserve values of short-term endowment insurances under all tables are very nearly alike.

The last Exhibit (VIII.) shows briefly the difference between reserve values on the ordinary life plan at 3 per cent. and 4 per cent. on American, New York Life Tropical, and a table prepared by taking three times the normal mortality of the Institute of Actuaries' Experience. It will be noticed that these differences decrease in accordance with the increasing mortality. This was to be expected.

The mortality tables which have been used throughout the

Exhibits are taken from experiences in which the age of the insured is not treated in exactly the same way. These experiences show the various values at age next birthday, age nearest birthday, and age last birthday. If tables were taken where the age treatment was exactly the same, there is little doubt that the approximation would be even closer.

The nearest cent has been taken throughout the calculations. It is useless to make any greater refinement, as this paper is intended only to show practical results.

To the actuary who is desirous of obtaining exact results, the foregoing information must be only a curiosity in mathematics. To the actuary who is called upon, at short notice, to give approximate results under many tables of mortality the method will be of the greatest assistance. I am led to believe that there are other lines of actuarial calculation, in which the approximation method will be found to work, and shall be glad if the paper induces others to pursue the investigation in all directions where it may seem available.

EXHIBIT I.

DIFFERENCE BETWEEN SINGLE PREMIUMS AT 3½% AND 4%.

Age.	Actuaries.	(Text Book) H^m	American.	Sprague's Select.	Carlisle.	Variation.	A_x 3½%. American.
20	37.69	37.82	37.20	37.59	37.53	.62	284.97
40	42.20	42.24	42.45	42.40	41.54	.91	410.03
60	35.74	35.86	36.46	36.57	36.01	.83	626.92
80	19.35	18.99	18.42	.....	21.06	2.64	849.97
	DIFFERENCE BETWEEN ANNUAL PREMIUMS AT 3½% AND 4%.						P_x 3½%. American.
20	.84	.85	.81	.80	.81	.05	13.48
40	1.14	1.14	1.15	1.16	1.07	.09	23.50
60	1.31	1.33	1.38	1.40	1.25	.15	56.83
80	1.57	1.53	1.69	.....	1.38	.31	191.56
	DIFFERENCE BETWEEN LIFE ANNUITIES AT 3½% AND 4%.						a_x 3½%. American.
20	1.56	1.59	1.58	1.52	1.55	.07	20.14
40	.97	.96	1.01	1.96	.98	.05	16.45
60	.37	.37	.38	.38	.40	.03	10.03
80	.07	.06	.06	.....	.09	.03	3.44

EXHIBIT II.

DIFFERENCE BETWEEN SINGLE PREMIUMS. 5% AND 6%.

Age.	Actuaries.	H^m	Carlisle.	Variation.	H^m A_x 5%.	
20	38.73	38.54	38.87	.33	187.50	
40	51.96	51.94	50.75	1.21	311.04	
60	53.57	53.71	53.33	.38	534.90	
80	33.81	33.26	36.20	2.94	788.44	
	DIFFERENCE BETWEEN ANNUAL PREMIUMS. 5% AND 6%.					
					H^m P_x 5%.	
20	1.05	1.08	1.03	.05	10.99	
40	1.69	1.70	1.57	.13	21.50	
60	2.24	2.26	2.12	.14	54.76	
80	2.87	2.87	2.56	.31	177.46	
	DIFFERENCE BETWEEN LIFE ANNUITIES. 5% AND 6%.					
					H^m a_x 5%.	
20	2.00	2.02	1.99	.03	16.06	
40	1.37	1.38	1.39	.02	13.47	
60	.60	.60	.64	.04	8.77	
80	.12	.11	.15	.04	3.44	

EXHIBIT III.

DIFFERENCE BETWEEN SINGLE PREMIUMS. 2½% AND 3%.

Age.	H^m	$H^{m\,(5)}$	A. F.	Variation.	H^m A_x 2½%.		
20	57.85	56.67	58.21	1.54	386.71		
40	56.96	56.50	56.77	.45	527.55		
60	42.74	42.46	41.71	1.03	715.48		
80	20.69	20.62	20.03	.66	882.56		
	DIFFERENCE BETWEEN ANNUAL PREMIUMS. 2½% AND 3%.						
					H^m P_x 2½%.		
20	1.11	1.04	1.13	.09	15.38		
40	1.34	1.33	1.36	.03	27.23		
60	1.47	1.45	1.45	.02	61.34		
80	1.55	1.56	1.62	.07	183.29		
	DIFFERENCE BETWEEN LIFE ANNUITIES. 2½% AND 3%.						
					H^m a_x 2½%.		
20	2.11	1.99	2.08	.12	24.15		
40	1.19	1.17	1.16	.03	18.37		
60	.43	.42	.40	.03	10.67		
80	.08	.08	.07	.01	3.82		

EXHIBIT IV.

DIFFERENCE BETWEEN SINGLE PREMIUMS. 3% AND 4%.

Age.	Actuaries.	English Life No. 3.	N. Y. Life Tropical.	$2 \times q_x$ Actuaries.	$3 \times q_x$ H^m	Variation.	A_x 3%. N. Y. Life Tropical.
20	84.12	84.40	85.30	85.95	85.95	1.83	398.31
40	91.10	89.35	87.74	84.76	78.32	12.78	530.18
60	74.71	73.80	67.41	59.40	50.03	24.68	721.29
70	57.59	57.00	49.53	41.31	32.48	25.11	815.72
	DIFFERENCE BETWEEN ANNUAL PREMIUMS. 3% AND 4%.						P_x 3%.
20	1.78	1.75	1.76	1.80	1.94	.19	19.28
40	2.37	2.30	2.35	2.43	2.39	.13	32.87
60	2.70	2.64	2.72	2.72	2.78	.14	75.38
70	2.80	2.72	2.88	2.97	3.11	.39	128.91
	DIFFERENCE BETWEEN LIFE ANNUITIES. 3% AND 4%.						a_x 3%.
20	3.35	3.16	2.80	2.42	1.94	1.41	19.66
40	2.03	1.92	1.63	1.33	1.00	1.03	15.13
60	.77	.76	.57	.40	.26	.51	8.57
70	.37	.37	.25	.16	.09	.28	5.33

EXHIBIT V.

DIFFERENCE BETWEEN SINGLE PREMIUMS ON TWO JOINT LIVES. 3% AND 4%.

Ages.	H^m	Actuaries.	Carlisle.	Variation.	A_{xy} Act. 3%.		
15 & 15	86.26	84.15	84.47	2.11	416.72		
15 & 55	79.32	77.94	79.08	1.38	644.98		
30 & 30	87.84	87.36	85.37	2.47	498.71		
45 & 60	68.79	68.49	69.09	.60	713.60		
60 & 60	60.40	59.98	60.46	.48	763.17		
DIFFERENCE BETWEEN ANNUAL PREMIUMS ON TWO JOINT LIVES. 3% AND 4%.							
					P_{xy} Act. 3%.		
15 & 15	1.85	1.64	1.68	.21	20.81		
15 & 55	2.64	2.55	2.53	.11	52.92		
30 & 30	2.12	2.10	1.93	.19	28.98		
45 & 60	2.68	2.65	2.52	.16	72.57		
60 & 60	2.75	2.74	2.35	.20	93.86		
DIFFERENCE BETWEEN LIFE ANNUITIES ON TWO JOINT LIVES. 3% AND 4%.							
					a_{xy} Act. 3%.		
15 & 15	2.83	2.67	2.64	.19	19.03		
15 & 55	.96	.93	.99	.06	11.19		
30 & 30	1.93	1.91	1.85	.08	16.21		
45 & 60	.61	.61	.65	.04	8.83		
60 & 60	.42	.41	.44	.03	7.13		

EXHIBIT VI.

DIFFERENCE BETWEEN SINGLE PREMIUMS ON LAST SURVIVOR. 3% AND 4%.

Ages.	H^m	Carlisle.	Variation.	H^m $A_{\overline{xy}}$ 3%.			
15 & 15	77.04	76.88	.16	200.49			
15 & 70	82.74	82.39	.35	290.87			
30 & 30	91.29	90.42	.87	290.44			
45 & 65	94.59	94.93	.34	490.39			
60 & 60	89.63	90.39	.76	584.28			
DIFFERENCE BETWEEN ANNUAL PREMIUMS ON LAST SURVIVOR. 3% AND 4%.							
				H^m $P_{\overline{xy}}$ 3%.			
15 & 15	1.88	1.86	.02	7.30			
15 & 70	1.84	1.77	.07	11.45			
30 & 30	2.35	2.31	.04	11.92			
45 & 65	2.84	1.79	.05	28.03			
60 & 60	3.28	3.19	.09	40.93			
DIFFERENCE BETWEEN LIFE ANNUITIES ON LAST SURVIVOR. 3% AND 4%.							
				H^m $a_{\overline{xy}}$ 3%.			
15 & 15	4.66	4.62	.04	26.45			
15 & 70	3.76	3.66	.10	23.35			
30 & 30	3.54	3.55	.01	23.36			
45 & 65	1.79	1.87	.08	16.50			
60 & 60	1.13	1.22	.09	13.27			

EXHIBIT VII.

ORDINARY LIFE.

DIFFERENCE BETWEEN TERMINAL RESERVES. 3% AND 4%.

FIFTH YEAR.

Age.	American.	H^m	Carlisle.	Variation.	${}_nV_x$ 3%. American.		
20	8.46	8.79	8.31	.48	37.73		
40	10.84	10.65	10.22	.62	83.54		
60	10.28	9.93	9.81	.47	169.58		
			FIFTEENTH YEAR.				
20	25.21	25.48	23.82	1.66	132.94		
40	26.88	26.02	26.10	.86	278.40		
60	18.83	18.32	18.41	.51	474.71		
			TEN-PAYMENT LIFE. 3% AND 4%. THIRD YEAR.				
20	26.11	26.41	25.95	.46	101.35		
40	27.12	27.20	26.72	.48	146.05		
60	19.20	19.20	19.72	.52	200.33		
			SEVENTH YEAR.				
20	61.90	62.42	61.15	1.27	254.85		
40	62.61	61.56	62.22	1.05	366.86		
60	42.26	42.23	44.17	1.94	503.08		
			TEN-YEAR ENDOWMENT INSURANCE. 3% AND 4%. THIRD YEAR.				
20	9.48	9.51	9.49	.03	261.91		
40	9.42	9.86	9.36	.50	260.41		
60	8.88	9.00	8.83	.17	249.31		
			SEVENTH YEAR.				
20	10.61	10.63	10.62	.02	659.80		
40	10.62	10.57	10.69	.12	657.47		
60	10.56	10.76	10.66	.20	636.90		

EXHIBIT VIII.

ORDINARY LIFE.

DIFFERENCE BETWEEN TERMINAL RESERVES. 3% AND 4%.

FIFTH YEAR.				FIFTEENTH YEAR.			
Age.	American.	N. Y. Life Tropical.	$3^x\ q_x$ H^m	Age.	American.	N. Y. Life Tropical.	$3^x\ q_x$ H^m
20	8.46	8.29	6.97	20	25.21	24.04	21.69
40	10.84	10.11	9.43	40	26.88	24.26	20.59
60	10.28	9.15	9.13	60	18.83	16.16	11.27

Note.

Valuation and Surrender Values.

E. W. SCOTT.

"WHETHER it be a note merely or an article of greater length," our Secretary writes, in soliciting papers to be sent in for the "Decennial Papers."

Of course, being an inhabitant of a small country, my paper will be of the same size, but I would not fail to send from "the old country" a warm greeting to my colleagues and fellow-members of the new one.

* * * * * *

I wish I could come across for the 18th of May! But a "note" you shall have! And a note from Holland always means something in favor of free trade and free competition, and measures to be taken in order to promote independent enterprise, combined with strength of growth. I should like to say a few words on Valuation and Surrender Values.

I am very well aware that this subject is not new, on the contrary it has been considered so many times and under so many different lights that there is scarcely anything left to say; but, nevertheless, I attempt to present a few remarks, because it is of almost overwhelming importance at this time and has so great influence in matters of competition.

It is, of course, the safest policy to adopt the same table of mortality used for the calculation of the premiums, as for the valuation of the premium reserve. But it is quite an impossibility at the present time to possess from the outset the net premium reserve, thus calculated. Too many expenses have to be disbursed; too much work has to be rewarded. And the work is hard—we know all about that at this side of the water also. The gross premium method would do, but, of course, leaves nothing for meeting future expenses and is, therefore, of no practical use. Net premium method leaving the company without the necessary funds for managing the business, gross premium method leaving the assured insufficiently protected, there must be sought for an intermediate method which protects the company and the assured as well, in the same measure. And, indeed, there is no

relax in seeking for the solution of this great problem, and the insurance literature of every country largely contributes to this. Here are my personal views :

The premium must provide for four items : (*a*) the risk, (*b*) the initial expenses, (*c*) the expenses for the second and further years, (*d*) profits. If the reserve is valued at net premiums, *b* is wholly left out of account and considered as a remaining debt of the company, while, in fact, *b* is paid, entirely paid off. Accordingly the reserve is too high and remains so (however, every year less considerably) till the end of the assurance. If the reserve is valued at gross premiums, *c* and *d* (which must be set aside for the future) are considered as already entirely paid, thus leaving nothing but *a*. In my opinion the only fair and reasonable mode, theoretically as well as practically, is to deduct from the gross premium *c* and *d*, and to make the valuation with the remaining *a* and *b* portions.

This mode gives a much more true, complete and honest photography of the position of a company—by far more than any other mode. And, moreover, the reserve resulting from this method of valuation gives a much more reasonable measure for calculating surrender values which, then, should not amount to much less than the office values. This is more equitable to the assured.

This method never gives a loss, neither to the company nor to the assured, in any case, provided the actual expenses do not exceed the equivalent loading. It is, in my opinion, the only fair one equitable to both parties.

* * * * * *

Amsterdam, 31st March, 1899.

NOTE.

J. H. NITCHIE.

THE accompanying tables show, by grouped ages, the exposures and expected and actual mortality during a single calendar year, in a company* having no recently acquired business, the policies being mostly for the term of life, about 40 per cent. premium-paying, the remainder paid-up.

Table I. shows the total amount of insurance exposed and the corresponding expected and actual mortality, by the Combined Experience Table.

Table II. shows the net "amount at risk" (in excess of the 4 per cent. reserve) and the corresponding net expected and actual mortality, or "cost of insurance."

It will be noticed that the rates $\left(\frac{d_x}{l_x}\right)$, based on the totals of these tables, are respectively .039785 and .035043, indicating an average age of 63⅔ for the exposures in Table I. and 62 for Table II.

It may be of interest to compare also the proportions of the several age groups as shown by the two tables, indicating the effect of the heavy reserves at advanced ages.

The actual mortality is also shown.

Ages.	TABLE I.			TABLE II.		
	Exposures.	Mortality.		Amount at Risk.	Cost of Insurance.	
		Expected.	Actual.		Expected.	Actual.
38–50	$131,094	$1,888	$914	$76,735	$1,103	$486
51–55	344,729	6,744	6,600	188,568	3,676	4,219
56–60	389,571	10,362	9,000	191,222	5,075	3,802
61–65	468,909	17,950	17,367	199,600	7,626	7,398
66–70	223,920	12,348	15,885	85,292	4,700	7,048
71–75	116,364	9,458	4,102	38,576	3,126	1,153
76–80	58,928	7,371	1,008	16,385	2,019	186
81–88	21,407	3,700	7,794	4,268	732	2,184
Totals	$1,754,922	$69,821	$62,670	$800,646	$28,057	$26,476

*National Life Ins. Co. of the U. S.

Abstract of the Discussion of Papers Read at Previous Meetings.

General Principles. Emory M'Clintock.

Page 283, Vol. V.

Mr. McClintock: There are one or two points in which this paper of mine requires a little amplification, in view of misunderstandings which, judging from items in the public press, and what I have heard from individuals, have arisen in regard to my views.

Referring to Dr. Sprague's suggestion that the reserve on life policies should begin to be accumulated out of the second premium, I said: "There is much to be said for this proposition, particularly in the case of companies newly organized, which have no assistance from any of the sources of miscellaneous profit, which an established company possesses, such as annuities and non-participating insurances, and no penalties contributed by retiring policy-holders toward the expense of replacing the discontinued risks." Then, after speaking of one effect of the State valuation system, namely, the universal slaughter of small companies, which took place about twenty-years ago, I said, "What is the remedy? The old life companies do not need one, and yet there should be no legal distinction between them and the newer organizations. As regards the latter class, at any rate, the remedy is plain. Make no attempt to enforce State valuations," etc. My general advice was contained in two words—"Freedom" and "Publicity."

My friends here all understood that this statement concerning the governmental system of valuation, and the suggestion that the smaller companies might be benefited if they had the privilege of adopting Dr. Sprague's system, was just what it purported to be, viz., a reference to the system of State valuations. But I find that in some quarters it has been misunderstood; that I have been quoted as favoring this system for use by any and every company; and in one instance a newspaper has published figures purporting to relate to policies issued by my own company, based on the system of beginning the reserves at the end of the second year, as if I had intended to refer to my own company in this matter, or to recommend that my own company or any other one company or class of companies should adopt that system. On the contrary, my language was that the old companies did not need a remedy. I guarded my statement in that way, so that I should not be misunderstood as intending to propose that this system should be adopted by the old companies. Such a thing was absolutely foreign to my thought, and I take this opportunity of making it clear.

Another point: there is no subject so important at the present time as the growing practice of promising to policy-holders that,

beginning after a certain period, they shall have at intervals the privilege of withdrawing the entire reserve and surplus then held by the company as derived from their premiums. I take this opportunity of placing on record a warning on this point, where it may be referred to at any time. The practice which I deplore consists in promising to pay at any time after a given period—say, fifteen or twenty years—the entire reserve held by the company as a guaranteed surrender value, and in addition thereto, every five years, or perhaps every year, as the case may be, the entire surplus in the hands of the company, to be paid over likewise as an additional element of the surrender value. This practice is neither more nor less than the abolition of the customary penalty upon discontinuance. I venture to repeat a few sentences from the paper now before us: "We have seen that a popular life company need not lose vitality through lapsing. When it incurs disfavor, and then only, it suffers loss in that way. Can it not afford to rely on the continuance of approval, and make itself still more popular by abolishing the penalty, even though it does admittedly grant a valuable option without consideration, and so give something for nothing? I answer no, for two reasons. First, it has no right to hazard its permanent existence on the rash assumption that its operations through all future time can never meet a temporary check. A life company has too many invaluable interests in its charge to take wantonly the risk of disaster and bankruptcy arising from the speedy disintegration which, in the absence of penalties upon discontinuance, must occur whenever the cloud of distrust may envelop it in consequence of some temporary mismanagement or misfortune. And, in the second place, we must recollect always that no popular life company has ever yet actually experienced the effects of the abolition of all penalties. The penalty in some form has heretofore stood as a barrier against the assaults of rival agents. Can any one suppose that the life companies could successfully have preserved in force their due proportion of good risks, against all the interested and insidious assaults of rival agents, if there had been literally no penalty upon lapsing; if, for example, every policyholder could have claimed at will, at any moment, his supposed technical proportion of the company's funds? No, you cannot presume that mere general popularity can protect you against rival agents if you impose no penalty; and, apart from that, you have no right to stake the future existence of your company on the presumption that temporary misfortune or mismanagement can never possibly occur."

Sometimes it happens that a company is compelled by competition or by law to promise surrender values on such a scale as to impose too small a penalty upon discontinuance. In such a case it appears to be the duty of the actuary to arrange for a reserve large enough to cover both the guaranteed value and the requisite surrender charge. On this point I shall take the liberty to quote from a paper of mine published twenty-eight years ago in the *Insurance Times*: "Where the surrender value of a policy has been

fixed by previous contract, the company should, year by year, make the reserve large enough to cover (with the aid of such undivided surplus as may pertain to the policy) both surrender value and surrender charge." I considered this statement so important that I repeated it word for word in a subsequent article published in 1876 in the same journal.

In the paper before us I dwelt at some length upon a class of policies providing for surrender without loss at one period only in the history of the policy—say, for example, twenty years after date. Contracts of this sort are outstanding in great numbers. They usually attract a specially good class of risks, but at the end of the twenty years the majority of the best risks withdraw, so that those remaining are understood to be below the average. One reason for this, apart from the appeals of agents, is that there has been a general education of the policy-holders to the belief that they are to expect to have their policies mature, as it is called, at the end of a given period. They regard these policies as endowments, and they are looking forward to their maturity to get the results of their investments. At the end of the twenty years the majority of the best risks withdraw, so that those remaining are understood to be below the average. The loss for the period may be, and doubtless is, balanced by the gain before the period, but it is clear that a reserve must be held against the expected loss. It has been usual either to contemplate the eventual formation of such a reserve, or to proceed farther and actually set it aside. There are certain advantages in providing beforehand, by contract, for the holding of whatever increased reserve may be considered necessary. As this form of contract is expressly declared as intended to enable the policy-holder to take with him at the given period the entire reserve and the entire surplus, fair dealing seems to require that whatever reserve is necessary should be stipulated beforehand.

In fact, we may regard a contract providing for the payment of the entire reserve and surplus at the end of twenty years as a twenty-year contract, so much term insurance and so much endowment insurance, to which is added the extraordinarily valuable privilege of leaving the endowment money in the company's hands and obtaining in lieu thereof a paid-up life insurance policy without examination. For example, if the policy be for $10,000, and the stipulated reserve $5,000, the contract may be considered to consist of an endowment insurance of $5,000 and a twenty-year term insurance of $5,000, with the further privilege of leaving the cash endowment in the hands of the company and obtaining in lieu thereof a paid-up life insurance of $10,000 without examination.

It may be replied that it is not necessary to regard it in this way. To this I say again, that if a contract can be looked at from two sides, we are bound to take it on its worst side, under penalty of being self-deceived. As a matter of fact, the holders of these policies do regard them as endowments, and those who choose to continue their insurances form the exception to the rule. Experi-

ence, so far as it goes, teaches us that they constitute a bad class of risks. It would appear, therefore, that the path of safety lies in framing these contracts, which custom at present requires us to frame, in such a way as to rob the option of its terrors. This way is easily found by increasing the guaranteed surrender value, called the reserve, so as to make this reserve sufficient to indemnify the company for the poor bargain which it makes in continuing the insurances upon those who claim that privilege. Thus, for example, instead of $10,000 insurance and $5,000 reserve at the end of twenty years, it may be desirable to provide for $5,500 reserve. To do this we must increase the net premium by the amount necessary to make up the increase of $500, regarded as a pure endowment.

MR. NICHOLS: As I understand Mr. McClintock, his theory is that the policy-holder, in the case of policies that he is speaking of, should in some form be made to pay the penalty of withdrawal. What I wanted to learn from Mr. McClintock is, whether I am correct or not in the assumption that that penalty is paid by the policy-holder by the reduction of his dividends during the period.

MR. MCCLINTOCK: The contract implies that the policy-holder shall receive the entire surplus accumulated, and also the entire reserve, in case of withdrawal. If the reserve is larger, the surplus will be smaller, the sum of the two being unchanged. I feel sure that I did not suggest that there should be a violation of the contract, which imposes no penalty at that time. My point is that a contract which says that at a particular period a man can withdraw without penalty is a contract which really terminates at that period, beyond which there is an extension which is at the option of the policy-holder. Now, if that option is a burden upon the company in the way of an average mortality worse than the table, my remark is that it is desirable for the company, in framing the contract, to make the endowment portion of it large enough to provide a sufficient reserve in the company's hands against the insurance continued afterward. As I said, by way of illustration, it is better to hold $5,500 of reserve, if you are having an under-average class of risks, than it is to hold the bare $5,000, which is the ordinary tabular reserve for the continued insurance.

MR. CRAIG: I would ask Mr. McClintock whether the extra reserve that he speaks of is not something indefinite—that is, whether he has positive data upon which to base a specific methematical amount?

MR. MCCLINTOCK: The suggestion I made is that the company would do well, seeing that the policy-holder has an option at his disposal, without an examination, to take life insurance at net cost, according to the ordinary tables, to add something to the single premium of ordinary tables as a farther reserve against that unfavorable option. There is as yet no mathematical certainty about the amount necessary to be added, which is a matter of individual judgment.

INDIVIDUALISM VS. COMMUNISM IN THE CONDUCT OF A LIFE INSURANCE COMPANY. ARCHIBALD A. WELCH.

PAGE 300, VOL. V.

MR. LEE: Mr. Welch says: "Life insurance differs from most other commercial interests in that it *requires* an association of many individual interests. It is impossible to conceive of the ordinary life insurance contract being issued except in connection with a large number of other similar contracts. Association of interests is indispensable to security, and this association is secured in the life insurance company."

In regard to this point, viz.: that life insurance *requires* an association of many individual interests, there can be but one opinion. The aggregation of individual interests is what forms the life insurance company and there can be no security without it.

Mr. Welch says: "The insurance contract, to be of value, cannot stand alone by itself uninfluenced except by the laws of mortality and interest."

Such being the case, it follows that the interest and security of the company, that is, the combined whole, ought always to be considered first, and it is the members who stay and complete the payments on their contracts who are the company. It is not an association formed for a few years and from which any member may withdraw at will, for such a privilege would imperil the safety of the purpose for which the company is formed, and render doubtful its power to fulfil the contracts of those who have no desire to withdraw and wish to carry out their original undertaking, insurance.

Every premium paid by a member has a margin or loading which is intended to provide for expenses over and above those arising from death claims.

A company once formed, there are certain expenses which must be incurred whether all the members remain in the company or not, and if a withdrawing member, ignoring his contract, ceases to contribute towards those expenses, those who remain must necessarily pay his share as well as their own.

Mr. Welch says: "Actuaries have, consciously or unconsciously, espoused the cause of individualism or communism." Individualism demands that each contract shall be considered as the unit of value. It seems to me that any one who has any practical knowledge of the business of a life insurance company must see that the communistic idea is a true one, which throws the interests of all together, making the unit all the individual interests.

I fully agree with Mr. Charlton T. Lewis when he states there could be no such thing as the science of probabilities which is not founded upon a conception of a definite aggregate made up of indefinite parts, and because the aggregate is definite, I do not see that it follows that the individual parts are definite, and while we calculate the reserve on one thousand or on one hundred thousand policies,

it does not follow that there is such a thing as a reserve belonging to an individual policy.

It is in considering this reserve as belonging to an individual policy that the main difference arises between the two systems. If we consider a reserve as belonging to an individual policy, it seems to me that it would be equally fair to assume that policy can stand alone and be able to carry out its contract by itself, but we see plainly that is impossible, and as it is so easily demonstrated that the future requires the communistic idea to carry out the contracts to completion, it must follow this principle is the true one at all times.

The company suffers from the withdrawal of members not only the loss of what they would contribute to the general expenses above cost of collecting premiums withdrawn, but it is held as a truism that, in time, the effect of withdrawals unreplaced will result in a high rate of mortality among those who remain, because the company is left with lives more or less impaired.

The successful conduct of a life insurance company demands that the members remaining in the company and completing their contracts should be protected against these sources of danger, and I agree with Mr. Welch that these factors of danger are not so constant that an algebraic formula can be made which would fit all cases.

Mr. Welch's deduction that the company alone has the moral as well as legal right of deciding how much of its earnings shall be expended for extending and caring for its business, and from what sources it shall appropriate to meet these expenses, seems to me logical and true.

The directors and officers of a company are actuated by the desire to do what is best for the welfare of the company, and must, from their position, be better able to judge what course to pursue than any State officer could be, no matter how able or sincere.

If the State should confine itself to showing the condition and standing of the company from year to year, competition would take care of the rest, and the companies would be saved from many burdensome restrictions, which now, in some States, are severely felt, and which place it out of the power of the companies to take action they know prudence demands.

I will quote what Mr. Sprague said some years ago before the Institute of Actuaries, in considering the subject of government supervision :

"Looking at the matter from whatever point we may, there is no doubt that the whole duty of the government, as regards the supervision of Life Assurance Companies, should be strictly limited to simply insisting on the offices publishing the fullest and most minute details of their working, but leaving them at the same time to conduct their business altogether in the way they consider best. When this is done, public opinion will surely become—as it has even already to a great extent become—the best and the surest controlling power."

MODIFICATION OF MR. M'CLINTOCK'S FORMULA FOR AN APPROXIMATION OF ACCUMULATED CONTRIBUTION TO SURPLUS. GARDNER LADD PLUMLEY.

PAGE 310, VOL. V.

MR. PEILER: To obtain results sufficiently accurate, from the method of approximation suggested by Mr. McClintock, it will sometimes be found advisable to adjust the form to be used according to the different conditions existing under different classes. For this purpose preliminary tests should be made under at least one age in each class and the true results should be recorded by individual years as shown in the illustration. This arrangement facilitates the selection of the form best suited for other ages of the same class.

To illustrate these suggestions the expression $r_1 V_1 + r_2 V_2 +$, etc., of Mr. McClintock's formula (7), page 141, Vol. I., was applied in three examples of different conditions, under the following general assumptions and with the following results, the latter being quoted for the ages 20, 40 and 60, respectively: Am. Ex.; i=.035; e=.015; s, for 10 years,=.25, .20, .15, .10, .15, .20, .25, .20, .15, .10; insurance=$1,000.

First example: Annual life, first ten years. The terms for adjustment were fixed at V_1, V_3, V_6, with the results, —8.62 (error= —.04); —19.55 (error= +.05); —36.77 (error= +.31). The absolute amount of error for intermediate ages may be interpolated for final correction. Errors by Mr. Plumley's modification= +.08, —.11, —.80.

Second example: Twenty-payment life, 11th to 20th year. A series, as suggested by Mr. Plumley, was adjusted on the basis of the terms V_1, V_6, V_{11}, representing the points of the greatest results, with the following scheme:

$$\frac{25r_1+18r_2+12r_3+7r_4+3r_5-2r_7-3r_8-3r_9-2r_{10}}{25} \times V_1$$

$$+\frac{9r_2+16r_3+21r_4+24r_5+25r_6+24r_7+21r_8+16r_9+9r_{10}}{25} \times V_6$$

$$+\frac{-2r_2-3r_3-3r_4-2r_5+3r_7+7r_8+12r_9+18r_{10}+25t_{11}}{25} \times V_{11}$$

Results: +15.04 (error=0), +26.48 (error= —.01), +41.10 (error= —.07). Errors by Mr. Plumley's modification: +.56, +1.00, +9.75.

Third example: Twenty-year annual endowment, 11th to 20th year. True results, +34.25, +34.16, +33.51. Approximation by Mr. McClintock's method may be dispensed with. Errors by Mr. Plumley's modification: +1.86, +3.27, +14.94.

Table showing results from application of Mr. McClintock's formula (7), page 141, Vol. I., for ascertaining contributions to surplus.

n	s_n	t	t_n	r_n	Annual Life, 1st to 10th Year, $r_n V_n$, Age 40	20-Pay't Life, 11 to 20th Year, $r_n V_n$ & $t_{10} V_{11}$, Age 40
1	.1000000	.11850	.1185000	—.1000000	—16.69	— 62.69
2	.1575000	.17025	.1787625	—.0390000	— 5.78	— 22.77
3	.2205000	.22200	.2447550	—.0417375	— 5.43	— 22.65
4	.2894063	.27375	.3168998	—.0446513	— 5.01	— 22.46
5	.2431012	.22200	.2698423	+.0737986	+ 7.00	+ 34.30
6	.1914423	.17025	.2172870	+.0784000	+ 6.11	+ 33.55
7	.1340096	.11850	.1588014	+.0832774	+ 5.12	+ 32.67
8	.2110650	.17025	.2395588	—.0522636	— 2.37	— 18.70
9	.2954910	.22200	.3279950	—.0559322	— 1.67	— 18.15
10	.3878320	.27375	.4246760	—.0598370	— .88	— 17.49
11						+110.88

Mr. Plumley: I have to say in reference to Mr. Peiler's careful examination of the formula suggested as a modification of Mr. McClintock's formula, that the modified formula was devised and used for the approximation of dividend contributions for five-year periods during the early history of a policy. It has not been used for ten-year periods, but within its limits it gives accurate results. As it depends for its accuracy upon the regular increase in the second differences of the reserve values, if these differences deviate too widely from such a law the results will be of no value. If for longer periods than five years approximations are desired, each five-year period can be calculated separately and these results accumulated and combined.

I should add that I have never considered it more than perhaps an interesting reduction to a form involving differences, and valuble for short periods only.

VALUATION OF REVERSIONARY ANNUITIES BY C COLUMNS. M. H. PEILER. PAGE 314, VOL. V.

Mr. Torrey: Mr. Peiler's very instructive paper on the "Valuation of Reversionary Annuities," is especially interesting at the present time from its bearing on the calculation of the premiums for continuous instalment policies, which are becoming so popular, and I shall confine myself strictly to the consideration of his article from that standpoint.

Every actuary whose company bases its premiums on 4 per cent. interest has no doubt had the question arise, as to whether it would not be better to assume a lower rate of interest for the instalment period (both certain and contingent) than for the insurance period. There is also another question which naturally arises,

would it be advisable to assume a lower rate of mortality for the beneficiary than for the insured? The method and formula which Mr. Peiler has presented accomplishes both of these objects, if desired, in a very neat, and, I believe, the easiest possible manner.

In the case of those companies which calculate their premiums on a 4 per cent. basis, and for their regular instalment policies discount the instalment at a lower rate of interest, say 3½ per cent., it is absolutely necessary that Mr. Peiler's plan, or an analagous one, should be followed, and the extra premium to be charged on account of the "contingent instalment" feature should be calculated on the supposition that 4 per cent. interest will be obtained during the life of the insured, and only 3½ per cent. thereafter. If with a 4 per cent. premium the "instalments certain" be discounted at 3½ per cent., and the extra premium for the contingent instalment feature be calculated in the usual way at 4 per cent. *throughout*, we would have upon the death of the insured the inconsistency of the "instalments certain" which are payable first, being based upon 3½ per cent. discount, while the "instalments contingent" would be based on 4 per cent.

In the case of companies which base their premiums and discount the instalments payable under their regular instalment policies at the same rate of interest, it will of course not be necessary to employ Mr. Peiler's method for calculating the extra premium to be charged on account of the "contingent instalment" feature, as the regular formula can be most advantageously used.

The second point mentioned which Mr. Peiler's method renders possible, namely, the employment of different rates of mortality for the insured and the beneficiary, I do not think of any particular importance; in fact, I cannot see any good reason for making any distinction between the insured and the beneficiary in the rate of mortality. The beneficiary under a contingent instalment policy should not be considered in the same category as a regular annuitant, as there is not the same opportunity for selection against the company, and the form of policy, in the great majority of cases, is selected by the insured and not by the beneficiary; therefore, there is little likelihood of being saddled with a class of beneficiaries blest with eternal life, as is the case with regular annuitants.

The chief objection to Mr. Peiler's method is, that it will require the preparation of entirely new commutation columns for each combination of ages, and as this is a work of considerable magnitude, it will hardly pay for the trouble, except in the case of those companies that employ different rates of interest for the insurance and the instalment periods. In such cases it is absolutely necessary that Mr. Peiler's plan, or a similar one, should be followed to avoid being inconsistent. If, however, the same rate of mortality is used for both the insured and the beneficiary, the work of preparing the new commutation columns rendered necessary by Mr. Peiler's method for each combination of ages will be somewhat shortened, as it will only be

necessary to prepare new $C_{1\,\overline{x.y}}$ and $M_{1\,\overline{x.y}}$ columns, according to the table of mortality and rates of interest selected for the insurance and instalment periods. The usual joint life $D_{x.y}$ and $N_{x.y}$ columns for the rate of interest assumed for the insurance period can be used with the following slight modification in certain instances. In the usual joint life, $D_{x.y}$ and $N_{x.y}$ columns the power of v used is that for the *oldest age of the two*, and not for the age of x, while in Mr. Peiler's new $C_{1\,\overline{x.y}}$ and $M_{1\,\overline{x.y}}$ columns the power of v used is always that for $x+1$; therefore, it will be necessary in those cases where the beneficiary (y) is older than the insured (x), to multiply the usual joint life $D_{x.y}$ and $N_{x.y}$ wherever used in the formula by $(1+i)^{y-x}$.

In considering the labor involved in the calculation of Mr. Peiler's $C_{1\,\overline{x.y}}$ and $M_{1\,\overline{x.y}}$ columns it should not be overlooked that they will be of very great assistance in calculating the reserves for the "contingent instalment" portion of the contract, and will therefore in a large measure repay for the labor involved in their calculation.

Mr. Peiler: This method has been found convenient in making calculations for special cases where it was deemed prudent to combine a mortality experience on male lives with one on female lives. It is applicable to contracts other than the continuous instalment contract, and is recommended for general use by several text-books. I understand that some companies have employed the method to determine premium rates for the continuous instalment plan as well as for other plans granting survivorship benefits. Wherever such use of the method is made the construction of commutation columns for quinquennial differences of ages would seem sufficient for practical purposes. It is evident that the powers of v may be applied according to the arrangement adopted by Jones and Chisholm or to that of De Morgan, or, in fact, in any manner convenient. With the modern aids to calculation such tables may be constructed in a comparatively short time, and they will be found serviceable in testing for deficient reserve values somewhat after the manner illustrated in Table II., page 317, Vol. V.

DISCUSSION OF THE PRESIDENT'S OCTOBER ADDRESS.

PAGE 360, VOL. V.

Mr. McClintock: There is an item in the President's address at page 361 referring to the payment of commissions. I quote: "Is it not possible that the better way to treat the difficulty would be to reduce initial expense by paying only a moderate rate of commissions for new business, and perhaps providing for a more liberal and lasting renewal interest?" I have no criticism to make concerning the first part of this sentence. The reduction of ex-

pense, paying only a moderate rate of commissions for new business, is a reform which every one would be glad to see pressed as far as it possibly can be. I refer, however, to the additional hypothetical, or contingent, suggestion of the President—"perhaps providing for a more liberal and lasting renewal interest." If the suggestion is to compensate agents for getting new business by increasing their renewal commissions, it appears to me that the suggestion is not in the direction of reform. Perhaps you will say whether that was your idea, Mr. President?

THE PRESIDENT: My idea was that, rather than pay sixty or seventy per cent. brokerage, I would rather pay a very much lower first commission and give a renewal interest. As to what the steps should be, I do not express any opinion.

MR. MCCLINTOCK: As compensation for obtaining new insurance?

THE PRESIDENT: Yes.

MR. MCCLINTOCK: As to that, I know that there can be two opinions about it, but I am unwilling to have the suggestion noted as a "reform," without raising, at least, a question-mark, to indicate that there is another view of it. The President went on to say, "It would be very difficult for any company to execute such a reform, but in my opinion it would not be at all difficult to do so if all the companies, or even the leading companies, would make the reform at the same time."

It appears to me that the direction of reform in this business is toward paying for work done at the time it is done. I am not now referring to the payment of collection fees to such extent as may be deemed proper for the purpose of making the collections and keeping up the business. I am referring only to such additional amount as may be added to this primary renewal commission for the purpose of compensating the agent for having done the business in the first place. As to that, I do not regard it as desirable to pay a man less than his work is worth to-day, and spread the remainder of the payment over the future. It seems to me that such a practice is uneconomical, and also that it does not conduce to a proper understanding of the position of the company. Where a company owes money for work done, the money ought to be paid, or the item ought to go in as a liability.

MR. FACKLER: I fully agree with Mr. McClintock that the truest economy is to pay immediately for the work that is done. But it seems to me that possibly nothing but experience will show what work has been done. A certain volume of business is brought in, and if it is done under high pressure certainly we know that it is work that is not done, in one sense. It is a common thing, as we know, for agents to give a large rebate, and instead of the persons insured having been really converted to life insurance and impressed with the excellence of that particular company, they have sometimes been brought in only as the result of an excessive rebate, and are not really properly brought onto the list of the company's policy-holders. In other words, the work of the agent has not

really been properly done ; and I suppose that the desire of any of those who would advocate giving a somewhat larger renewal commission, and cutting down in a corresponding degree the first year's commission, is to lessen the temptation to the agent to put business on the books merely for the sake of getting a large first year's commission, and at the same time to do away with the great temptation to make a rebate in order to bring the man in. On the other hand, to make it the agent's interest to fully convert men or fully persuade them of the advantages of the form of insurance on which the application is made, and of the substantial advantages of continuing in that company ; so fully to persuade the man, I say, on these points, that he will remain insured in the company. And it seems as though the greatest incentive to the agent to do that work thoroughly and well is to give him a fair renewal commission. At least, experience would seem to show something of that kind. Possibly the renewal commission need not be extended a very long time. However, as to that, opinions may differ. Much of the good may be effected by limiting the increase of renewal commission to the second or the third year of the policy. But certainly the business that was obtained in the olden times, when commissions were ten per cent. the first year and five per cent. on renewals (the proportion of the renewal commission being vastly greater than now), was often kept up vastly better than the business that is obtained to-day, on a brokerage of nobody knows how little under a hundred per cent., and without any renewal or collection commission to speak of.

Abstract from the Minutes of the Decennial Meeting of the Actuarial Society of America, held in the Directors' Room of the Metropolitan Life Insurance Company, New York, on Thursday and Friday, May 18th and 19th, 1899.

FIRST SESSION.

Thursday, May 18th, 1899, 10.30 A. M.

The meeting was called to order by the President, Mr. Bloomfield J. Miller, who, after the roll call, addressed the Society :

Gentlemen of the Actuarial Society :

For the last time it is my duty, under the rules of the Society, to open our meeting with a presidential address. I propose to point out some dangers and difficulties which I think American Life Insurance Companies will be obliged to meet in the comparatively near future, and to suggest some thoughts which I hope may not be entirely without value, in regard to the general policy which seems to me advisable to adopt to minimize the dangers and mitigate the difficulties. Any opinions that I may express, although embodied in a presidential address, are, of course, to be considered simply in the light of personal, unofficial views and will be entitled to only such courteous consideration as kindred views expressed by other members in their purely personal capacity.

The past ten years has been a period of exceptional prosperity for our American companies. A good average rate of interest has been realized upon investments, little trouble has arisen from adverse or hostile legislation or litigation, and the companies have increased vastly in volume of business as well as in financial strength. It is a period over which we can look with satisfaction and congratulation. Competition, it is true, has been severe, and has somewhat increased the cost of new business. Incidentally, the payment of large commissions on first premiums and the occasional practice of offering bonuses and prizes to agents securing an unusual amount of business, has encouraged the pernicious practice of rebating, and resulted in too high a ratio of lapses among young policies. On the whole, however, very satisfactory results have been achieved for the policy-holders in general; and the extension of the benefits of insurance has greatly promoted the public welfare.

The tendency to the increase of the expenses of securing new business seems to have been checked of late years, but I think there are signs which indicate trouble of a more serious nature in the future than has been experienced in the past.

Enormous sums of money accumulated with astonishing rapidity by our life insurance companies, have, I think, rendered inevitable a competition between companies in the investment of funds, which has necessarily contributed more and more to a reduction in the rates of interest obtainable on desirable securities. I think it safe to say that in the past ten years the current rates of interest on railroad bonds have fallen from about four per cent. to about three and one-half per cent., and the rates obtainable on municipal bonds have fallen from about four and one-half per cent. to three and one half per cent. The rates of interest on bonds and mortgages have easily fallen off one per cent. within the same period, and it is within the truth to say that upon the new investments which the companies are now making, they net on the average nearly one per cent. less than they did ten years ago. I fear that there will be still more shrinkage in the current rates of interest obtainable on new investments in the near future, and that before long the average interest earnings of the companies will be over one per cent. less than they were ten years ago. When it is remembered that the some thirty companies doing business in the United States have assets amounting to about $1,500,000,000, it will be seen that a difference of one per cent. in the average interest earnings of the companies amounts to about $15,000,000 a year. Any diminution in interest rates necessarily carries with it a corresponding reduction in dividend earnings and dividend payments. According to the Connecticut Insurance Reports, the net rate of interest earned by companies doing business in that State fell from 4.91 per cent. to 4.32 per cent. between the years 1889 and 1898. When it is borne in mind that the companies' investments still include many securities yielding high rates of interest which were purchased in past years and are constantly approaching maturity, when the proceeds will probably have to be reinvested at lower rates, the question as to whether the reserve basis for new business should not be raised from four per cent. to three and one-half per cent. or three per cent., becomes well worthy of consideration. If, as is possible, the States may incline to a change in the rate of interest applicable to the computation of legal reserves, which has hitherto been almost uniformly four per cent., I earnestly recommend the members of our Society to use what influence they can to have the American Experience Table of Mortality substituted at the same time for the Actuaries' Table, which has hitherto been generally adopted. I believe it to be a much better index of the laws of American mortality than the English Table.

Decreased rates of interest will have a comparatively more important effect on the dividends earned by policies where the investment feature predominates, such as Single-Premium Policies, 10-Premium Policies and Short Endowments, than it will have in case of policies with smaller premiums, where the insurance element exceeds in importance the investment feature, and I believe it would be good policy for the companies to adjust their scale of commissions so as to favor the issue of low-premium long-term policies

rather than the issue of high-premium investment policies. I believe it desirable to reduce the expenses attached to investment policies in order that the results realized on such policies may continue to be satisfactory to the insured. A good way to do this in my opinion is to name a class of policies upon which the company will pay maximum commissions and to extend these commissions to all policies having smaller premiums, and in case of policies having higher premiums to treat the excess premiums in the light of mere deposits for investment and to pay on such excess premiums a very moderate collection commission which, I think, might well be the same for first premiums as for renewals.

In the construction of new premium rates I should recommend a uniform and quite high loading on Ordinary Life Policies and the same loading in dollars and cents for higher premiums, with an additional and comparatively small uniform loading on the excess of such premiums over the net premiums for Ordinary Life Policies. I have found that very satisfactory and well-graded loadings can be provided for by this simple rule. Arbitrary loadings, such as thirty per cent. on Ordinary Life Policies and twenty-five per cent. on 20-Payment Life Policies, are objectionable because at age 70, for instance, the two classes of policies are substantially identical and the gross office premium for the Ordinary Life Policy at advanced ages might thus exceed the 20-Payment Life rate. A great deal of trouble and inconvenience can be saved, and all inconsistency avoided by the adoption of a uniform rule which will produce a well-graded system of loadings for all classes of policies.

I think I have noticed a growing tendency on the part of the legislatures of the forty-five States to harass and oppress other State companies doing business within their several territories. It has also seemed to me that there was a growing tendency on the part of policy-holders to litigation based on the most frivolous pretexts, and a growing tendency in the courts to favor the policy-holder at the expense of the company, when such a course could be sustained by strained and ingenious arguments. The complicated contracts which the companies are now issuing are especially likely to afford litigants the hope of realizing on an unjust claim upon some little ingenious point which may be magnified by the court to an extent sufficient to overbalance a dozen other points which are of much more real importance to a just decision of the case. It seems to me that our American companies are in danger of introducing too many complications into their contracts, and that simplification rather than complication should be the future policy of the companies. Another thing that makes me apprehensive for the future is that competition for business has recently seemed to take the form rather of a competition in guarantees than of competition in commissions and expenses, and that this has led in the direction of a departure from the principle of net reserve as a safeguard to stability. For nearly forty years the propriety of refusing permission to a company with an impaired net reserve to write new business until the impairment was made up, has been almost uni-

versally recognized by our legislatures and by insurance experts. For my part, I am a firm believer that a company should have intact a net reserve computed upon a conservative basis in order to be entitled to solicit new business from the public. I believe that a solvent company in good standing should charge itself with a liability equal to the net reserve computed on the same basis as that on which its premiums and dividends have been determined, and that surrender values should be at all times somewhat within the policy reserve as so computed. A policy-holder should see the propriety of being subjected to a moderate penalty if he avails himself of the privilege of surrendering his policy before maturity in view of the fact that the company has no such option of cancelling the policy before maturity, even if it should desire to do so. I think that while a very small surrender charge is sufficient to protect a company from damage by withdrawals from its membership, it would be inexpedient, if not dangerous, to invite and encourage withdrawals by the total abolition of the surrender charges.

It is true that highly esteemed and respected members of our Society do not at present believe in the enforcement of the principle of net reserves. Their opinions are, of course, entitled to careful and candid consideration. One of the arguments advanced by them, however, seems to me to be of little practical value. It is to the effect that in some complicated forms of policies the contract would have, at times, a negative reserve if the same were computed upon the usual formulæ, and that in other cases the conditions of policies are so complicated as to make computation of the net reserve so laborious and complex as to be practically impossible. It seems to me that the principle of net reserves is of far more importance than any principle involved in those complicated contracts. If we must give up one or the other, it is my opinion that we had better keep the net reserves and let the complicated contracts go. I can see nothing to be gained in the long run by the introduction of too much irregularity or complication into a policy contract.

I do not wish to be understood as holding any pessimistic views in regard to the future of life insurance companies in America. I have no doubt that the life insurance companies of Canada and the United States will prove themselves fully able to cope with any difficulties or dangers that may arise, and that they will continue to grow in health, strength and usefulness, and will, little by little, correct any comparatively trifling evils which have naturally developed as the result of an almost abnormal growth. I confidently rely on the Actuarial Society of America to exercise wholesome and efficient, if not a controlling, interest on the true welfare of life insurance in America for all time to come.

In conclusion, it is my melancholy duty to announce that since our last meeting another of our Brothers has been called away and has obeyed the irresistible summons that in time will come to each of us. Wm. D. Whiting, who has contributed much to our social pleasures and professional discussions, went to his rest on April 20th, 1899. We shall miss his presence, and, though absent, he

will not be forgotten. His memory is enshrined in our hearts and his personality is indelibly pictured in our "Transactions." A suitable minute of his death will be entered in our "Transactions" of this date.

Officers and members present at roll call and during the sessions:

Bloomfield J. Miller, President; Oscar B. Ireland, 2d Vice-President; Israel C. Pierson, Secretary; John Tatlock, Jr., Treasurer. Other members of Council—David Parks Fackler, Howell W. St. John, Emory McClintock, William McCabe, William E. Starr, John B. Lunger, Elbert P. Marshall, James M. Lee, Thomas B. Macaulay. Members—David Griscom Alsop, Jesse John Barker, Thomas Bradshaw, John Jones Brinkerhoff, Hubert Cillis, James McIntosh Craig, Joseph Arend De Boer, Robert Patterson Field, John Kinsey Gore, Clayton Colman Hall, Robert George Hann, Charles Hildebrand, John Marshall Holcombe, Robert Watkinson Huntington, Jr., Charles Nelson Jones, George Halsey Kirkpatrick, Charlton Thomas Lewis, Charles Alvin Loveland, William Campbell Macdonald, William Andrew Marshall, James Weir Mason, Hiram John Messenger, Walter Smith Nichols, Sydney Norris Ogden, John Sherman Paterson, Maximilian Heinrich Peiler, Gardner Ladd Plumley, Charles Grant Reiter, John George Richter, George Ferry Salter, George White Sanders, Frank Sanderson, Edward James Sartelle, William Simeon Smith, Morris Whittemore Torrey, Joel Garretson Van Cise, George Wegenast, Archibald Ashley Welch, Daniel Halsey Wells, George Badger Woodward. Associates—Charles Hart Angell, Samuel Stickney Hall, William Richmond Hitchins, Henry Nicholas Kaufman, John Francis Roche, Douglas Hall Rose.

The President read a letter from Mr. John R. Hegeman, President of the Metropolitan Life Insurance Company, inviting the members to lunch on Thursday and Friday, and proposing a visit to the several departments of the Company during office hours. On motion of Mr. McClintock the invitation was accepted with thanks and Friday, at 11 A. M., was selected as the hour for inspecting the workings of the Metropolitan office.

The Treasurer presented his report which, having been audited by Mr. Jones, was accepted. The Secretary read a résumé of the history and work of the Society during the ten years just completed. It was voted that this report be included in the minutes. See page 117.

Mr. Nichols presented a minute concerning Mr. William D. Whiting, which was adopted. See page 142.

Mr. Lunger offered the following resolution in acknowledgment of the gift of William D. Whiting's library to the Society. It was unanimously adopted:

"The Actuarial Society of America, at its Annual Meeting on May 18th, 1899, having been informed of the generous bequest of

his library to the Society by Wm. D. Whiting, places on its records the following minute:

"With grateful appreciation and as a fitting memorial of their late esteemed member, this Society accepts the valuable library presented to them by Mrs. Wm. D. Whiting at the request of her husband. The Secretary of the Society is instructed to send to Mrs. Whiting a copy of this minute, and to place in each book in the library a plate containing the following inscription:

"PRESENTED TO
THE ACTUARIAL SOCIETY OF AMERICA,
IN ACCORDANCE WITH THE BEQUEST OF
WILLIAM D. WHITING,
AND ACCEPTED BY THE SOCIETY AS A MEMORIAL OF HIM."

The Society then proceeded to the election of Officers and Members of Council, with this result: President, THOMAS B. MACAULAY; 1st Vice-President, OSCAR B. IRELAND; 2d Vice-President, ISRAEL C. PIERSON; Secretary, JOHN TATLOCK, JR.; Treasurer, JOHN B. LUNGER.

Adjourned at 12.45 P. M.

SECOND SESSION.

THURSDAY, May 18th, 1899, 2.30 P.M.

President Miller in the chair.

The election was continued with the following result: For Members of Council, WILLIAM C. MACDONALD and JAMES M. CRAIG for three years, in place of WM. MCCABE and WM. E. STARR, whose terms expire; WALTER S. NICHOLS for two years, in place of THOS. B. MACAULAY; JOEL G. VAN CISE for one year, in place of JOHN B. LUNGER.

The members then listened to the reading of papers as follows: See pages 9–95.

THIRD SESSION.

FRIDAY, May 19th, 1899, 10 A. M.

President Miller in the chair.

Mr. H. W. St. John offered the following resolution, which was adopted:

"The first Secretary of the Actuarial Society, Israel Coriell Pierson, having completed a decennial term of continuous service and declined a reëlection, and having presented a historical review of this period, in which he has revived, with facile pen, the causes and incidents in the life of the Association, and quickened in graceful terms remembrance of the dead, the members assembled at this, the decennial meeting, desire to unite in expressing herewith, for permanent inscription on its records and as a fitting supplement to this history, their sincere esteem for the character of

Secretary Pierson and their cordial appreciation of his zeal and efficiency in the performance, at home and abroad, of the arduous duties of his office."

It was moved, seconded and carried that the resolution be entered in the minutes and that an engrossed copy be presented to the Secretary.

After the reading of papers was completed the members began the discussion of papers read at the previous meeting. See pages 96–107.

Adjourned at 11 A. M.

During the recess a photograph of the members was taken in the court of the Company's building. See page 141.

FOURTH SESSION.

FRIDAY, May 19th, 1899, 2.15 P. M.

President Miller in the chair.

The following minute was offered by Mr. Oscar B. Ireland and unanimously adopted :

"The members of the Actuarial Society of America tender sincere thanks to Mr. John R. Hegeman, President, and the other officers of the Metropolitan Life Insurance Company, for their generous hospitality, both official and personal, for the use of their beautiful building in which to hold the decennial meeting, and for the many ways in which their taste and liberality have contributed to make the meeting an enjoyable success, both as to professional work and as to the social accompaniments which so pleasantly stimulate and encourage the graver labors of the Society.

"The Secretary is requested to have this minute engrossed and presented to Mr. John R. Hegeman, the President of the Metropolitan Life Insurance Company."

The discussion of papers was continued.

After announcements concerning the Banquet at Delmonico's at 7 P. M., the Society adjourned.

ISRAEL C. PIERSON,
Secretary (*retiring*).

JOHN TATLOCK, JR.,
Secretary,
32 Nassau St., New York.

SOCIAL FUNCTIONS.

At the noon recess on Thursday and Friday the members were entertained at lunch by the officers of the Metropolitan Life Insurance Company.

On Thursday evening, May 18th, 1899, a reception was given to the members of the Actuarial Society of America by the officers of the Metropolitan Life Insurance Company in the Directors' room and Marble Court. The latter was elaborately decorated with

palms and flowers and illuminated with electric lights. Mr. JOHN R. HEGEMAN, President of the Company, assisted by Mr. BLOOMFIELD J. MILLER, President of the Society, received the guests. About three hundred and fifty were present. A musicale followed the reception at nine o'clock.

An elaborate supper was served by Delmonico in the large room of the annex of the building. The room was handsomely decorated with flags of all nations, and the orchestra discoursed music, including "The Star-Spangled Banner," "God Save the Queen" and "Wacht am Rhein."

The formal Decennial Banquet of the Society was held in the Red Hall at Delmonico's, on Friday, May 19th, 1899, at 7 P. M., when the Presidents of the Companies, as guests of honor, and other invited guests, were present. Mr. BLOOMFIELD J. MILLER, the President, presided as toastmaster. The toasts were as follows:

I. { "The President of the United States."
   { "The Queen of Great Britain and Ireland."

II. "The Guests." Mr. JAMES W. ALEXANDER, President Equitable Life Assurance Society.

III. "The Institute of Actuaries and Kindred Organizations." Mr. JAMES CHATHAM, Actuary Scottish Life Company, Edinburgh.

IV. "The Popularization of Life Insurance." Mr. JOHN R. HEGEMAN, President Metropolitan Life Insurance Company.

V. "The Life Companies." Hon. JOHN A. McCALL, President New York Life Insurance Company.

VI. "Germany, the Fatherland of the Anglo-Saxon Race." Herr VON KNEBEL DOEBERITZ.

VII. "Life Insurance a Factor in Civilization." Mr. JOSEPH ASHBROOK, Manager Provident Life and Trust Company, Philadelphia.

VIII. "The Society an International Bond." JOHN L. BLAIKIE, Esq., President North American Life Assurance Company, Toronto.

IX. "The Decennial of the Society." Mr. CHARLTON T. LEWIS.

The President, in making his opening postprandial remarks, referred to Mr. DAVID PARKS FACKLER as being, more than any one else, the founder of the Actuarial Society of America, and asked him to give a brief account of its organization. After the first toast, which was proposed without response, the President introduced in their order those who were to speak to the other toasts. Mr. JAMES CHATHAM, of the Scottish Life, Edinburgh, was appointed as a delegate to the Decennial of the Actuarial Society of America by the Institute of Actuaries, Faculty of Actuaries and Actuarial Society of Edinburgh, respectively. His name was on the list to respond to the toast "Institute of Actuaries and Kindred Organizations." At the last moment word came that he was detained at Jamaica,

West Indies. Messrs. Emory M'Clintock and Thos. B. Macaulay, both Fellows of the Institute, were requested to respond to the toast assigned to him.

At the end of the responses to the toasts, the President called upon the retiring Secretary, who replied briefly, thanking the members of the Society for their kindness and cordial coöperation during his term of office and for the honor conferred by election to the 2d Vice-Presidency.

An orchestra discoursed music during the banquet. Selections by the Lotos Glee Club were interspersed between the toasts. The banquet souvenir was a small imitation volume, "Actuarial Society of America, Decennial Number, May 19th, 1899," which contained the "glaces fantaisies."

Two cablegrams arrived during the evening. They were received with hearty enthusiasm.

Amsterdam, May 19th, 1899.—"Three cheers from little Holland for the success and splendor of the Actuarial Society of America. Sorry we cannot be amongst you all to-night, sons of the Pilgrim Fathers." Schevichaven, Scott, Blankenberg.

[Mr. Scott is a member of this Society. He with the other two gentlemen are managers of the Algemeene Maatschappij van Levensverzekering en Lijfrente, Amsterdam, and members of the First and Second International Congress of Actuaries.]

Brussels, May 19th, 1899.—"Association Actuaires Belges souhaite longue prosperité à l'Actuarial Society of America."

Messrs. Omer Lepreux and Am. Bégault, President and Secretary, respectively, of the Permanent Committee of International Congresses and members of the Belgian Association of Actuaries and of this Society, expected, until the last moment, to sail on May 6th to attend the Decennial.

Secretary's Report,

For the Ten Years beginning April 25th, 1889.

To the Members of the Actuarial Society of America:

At this decennial meeting let us review the first decade of the Actuarial Society of America. The history of organization on April 25th and 26th, 1889, and of development, growth and work since then is recorded for the most part in the five volumes of nearly 2,100 printed pages of the "Papers and Transactions." Prior to ten years ago the actuaries of the United States and Canada were isolated geographically and lacked personal acquaintance. The Institute Journal was at command and occasionally in this country an article from the pen of some one of the prominent actuaries appeared in the insurance journals but the actuaries had no common medium or opportunity for the exchange of views and discussion of actuarial questions.

There had existed for a long while a desire for some kind of a permanent association. Such a thing had been suggested and some futile efforts to secure it had been put forth but the opportune time for making an active movement with certain promise of success did not seem to have arrived until February 1889 when five of the oldest actuaries in the United States almost simultaneously proposed to form an Actuarial Society. Named in alphabetical order, they were Messrs. David Parks Fackler, Clayton C. Hall, Sheppard Homans, Emory McClintock and Howell W. St. John. Mr. Fackler, not being officially connected with any company, was requested by the others to communicate on the subject with "those persons now actually serving as the actuaries or the consulting actuaries of companies." This he did on March 7th, 1889 and solicited opinions as to the desirability of holding a meeting at an early date in New York to consider the question of organization. The responses to his letter having been prompt and unanimously favorable to the suggestion, Mr. Fackler, on March 22d, issued a call for a meeting to be held on April 25th at the Astor House, New York, the visitors to be the guests of the New York actuaries. Accordingly, on April 25th at one o'clock the actuaries met, many as strangers to each other, and participated in an informal luncheon during which and afterward there was opportunity for those present to become acquainted. A little later Mr. Fackler called the meeting to order and nominated for Chairman Mr. Sheppard Homans who, having been chosen, took the chair. The present Secretary was requested to act as Secretary of the meeting. When the roll was called, the following named gentlemen were found to be present:

Messrs. Barker, Cillis, Ellis, Fackler, Hall, Hendry, Homans, Lee, Lunger, Macaulay, McCabe, McClintock, Macdonald, E. P.

Marshall, W. A. Marshall, Miller, Pierson, Phillips, Sanders, Smith, Standen, Starr, Stebbins, Weeks, Wells, Woodward and Wright. Messrs. Ireland and Wing came in at a later hour, making twenty-nine in all.

Mr. Fackler read letters from several actuaries who, though unavoidably absent, expressed themselves as heartily in accord with the movement to organize an actuarial association. He related the story of the origin of the proposal to organize and stated the object of the meeting. Those present were called upon in alphabetical order to express their views as to the desirability of an organization, its name, its object and nature and extent of membership. On all these points there was a remarkable unanimity.

It was resolved to appoint a committee of six, besides the Chairman, to prepare a form of constitution for an actuarial association. That task was committed to Messrs. Fackler, McCabe, McClintock, Miller, Wright and Wells in addition to the Chairman, Mr. Homans.

It having been voted to designate a committee to make nominations for officers, the Chairman named Messrs. Barker, Hall, Weeks, Woodward, Ireland and H. W. Smith.

These two committees were requested to be ready to report at an adjourned meeting to be held on the following day, April 26th, at 10.30 A. M.

Mr. Barker moved a vote of thanks to the Organizing Secretary, Mr. Fackler, and to the Chairman and Secretary of the meeting, which was unanimously carried.

Mr. McClintock said, "We ought to have a good rousing speech from Mr. Fackler. He has done everything himself, every particle of it. It is his own devising and we owe him the vote of thanks we have given." Mr. Fackler replied: "I feel most thoroughly compensated for any trouble I may have had in the sight which is before me and I accept with hearty appreciation your expression of commendation for what I have done."

The meeting then adjourned until 6.30 P. M.

At 7 P. M. the members sat down to dinner. The New York City Actuaries were the hosts of the occasion and Mr. Fackler officiated as toastmaster. The toasts were:

"The Time-tried System of Life Insurance, sound in principle and correct in practice; may it last until death is no more."

Responses by Mr. Sheppard Homans, Mr. Walter C. Wright.

"The Institute of Actuaries. Well have its members paid the debt which, according to its Baconian motto, every man owes to his profession."

Responses by Mr. Emory McClintock, Mr. Thos. B. Macaulay, Mr. William McCabe.

"The Actuarial Profession in America. May association aid and stimulate to earnest individual effort in the interest of life insurance, and may all both deserve and maintain a high professional standing."

Responses by Mr. Howell W. St. John (detained by illness), Mr. Bloomfield J. Miller, Mr. Clayton C. Hall.

Other gentlemen who replied to impromptu calls were Messrs. Wm. Hendry, Geo. B. Woodward, Geo. W. Sanders, Asa S. Wing, Rufus W. Weeks, I. C. Pierson, H. W. Smith and Jesse J. Barker. The postprandial speeches may be found in Vol. I., pp. 33–53. The meeting then adjourned until the following day.

A meeting was held on Friday morning, April 26th, 1889, pursuant to adjournment. The Committee on Organization reported the draft of a constitution which, after being amended, was adopted. (See constitution, pp. 29–30, Vol. I.)

The constitution was then signed by those present and subsequently by the nine others who had expressed their adherence to the proposal of organization.

CHARTER MEMBERS.

J. J. Barker.	James M. Lee.	H. W. St. John.
Hubert Cillis.	C. A. Loveland.	G. W. Sanders.
J. M. Craig.	J. B. Lunger.	Edward B. Smith.
Joseph A. DeBoer.	T. B. Macaulay.	H. W. Smith.
Geo. Ellis.	Wm. McCabe.	Wm. T. Standen.
D. P. Fackler.	Emory McClintock.	Wm. E. Starr.
Robt. P. Field.	W. C. Macdonald.	S. N. Stebbins.
Wm. O. Gould.	E. P. Marshall.	R. W. Weeks.
C. C. Hall.	W. A. Marshall.	D. H. Wells.
Wm. Hendry.	Bloomfield J. Miller.	Asa S. Wing.
J. M. Holcombe.	J. H. Nitchie.	Geo. B. Woodward.
Sheppard Homans.	Geo. W. Phillips.	Walter C. Wright.
Oscar B. Ireland.	Israel C. Pierson.	

(See at end of this report facsimile of the original signatures, pages 137 and 138, and pictures, page 139, of the charter members.)

The Committee on Nominations then made its report which was unanimously approved by the election of the following officers and council:

President: - - - -	Sheppard Homans.
1st Vice-President: - -	David Parks Fackler.
2d Vice-President: - -	Howell W. St. John.
Recording Secretary: -	Israel C. Pierson.
Corresponding Secretary: -	Wm. T. Standen.
Treasurer: - - -	Bloomfield J. Miller.

Other Members of the Council:

Oscar B. Ireland, George W. Phillips,
William McCabe, Henry W. Smith,
Emory McClintock.

Much enthusiasm and a friendly spirit were manifest throughout the proceedings. A resolution of thanks to the officers, moved by Mr. Hendry and seconded by Messrs. Smith and Hall, was

unanimously adopted. Messrs. Fackler and Homans responded.

The meeting adjourned *sine die* at 1 P. M., April 26th, 1889.

This was the inception of the Actuarial Society of America, founded somewhat as an experiment but in confidence that there would be an evolution and certain gradual development.

During the ten years now ended twenty meetings of the Society have been held on the dates and at the places given in Table I. Twenty-nine members were present at the charter meeting, the other nine having been absent unavoidably. Of the attendance at any one meeting, the smallest number was 24, on Oct. 3d, 1889; the largest number was 55, on April 26th, 1894. Four of the members have answered to the roll-call at every meeting, Messrs. Fackler, Ireland, Starr and Pierson. There have been absent once, Messrs. Lee and Macaulay; absent twice, Messrs. Miller and Craig.

The officers and members of the Council have been active and efficient in the discharge of their several duties and zealous in their endeavors to advance the interests of the Society. The Council have held thirty-eight meetings, the first one on June 18, 1889. At that time, although there was no provision in the Constitution for more than one stated meeting of the Society annually, it was decided to call a second meeting which was held on October 3, 1889. This was the origin of the fall meetings which have been since continued and, at least in a social way, one of the most interesting features of the Society.

I. Dates and Places of Meetings of the Actuarial Society of America and Attendance.

	DATE.	PLACE.		NO. PRESENT.
1st.—1889	April 25 and 26	New York	Astor House. Charter meeting.	28
2d.—1889	Oct. 3	New York	Board Room, Home Insurance Co.	24
3d.—1890	April 24	New York	Astor House.	44
4th.—1890	Oct. 23 and 24	Hartford	Senate Chamber of Capitol.	39
5th.—1891	April 29 and 30	New York	Hall of Board of Fire Underwriters.	49
6th.—1891	Sept. 30 and Oct. 1	Toronto	Board of Trade Council Chamber.	33
7th.—1892	April 28 and 29	New York	Hall of Board of Fire Underwriters.	43
8th.—1892	Oct. 13 and 14	Boston	Hotel Vendome.	37
9th.—1893	April 27	New York	Dining Room, Mills Building.	43
	April 28	New York	Adjourned to Hall of Board of Fire Underwriters.	
10th.—1893	Oct. 19 and 20	Philadelphia	Directors' Room, Penn Mutual Life Insurance Co.	43
11th.—1894	April 26 and 27	New York	Directors' Room, Metropolitan Life Insurance Co.	55
12th.—1894	Oct. 11 and 12	New York	Room 303, Mutual Life Insurance Co. Building.	46
13th.—1895	April 25 and 26	New York	Hall of Board of Fire Underwriters.	43
14th.—1895	Oct. 10 and 11	Montreal	Windsor Hotel.	25
15th.—1896	April 30 and May 1	New York	Imperial Hotel.	43
16th.—1896	Oct. 8 and 9	Pittsfield	"The Maplewood."	32
17th.—1897	April 29	New York	Directors' Room, Metropolitan Life Insurance Co.	49
18th.—1897	Oct. 7 and 8	Springfield	Massachusetts Mutual Life Insurance Co. Building.	34
19th.—1898	April 28 and 29	New York	College Room, "The Arena."	40
20th.—1898	Oct. 6 and 7	Worcester	Directors' Room, State Mutual Life Assurance Co.	24

The Constitution and By-Laws have been amended and developed as the circumstances and growth of the Society required, as shown by Table II.

II. Constitution and By-Laws.

1889, April 26. Constitution adopted.
1890, April 24. By-laws adopted.
1890, Oct. 23. Constitution amended, consolidating the offices of Rec. and Cor. Secretary in Secretary; increasing number of elective members of Council from five to six, and making three classes of two each, to serve for three years.
1891, Sept. 30. Constitution amended, making ex-Presidents members of Council *in perpetuo*.
1893, April 27. By-laws amended; members delinquent in dues to be dropped from the roll.
1896, April 30. Constitution amended, providing for Associate Membership. (This was proposed in 1892, but no action was taken.)
By-laws amended; dues of Associates, $5.00.
1898, April 28. By-laws amended; dues of Foreign Members changed to $5.00.
1898, Oct. 6. Constitution amended; date of annual meeting changed to first Thursday after May 14th in each year.

It may be interesting to know who have been the officers and members of Council during this decade. Table III. contains the data.

III. Offices Held During the 10 Years.

		FROM	TO
Sheppard Homans,	President, .	1889, April 26	1891, April 30
David Parks Fackler,	1st Vice-Pres.,	1889, April 26	1891, April 30
	President, .	1891, April 30	1893, April 28
Howell W. St. John,	2d Vice-Pres.,	1889, April 26	1891, April 30
	1st Vice-Pres.,	1891, April 30	1893, April 28
	President, .	1893, April 28	1895, April 25
Emory McClintock,	Councillor,	1889, April 26	1891, April 30
	2d Vice-Pres.,	1891, April 30	1893, April 28
	1st Vice-Pres.,	1893, April 28	1895, April 25
	President, .	1895, April 25	1897, April 29
B. J. Miller, . . .	Treasurer, .	1889, April 26	1893, April 28
	2d Vice-Pres.,	1893, April 28	1895, April 25
	1st Vice-Pres.,	1895, April 25	1897, April 29
	President, .	1897, April 29	1899, May
Geo. W. Phillips, .	Councillor,	1889, April 29	1893, April 28
	"	1894, April 26	1895, April 25
	2d Vice-Pres.,	1895, April 25	1897, April 29
	1st Vice-Pres.,	1897, April 29	*1898, Sept. 27

* Died.

Offices Held—*Continued.*

		FROM	TO
Oscar B. Ireland,	Councillor,	1889, April 26	1892, April 28
	Treasurer,	1893, April 28	1897, April 29
	2d Vice-Pres.,	1897, April 29	1899, May
Israel C. Pierson,	Rec. Sec'y,	1889, April 26	1890, Oct. 23
	Secretary,	1890, Oct. 23	1899, May
Wm. H. Standen,	Cor. Sec'y,	1889, April 26	1890, April 24
Rufus W. Weeks,	Cor. Sec'y,	1890, April 24	1890, Oct. 23
	Councillor,	1890, Oct. 23	1894, April 26
	"	1895, April 25	1897, April 29
John Tatlock, Jr.,	Treasurer,	1897, April 29	1899, May

Members of Council.

	FROM	TO
Oscar B. Ireland,	1889, April 26	1892, April 28
William McCabe,	"	1893, April 28
Emory McClintock,	"	*1891, April 30
George W. Phillips,	"	1893, April 28
Henry W. Smith,	"	1891, April 30
Rufus W. Weeks,	1890, Oct. 23	1894, April 26
Daniel H. Wells,	†1891, April 30	1892, April 28
Sheppard Homans,	"	‡1891, Sept. 30
Clayton C. Hall,	‡1891, Sept. 30	1894, April 26
George B. Woodward,	1892, April 28	1895, April 25
Asa S. Wing,	"	1895, April 25
Thos. B. Macaulay,	1893, April 28	1896, April 30
Daniel H. Wells,	"	1896, April 30
Chas. A. Loveland,	1894, April 26	1897, April 29
Geo. W. Phillips,	"	**1895, April 25
Rufus W. Weeks,	§1895, April 25	1897, April 29
Jesse J. Barker,	"	1898, April 26
Walter C. Wright,	"	1898, April 28
William McCabe,	1896, April 30	1899, May
William E. Starr,	"	1899, May
John B. Lunger,	1897, April 29	1900, May
Elbert P. Marshall,	"	1900, May
James M. Lee,	1898, April 28	1901, May
Thos. B. Macaulay	"	1901, May

Ex-Presidents, by Constitution, Art. V., are members of the Council *in perpetuo*.

*Elected 2d Vice-President.

†To fill vacancy caused by election of Mr. McClintock, 2d Vice-President.

‡Mr. Homans became a Councillor by Amendment to Constitution, Art. V., making ex-Presidents members of the Council *in perpetuo*. Mr. Hall was elected to fill vacancy thus caused.

**Elected 2d Vice-President.

§To fill vacancy caused by election of Mr. Phillips, 2d Vice-President.

The publications of the Society have been twenty numbers of "Papers and Transactions," which constitute five volumes, with title pages and indexes.

Vol.	I.	contains	199 pages.
"	II.	"	438 "
"	III.	"	524 "
"	IV.	"	490 "
"	V.	"	422 "
	Total,		2,073 "

The Secretary acknowledges here the valuable assistance of Mr. Stabler and others in editing the papers and discussions; also, the services of Messrs. A. A. Welch, actuary of the Phœnix Mutual, Geo. F. Salter, of the actuarial staff of the Prudential, and M. W. Torrey, actuary of the Manhattan, in preparing the indexes to Volumes II., III., IV. and V., and generally the generous and most willing aid and advice of officers, councillors and members of the Society, in many ways and at all times, without which it would have been impossible to make the five volumes of "Papers, etc.," as meritorious and valuable as they are as to subjects and matter, and as creditable in appearance. Universal coöperation, kindness and courtesy have made all the work easy and a pleasure.

As to the correspondence it has been a delight, whether with members or others, whether domestic or foreign. The books contain 1,737 pages of copies of letters written, not including the many notes and communications which it was not deemed necessary to copy. There have been issued also 104 circular letters and printed documents. Seven file boxes contain the letters received which seemed worth preserving. They include communications in English, French, German, Dutch, Spanish and Italian.

The Library has attained a considerable size. We are indebted to the "Institute of Actuaries" for the Journal from 1889, the "Faculty of Actuaries," "Actuarial Society of Edinburgh" and "Institute of New South Wales" for their publications. This Society is also under obligations to the "Institut des Actuaires Français," the "Association des Actuaires Belges," the "Vereeniging van Wiskundige-Adviseurs bij Nederlandsche Maatschappijen van Levensvezekering" and the "Associazione Italiana per l'incremento della Scienza degli Attuari" for their Bulletins, from the date of their organization, severally, and to many others for books, reports, documents, etc.

Complimentary copies of the Papers and Transactions have been mailed regularly to all of these organizations, to the insurance journals, to the insurance commissioners, to the prominent libraries, etc.

But the most important addition to the Library is that recently received from Mrs. Whiting as the bequest of our lamented brother member, Wm. D. Whiting. This gift includes a complete set of the Institute of Actuaries' Journal and many other valuable works, together with a library case to contain them. A catalogue of that Library will be published with No. 21 of Papers, etc. See page 136.

At this meeting a resolution will be presented, expressing to Mrs. Whiting the Society's appreciation of this valuable contribution and the thoughtful kindness which prompted it.

The membership has so increased that notwithstanding the losses it is almost three times as large as it was at the outset. Table IV. shows the increments and decrements by years.

IV.—Membership.

At the founding of the Society, April 25th, 1889, there were charter members,			38
Added during the 10 years:			
'89, October 3,		7	
'90, April 24,		15	
October 23,		6	
'91, April 29,		2	
September 30,		1	
'92, April 28,		3	
October 13,		8	
'93, April 27,		8	
October 19,		4	
'94, April 26,		2	
October 11,		2	
'95, April 25,		1	
October 10,		9	
'96, April 30,		3	
October 8,		1	
'97, April 29,		1	
" Associate,		3	
October 7,		1	
'98, April 28, Associate,		4	
October 6,		10	
Full members,	84		
Associate,	7		91
Total charter and elected members,			129
Died during the 10 years:			
Charter members,	6		
Elected members,	7	13	
Resigned,		2	
Discontinued,		2	17
Present membership,			112

Dr. J. A. Fowler having offered his resignation as a member of the Society, it was accepted April 26, 1894.

Mr. Samuel E. Stilwell, having given up actuarial pursuits and assumed the general agency of a life insurance company, offered his resignation as a member of the Society. The resignation was

accepted on October 11, 1894. In each case there was an expression of regret.

During the ten years our ranks have been broken and the ties of friendship formed here have been severed, as stated, by the death of six of the thirty-eight charter members and seven of the other members, thirteen in all. They are :

Samuel Newell Stebbins, August 26, 1889, within four months after the organization of the Society. A charter member, "whose warm heart and broad mind, despite his advanced age, had kept him young in spirit and led him to take a warm interest in the formation of the Actuarial Society."

Edward Buckey Smith, July 31, 1890. "An accomplished mathematician and beloved in all walks of life."

Edmond-Auguste Cosmao-Dumanoir, November 26, 1894. "One of the three or four oldest actuaries in France."

Horace Cooper Richardson, February 12, 1895. "His associates in the New York Life office, as well as the many friends he left, will feel his removal long and deeply."

Charles Benjamin Perry, August 23, 1895. "He was of a quiet and retiring disposition but of great depth of feeling and was warmly attached to his friends and intimate business associates."

Léon Mahillon, January 19, 1896, in a little over three months after his election. President of the first International Congress of Actuaries. "In addition to his talent, his acquirements, his activity, that which was perhaps the most distinguishing feature of his character was a personal magnetism and enthusiasm for high ideals which attracted and charmed all with whom he had to do."

Jacques Martin-Dupray, September 12, 1897. "He was an actuary of unusual distinction, prominent also in philanthropic matters."

Sheppard Homans, January 8, 1898. The first President of the Society. "The oldest in service of American actuaries. It would be difficult to name a man who had fewer enemies and more friends. He will be missed socially as much as officially and his genial, kindly personality will abide in the heart of each member of this Society."

"Oh, for the touch of a vanished hand
And the sound of a voice that is still!"

Henry Worthington Smith, February 3, 1898. "He was an active contributor to the columns of insurance journals; afterward became the editor of one. 'Principles and Practice' is a monument to his mathematical and actuarial acquirements. He will be remembered as one of the charter members."

George Ellis, June 25, 1898. "It is not, however, the success which crowned his professional and personal activities that the friends of George Ellis will longest remember, but rather certain traits of high character expressed in daily conduct and lending a color of dignity to his demeanor; with a favored few will ever abide the memory of the courage, serenity and faith which illumined his closing hours to the end."

Joseph Hussey Sprague, July 21, 1898. "Keenly discriminating in his estimate of men, measures and events, he was considerate in forming and expressing individual convictions and inflexibly faithful to the claims of friendship as well as the demands of personal and official obligations."

George William Phillips, September 27, 1898. "Though ordinarily reserved in mien, he became genial and kindly on fuller acquaintance and with his great abilities and conscientious work won the esteem of all. His share in the upbuilding of the Equitable will be his enduring monument."

William Dunbar Whiting, April 20, 1899. "He united the keen insight of the lawyer to the analytic powers of the mathematician. * * * Only those who enjoyed more than a mere professional acquaintance suspected how often he extended a helping hand to those in need."

We should be very glad were these thirteen members here today and are very sure that they would enjoy participating with us in the celebration of the completion of the tenth year of the Society which some of them aided much to build up. They will ever remain in pleasant and tender memory and be associated with the first decade of the Actuarial Society of America.

"Let fate do her worst, there are relics of joy,
Bright dreams of the past, which she cannot destroy;
Which come in the nighttime of sorrow and care
And bring back the features that joy used to wear.
Long, long be my heart with such memories filled!
Like the vase, in which roses have once been distilled—
You may break, you may shatter the vase, if you will,
The scent of the roses will hang round it still."

Passing to statistics, as to ages on entering the Society, the oldest was 76, the youngest, including the Associates, 23. The average age at entry was 43. As to present age, the oldest is 86, the youngest is 29. The present average age of all the members is 46.

Combining all the data, the total number of years lived or exposed to risk from April 25, 1889, to April 25, 1899, was 827; the probable number of deaths by the Actuaries' table of mortality was 15.5; the actual number of deaths was 13, or 84 per cent. of the probable deaths.

May we not hope that during the next decade the mortality will be even more favorable, and especially that as to one member* there will be at the end of the second decennium a strong probability in favor of his rounding out the century and surviving the end of the Carlisle table?

In addition to the papers presented by the members and the discussion of them, the minutes of the meetings and other matters which fill the pages of the five volumes of the Papers, etc., there are other items which need to be mentioned in order to make the

* Mr. William E. Starr.

history of the first decade more nearly complete. For example, there has been the dinner at each meeting, with its informal after-dinner talks, sometimes on subjects scientific or actuarial, sometimes on subjects general. This has been the opportunity for conference when the members were brought closer together as friends and brothers. Most of the postprandial speeches are preserved in the stenographer's reports now on file. The informal social functions also have contributed much to the pleasure and profit of the meetings.

Fortunately, a full report of the proceedings at the formation of the Society was preserved. Surely the early history, especially that of the first decennium, will ever be regarded as of great interest and value.

A brief résumé of the meetings, excluding the papers and discussion, may be of interest.

1889, April 25 and 26.—In addition to being the charter meeting, it was the occasion of becoming acquainted and the first dinner. Mr. Fackler presided.

1889, October 3—was the first fall meeting and an experiment. A pleasant feature was the dinner at the round table, Union League Club, by courtesy of Mr. Homans, who presided.

1890, April 24—the meeting at which it was first felt that the Actuarial Society of America had really been organized and would be permanent. Dinner was given at the Astor House by the New York and Newark members, Mr. Homans presiding.

The minutes of the second and third meetings, October 3, 1889, and April 24, 1890, not having been printed in the "Papers, etc.," of those meetings, they are appended hereto to furnish a complete printed record. See pages 132–135.

1890, October 23 and 24.—On the invitation of the Hartford members, the first meeting outside of New York was held in the Senate Chamber of the Capitol by the courtesy of Governor Morgan G. Bulkley. Mr. H. W. St. John, Second Vice-President of the Society, presided gracefully at the dinner given in the Banquet Room of the Hartford Club. It was notable as to the pleasing social qualities and postprandial speeches.

1891, April 29 and 30—New York.—Prize for best essay on "Legislative Interference with Impaired Life Insurance Companies," offered by Mr. Fackler, was reported at this meeting. The dinner at the Lawyers Club was made memorable by the members telling how they came to be actuaries and by the relation of his personal experience by the lamented Edwin W. Bryant.

1891, September 30 and October 1—Toronto.—As guests of the Canadian members, the meeting in the board of trade Council Chamber, the banquet at "The Queen's Hotel," enlivened by songs and recitations and ending with "Auld Lang Syne," the drive and afternoon tea at the residence of Mr. and Mrs. Samuel Nordheimer will long be remembered.

1892, April 28 and 29—New York.—An interesting circumstance was the selection of a motto for the Society. It was done chiefly

by correspondence. "The work of science is to substitute facts for appearances and demonstrations for impressions" (Ruskin), was formally adopted at this meeting. Fackler prize essay, first prize awarded to Mr. A. A. Welch. Dinner at the Lawyers Club.

1892, October 13 and 14—Boston.—As guests of the officers of the John Hancock Mutual Life Insurance Company and Mr. Geo. B. Woodward, at the Hotel Vendome. Those present will not forget the charming drive through Cambridge, Brookline and other suburbs, the dinner at which Mr. Woodward presided, and the breakfast at the Algonquin Club, on Friday at 12, when Gen. Francis A. Walker and others spoke.

1893, April 27 and 28—New York.—The meeting began with a dinner in the Mills Building dining room, Mr. Fackler presiding. Business meeting on the 28th.

1893, October 19 and 20—Philadelphia.—As guests of the Philadelphia members. The first item was a lunch in the breakfast room of the Continental Hotel. At the dinner at the Continental Hotel, Mr. J. J. Barker presiding, the menu was unique as to design and particularly as to the subject-matter, abounding most elaborately in quotations from Shakespeare appropriate to each course. After adjournment there was the delightful drive through Fairmount Park and the reception at the residence of Mr. W. L. More.

1894, April 26 and 27—New York.—As guests of the Metropolitan Life Insurance Company. Lunch was served between the business sessions. It was the fifth anniversary of the Society. The members of the Society were entertained at a banquet given at Delmonico's, on Thursday evening, Mr. John R. Hegeman, the president of the Metropolitan, presiding. "The scene was a brilliant one, music delighted the ear and flowers charmed the eye. The souvenir was the elaborate menu of white morocco with silver ornamentation." At the close of the second day's session came the sail by Mr. Hegeman's steam yacht and reception by Mrs. Hegeman, at "Rosedene," Mamaroneck-on-the-Sound.

1894, October 11 and 12—New York.—Informal dinner at the Insurance Club. Mr. Gerald H. Ryan, editor of the *Journal of the Institute of Actuaries*, was present.

1895, April 25 and 26—New York.—Annual meeting. Dinner at the Insurance Club, Mr. St. John presiding.

1895, October 10 and 11—Montreal.—As guests of the officers of the Sun Life Assurance Company. Dinner at the Windsor Hotel, the President, Mr. Emory McClintock, presiding. The early morning trip on Friday to Lachine and the descent of Lachine Rapids by steamer, luncheon at noon at the Windsor, and the drive through the city and Mount Royal Park, are among the pleasantest recollections in connection with meetings of the Society.

1896, April 30 and May 1—New York.—Amendment to Constitution providing for associate membership was adopted. Meeting at the Hotel Imperial, also the dinner at which the President, Mr. McClintock, presided.

1896, October 8 and 9—Pittsfield.—As guests of the officers of the

Berkshire Life Insurance Company. This was unique, in that the meetings were held in the large parlor of "The Maplewood," with a wood fire blazing in the open fireplace to temper the October chill. Mr. McClintock, the President, presided at the dinner, the Mayor of Pittsfield and officers of the Berkshire being present. Every one will remember the most delightful trip on the second day by tally-ho to Lenox, the clear, crisp atmosphere giving zest and the beautiful autumnal foliage adding charm to the drive. There are vivid recollections of the bountiful lunch which was served on the return.

1897, April 29—New York.—Directors' room of the Metropolitan Life Insurance Company. Mr. McClintock having met with an accident, Mr. Miller, the 1st Vice-President, presided. A message of sympathy was sent to Mr. McClintock. Mr. Hegeman entertained the members at lunch. Dinner at "The Imperial."

1897, October 7 and 8—Springfield.—As guests of the officers of the Massachusetts Mutual Life Insurance Company. The dinner occurred at the far-famed Massasoit House, President Miller presiding; on the same evening Mrs. Ireland entertained the ladies at dinner at her home. Mr. Ireland had made ample provision for the entertainment of the guests. The visit at Springfield was concluded with a trip by electric cars to the summit of Mt. Tom, an elevation of 1218 feet, where a lunch was served.

1898, April 28 and 29—New York.—The meeting and dinner were held in the college room of "The Arena," President Miller presiding.

1898, October 6 and 7—Worcester.—As guests of the officers of the State Mutual Life Assurance Company. The meeting was held in the very comfortable Directors' room. The date of the annual meeting was changed from the last Thursday in April to the first Thursday after the fourteenth day of May in each year. The usual dinner was given in the large dining room of the State Mutual restaurant, President Miller presiding. Postprandial speeches were made by General A. G. Bullock, President of the State Mutual, and others. What was pronounced by everybody as one of the most enjoyable and profitable of the fall meetings terminated with a drive about the city and a lunch.

In a large scrap-book are press notices of meetings, obituaries, copies of menus of all the dinners, and other printed material pertaining to the Society and the members. Mr. Geo. F. Salter and Mr. Samuel S. Hall have rendered valuable aid in keeping this book.

The photograph album was suggested by our good friend, Mr. Homans. It contains the pictures of nearly all who have been members of the Society. We also have duplicates of most of these pictures, to be made up into a group at some time in the future. The members have very willingly responded to the request for their photographs. Some of the members will remember the fortunate ruse or accident by which the photograph of Mr. Phillips was secured—the only picture in existence.

It must be mentioned with grateful appreciation that the execu-

tive officers of the Life Insurance Companies have extended cordial and very substantial encouragement and assistance.

Certain cardinal principles on which the Society was founded were emphasized in the conference preliminary to the organization of the Actuarial Society of America :

No politics.

The members are such as individuals, not as representatives of companies.

"No resolution expressive of opinion shall be entertained at any meeting," is an article of the Constitution.

"This Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these 'Papers and Transactions,'" is printed at the head of the table of contents of each number of Papers, etc.

These principles were incorporated in the Constitution and have always guided the Society. A spirit of good fellowship and harmony has characterized the meetings of the Society and the intercourse of members. Thus may it ever be.

May the Secretary refer to special expressions of kindness on the part of the officers and members, among them being the surprise dinner on June 6, 1892, and the honor of election as a delegate to the First and Second International Congresses of Actuaries. These tokens cannot be valued by any commercial standard. Your unstinted goodness will be gratefully remembered so long as memory lasts. "*Forsan et hæc olim meminisse juvabit.*"

There can be no better wish for my successor in office than that he may have the same experiences of courtesy and favor which have been my lot as Secretary. Concluding these ten years of service, let me again thank you all for the uniform and constant kindness, coöperation and consideration which you have so generously accorded.

When this Society was organized the Institute of Actuaries had been in existence forty-one years and the Faculty of Actuaries thirty-three years, the Jubilee of the former having been celebrated at the Second International Congress of Actuaries in London, May, 1898.

The inestimable benefits conferred by the Actuarial Society of America upon the business of life insurance, as well as upon its members, during its first decennium, have fully justified the founding of the Society, which stands to-day as a monument to the faith, zeal and wisdom of the five gentlemen who projected it in 1889. The history of the progress of the past ten years is an earnest and prophecy of continued advance in the future. What the Actuarial Society of America will become by the time of the celebration of its semi-centennial who can foretell?

ISRAEL C. PIERSON,

NEW YORK, May 18th, 1899. *Secretary.*

Minutes of the Meetings Oct. 3d, 1889, and April 24th, 1890.

Board Rooms of the Home Fire Ins. Co.,
115 Broadway, N. Y., Oct. 3d, 1889, 2 P. M.

In pursuance of the call issued Sept. 12th this Society met at the place and time mentioned, Mr. Homans, the President, in the chair. Before proceeding to business the President invited the members to join him in a dinner in the evening at the Union League Club. It being necessary to make arrangements at once, Messrs. Miller, Fackler and Pierson were appointed a committee for that purpose.

Mr. Miller reported in a very short time, and recommended that the invitation to dine at 7 P. M., at the Union League, be accepted. The recommendation was adopted.

After a brief address of welcome by the President, the roll was called, to which the following members responded:

Messrs. Ellis, Fackler, Hendry, Holcombe, Homans, Ireland, Lee, Lunger, Marshall, E. P.; Marshall, W. A.; Miller, Phillips, Pierson, St. John, Smith, H. W.; Smith, E. B.; Standen, Weeks, Wells, Wright, Craig, Field, De Boer, Starr—24 in all.

On motion of Mr. Ellis, the following order of procedure, recommended by the Council, was adopted for this meeting:

1. Minutes of the last meeting.
2. Summary of Council proceedings.
3. Candidates for membership voted upon.
4. Reports of officers and committees.
5. Unfinished business.
6. New business.
7. Reading papers.
8. Discussion of papers.
9. Review of minutes of meeting.

On motion, the published minutes of April 25th, '89, meeting were adopted as the official minutes of the Society.

The Secretary read a summary of the proceedings of the Council. On motion of Mr. Field they were approved.

On motion of Mr. Miller, the Society then discussed informally the question, "Is a change in the Constitution desirable?" After a free consideration an expression of opinion was taken: Is it advisable to propose an amendment to the Constitution at the present time? Aye, 6; nay, 18.

On motion of Mr. H. W. Smith, the Society took a recess to allow the Council to act upon the names proposed for membership.

RECESS.

After recess the Secretary read the names proposed for membership, as follows:

Charles Hildebrand,	Walter S. Nichols,
Charles N. Jones,	Horace C. Richardson,
Christopher Kyle,	Wm. D. Whiting,

James Weir Mason.

It was voted to proceed to ballot on these nominations, Mr. Standen acting as teller. All the persons named were declared elected.

Mr. H. W. Smith formally announced the death of Mr. S. N. Stebbins, and moved that a committee of five be appointed to formulate a minute to be placed upon the record and sent to his family. It was so ordered. The President appointed Messrs. H. W. Smith, Fackler, Phillips, Weeks and Wright.

There being no further business, papers were read as follows:

"History of Dividend Systems in the United States." D. P. Fackler.

"Are our Mortality Tables Reliable?" Wm. Hendry.

"Accumulation Formulæ." Edward B. Smith.

On motion of Mr. Weeks, it was voted that the Council be instructed to print all papers read before the Society in such form as they think best, except those the writers of which have particularly requested that they should not be printed.

On motion of Mr. Fackler, it was resolved that the thanks of the Society be tendered to the President of the Home Fire Ins. Co. for the use of the Board room.

On motion of E. B. Smith, it was voted that the By-laws recommended by the Council should be printed and sent to each member of the Society.

On motion, the Society adjourned.

Israel C. Pierson,
Recording Secretary.

The dinner at 7 p. m., at the Union League Club, was attended by Messrs. Homans, Ellis, Fackler, Lunger, Marshall, E. P.; Miller, Pierson, St. John, Wells, Wright, Craig, Field, Ireland, Marshall, W. A.; De Boer.

Thanks were due and expressed to Mr. Homans for the pleasure of having the dinner in the Club private dining-room.

Second Annual Meeting of the Actuarial Society of America, held at the Astor House, New York City, April 24th, 1890, 10 a. m., Mr. Sheppard Homans, President, in the chair.

The meeting was called to order by the President.

The Secretary called the roll, to which the following-named members responded:

Messrs. Barker, Craig, De Boer, Fackler, Field, Hall, Hendry, Holcombe, Homans, Ireland, Lee, Loveland, Lunger, Macaulay, McCabe, McClintock, Marshall, E. P.; Marshall, W. A.; Miller, Pierson, St. John, Smith, H. W.; Starr, Weeks, Wells, Woodward, Hildebrand, Kyle, Nichols, Richardson, H. C.

The minutes of the meeting of October 3d, 1889, were read and approved.

Mr. FACKLER presented the report of the Committee on By-laws. On motion of Mr. MILLER, the order of business as proposed in the form of "By-laws" was adopted for the present meeting of the Society. The full report was taken up and considered later.

The Secretary read a résumé of the transactions of the Council.

The Society then proceeded to the election of members by ballot. All of the nominees were elected as follows:

A. A. WELCH, . . . *Ass't Actuary, Travelers.*
J. TATLOCK, JR., . . *1st Ass't Actuary, Mutual.*
C. B. PERRY, . . . *2d Ass't Actuary, Mutual.*
J. S. PATERSON, . . . *Actuary, N. Y. Ins. Dept.*
W. S. SMITH, . . . *Deputy Mass. Ins. Commissioner.*
E. L. STABLER, . . . *Actuary, Manhattan.*
C. T. LEWIS, . . .
H. J. MESSENGER, JR., . *Prof. of Mathematics, N. Y. University.*
E. W. BRYANT, . . .
A. K. BLACKADAR, . . *Actuary, Canada Ins. Dept.*
GEO. WEGENAST, . . *Ass't Actuary, Ontario Mutual.*
J. H. RICHARDSON, A. I. A., *Deputy Com., New Zealand Life Ins. Dept.*
J. G. RICHTER, . . . *Actuary, London Life, Canada.*
J. H. SPRAGUE, . . . *Actuary, Conn. Ins. Dept.*
J. A. FOWLER, . . . *Editor Am. Ex. & Review.*

On motion of Mr. HALL, the Secretary was requested to invite the newly-elected members who were in town to attend the afternoon session.

The report of the Committee on "By-laws" was then taken up. After remarks and some amendments, "By-laws" were unanimously adopted. (See Vol. I., p. 109.)

The President read an invitation from Mr. Herman Hollereth to examine his "electric tabulating machine." On motion, it was voted to accept the invitation for April 25th at 2 P. M.

The Treasurer's report was then read. On motion, it was accepted.

The President made an informal verbal report.

On motion, a committee of five was appointed to make nominations of Officers and Council for the ensuing year. The Committee, consisting of Messrs. LUNGER, E. P. MARSHALL, BARKER, WOODWARD and HALL, then retired.

Several amendments to the Constitution were proposed. After informal discussion, it was agreed to refer them for action to the next meeting, in accordance with Art. XIII. (See Vol. I., p. 108.)

At 11.30 A. M. a recess was taken until 12.

At that hour the Committee on Nominations reported and submitted the names of retiring Officers and Members of Council for reëlection.

They were elected by ballot as follows:

Sheppard Homans, ...	*President.*
D. P. Fackler, ...	*1st Vice-President.*
H. W. St. John, ...	*2d Vice-President.*
Israel C. Pierson, ...	*Rec. Secretary.*
Rufus W. Weeks, ...	*Cor. Secretary.*
Bloomfield J. Miller, . . .	*Treasurer.*

OTHER MEMBERS OF COUNCIL.

Oscar B. Ireland, Emory M'Clintock,
Wm. McCabe, Geo. W. Phillips,
H. W. Smith.

On motion, at 12.30, the Society adjourned until 2.30 P. M.

The President called the Society to order at 3 P. M. The following newly-elected members were welcomed to the meeting of the Society: Messrs. Bryant, Lewis, Messenger, Perry, Richter, Sprague, Stabler, Tatlock and Wegenast.

Papers were then read. (See Vol. I., pp. 5–37.)

At 5.30 the Society adjourned to meet at 6.30 P. M. as guests of the New York and Newark members of the Council at dinner.

At 7 P. M. the members sat down to dinner in Room 74 of the Astor House, Mr. Homans presiding. Messrs. Wing and Mason were present, having been unavoidably detained from the business sessions. Mr. Tatlock was obliged to be excused.

At 9 o'clock cigars and coffee were served. Informal speeches were then made by the new members and others.

On motion, Messrs. Homans, Weeks, Lewis, Phillips and McClintock were appointed a committee to report concerning a library and coöperation with the Equitable in the matter.

On Mr. Ireland's motion, Mr. St. John seconding, the Society passed a vote of thanks to the New York and Newark members of Council for their hospitality.

Mr. St. John, on behalf of the Hartford members, invited the Society to meet in October in their city.

On motion, the place and date of the next meeting was referred to the Council.

At 11 P. M., April 25th, 1890, the Society adjourned.

Israel C. Pierson,
Recording Secretary.

William D. Whiting Library.

PRESENTED TO THE SOCIETY.

Actuarial Society of America. Papers, etc., Vols. I.–V.

Institute of Actuaries' Journal. Vols. I.–XXXII., bound.

" " " Vol. XXXIII. to present time.

" " Text Books. Parts I. and II.

" " Graduated Exercises Text Books. Parts I. and II.

Actuarial Society of Edinburgh. Parts II. and III.

Actuarial Society of Glasgow. Series I. and II.

Doctrine of Life Annuities. Baily.

Assurance Premiums. Orchard.

Life Tables. Edmonds.

Value of Life Policies. Hillman.

Agents' Tables. Fackler.

Principles and Practice. Willey, Smith, Tatlock.

System and Tables of Life Insurance. Meech.

Mortality Experience, Institute of Actuaries. 1869.

Institute of Actuaries' Life Tables. 1872.

Mortality Experience, Several Companies.

Tables, American Experience Three Per Cent. Connecticut Mutual.

Least Squares. Chauvenet.

Course in Mathematics. Hutton.

Finite Differences. Sunderland.

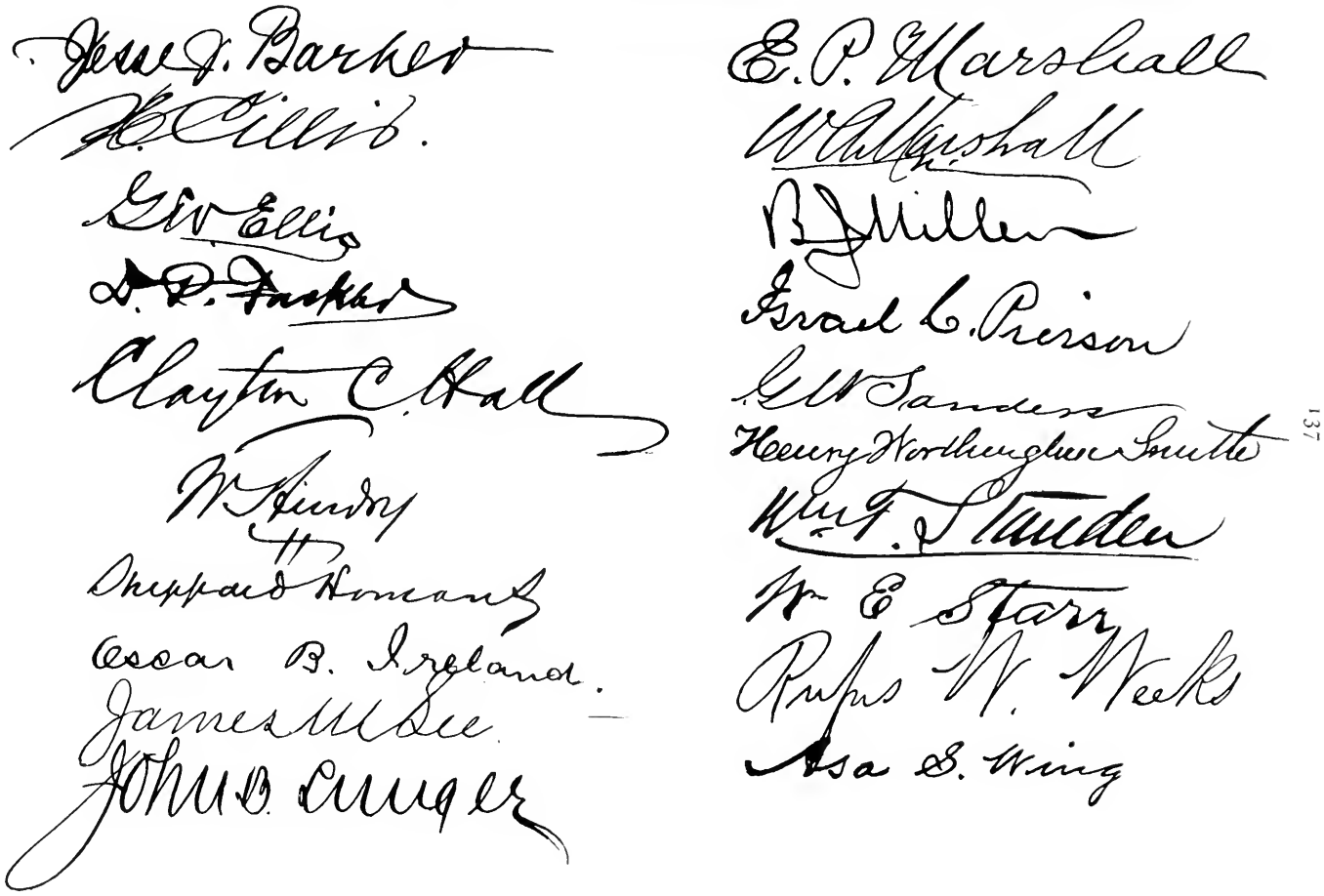
Calculus.

Tables of Logarithms.

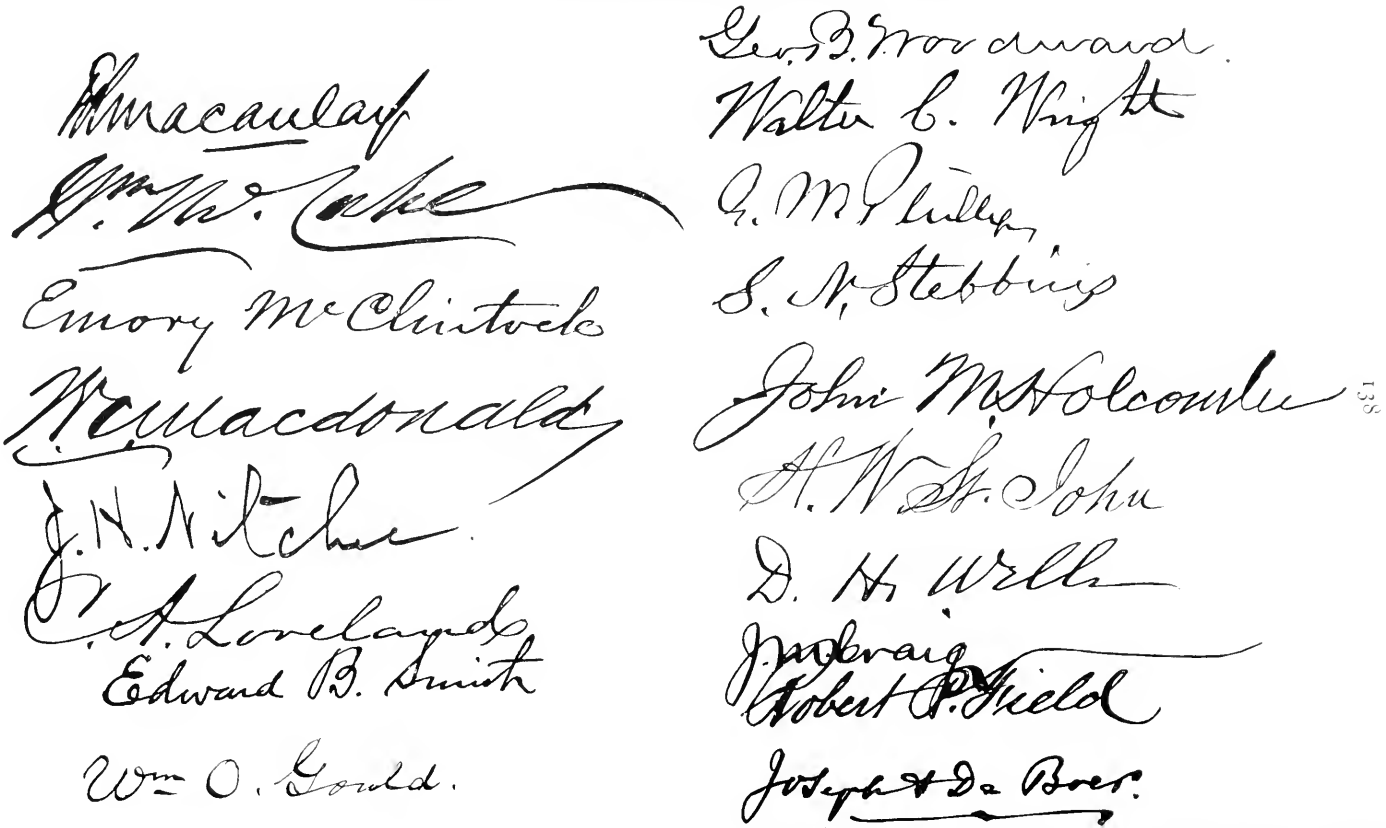
Massachusetts Reports. 1859–1866.

State Insurance Reports.

Miscellaneous Books and Papers.

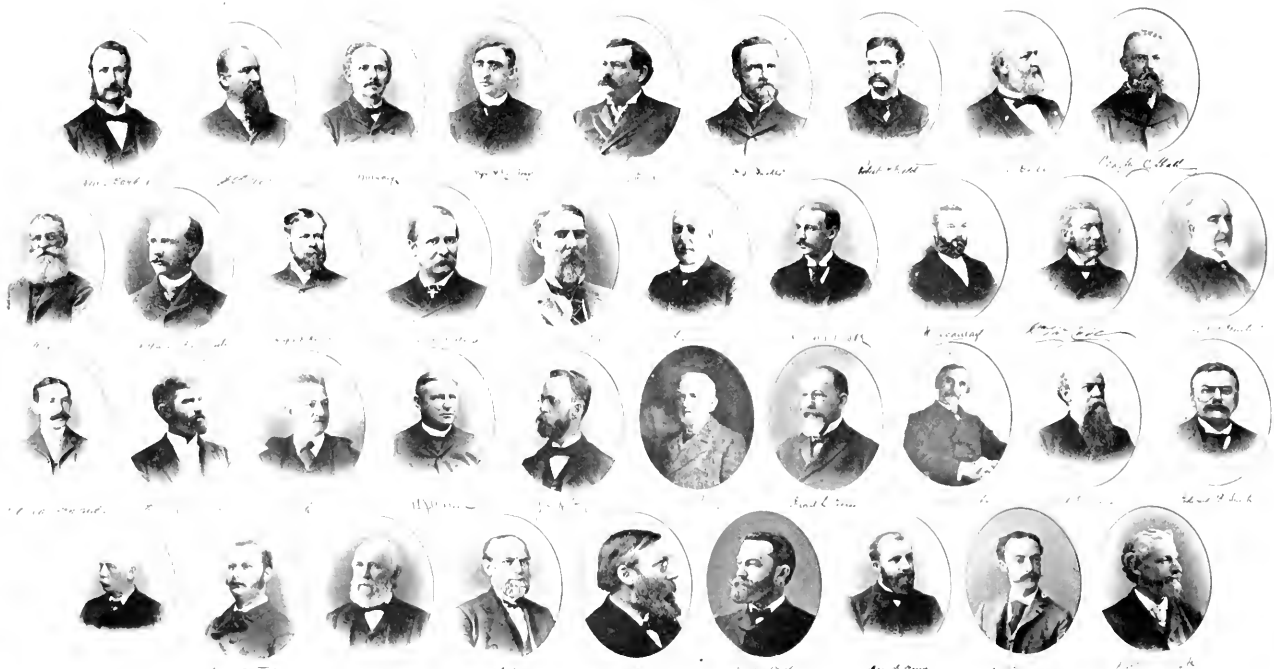


CHARTER MEMBERS. Continued.



Thomas Macaulay
Wm. McCabe
Emory McClintock
W. C. Macdonald
J. H. Nitchie
C. A. Loveland
Edward B. Smith
Wm. O. Gould.

Geo. B. Woodward.
Walter C. Wright
G. M. Phillips
S. N. Stebbins
John M. Holcombe
H. W. St. John
D. H. Wells
J. McCraig
Robert P. Field
Joseph A. De Boer.



CHARTER MEMBERS ACTUARIAL SOCIETY OF AMERICA.

GROUP—DECENNIAL MEETING—COURT, METROPOLITAN LIFE BUILDING MAY 10, 1866.

WILLIAM DUNBAR WHITING,

Consulting Actuary.

Born, New Orleans, La., December 13th, 1844.

Died, New York, April 20th, 1899.

WE occasionally meet in the ranks of professional life with men whose exceptional genius and diversified talents give them a unique individuality. Such was William Dunbar Whiting, who was among the first to be elected to membership in this Society after its organization. He united the keen insight of the lawyer to the analytic powers of the mathematician, and, turning aside from the beaten tracks of authority, delighted in blazing his own independent pathway through every subject which he investigated.

With these traits he combined a disposition naturally polemic and a sincere desire to discover truth, fearless as to where the quest might lead. He was never satisfied until he had gone to the bottom of the matter he was investigating, and his mental activity was tireless in its pursuit. It was the combination of these qualities which gave to Mr. Whiting the power he exerted in the insurance world, and compelled even those who dissented from his views to recognize his genius. As a professional adviser of insurance departments, his influence was strong.

There was another side to his nature, however, which was not so generally understood. Beneath the exterior of this man was masked a kind and generous heart. His antagonisms were on the surface and never degenerated into personal dislikes. Only those who enjoyed more than a mere professional acquaintance suspected how often he extended a helping hand to those in need.

Mr. Whiting was the descendant of an old Colonial family of Massachusetts, but was born in New Orleans, December 13th, 1844, his childhood being spent in that city. He passed his examination for West Point in 1861, but was led to abandon his plans by the outbreak of the Civil War, and utilized his training as a civil engineer for some years in the West Indies. Coming to New York, he joined the United States Life Insurance Co., and became at once interested in the actuarial features of the business, from 1869 to 1876 being the actuary of the company. Meanwhile he pursued the study of law and graduated from New York University with the degree of LL.B., and was admitted to the bar in 1877. He then took up the study of medicine, and received the degree of M. D. from the Bellevue Medical College in 1881. In 1878 he was appointed actuary of the Maine Insurance Department, and occupied the position until his death, on April 20th last. He made the profession of a consulting actuary his specialty, and enjoyed an extended and valuable clientage.

The death of Mr. Whiting has taken from us a member whose attainments in their variety and scope were exceptional.

Yours Truly
Wm D. Whiting

VOL. VI. NO. 22.

PAPERS AND TRANSACTIONS.

OCTOBER 19TH AND 20TH, 1899.

CONCERNING SOME RECENT MORTALITY TABLES ON ANNUITANTS.

RUFUS W. WEEKS.

IN VOL. VI. of the Papers and Transactions of this Society, on pages 13 to 23, we have a very interesting and valuable contribution by Mr. MClintock, being two mortality tables on the lives of annuitants, male and female, respectively. In devising these tables Mr. McClintock made use, as he states, of the data furnished in the paper (Vol. IV., pages 275 to 307) by the undersigned, on "Mortality Experience on Annuitants (European and American) in [15] American Life Insurance Companies"; and, on taking up Mr. McClintock's tables my first query was how nearly parallel his finely graduated mortality rate comes to the raw material of that experience. Mr. McClintock's tables themselves give the comparisons, age by age, and it is easy to show to the eye how the two compare, by summing the expected deaths, as he gives them, according to his mortality rate, in groups of five successive ages, and summing also the actual deaths in the same way. The following two tables show the result:

TABLE I.—MALES.

EXPERIENCE, 15 AMERICAN COMPANIES; EXPECTED DEATHS, McCLINTOCK'S STANDARD, COMPARED WITH ACTUAL DEATHS.

Age Groups.	Expected Deaths McClintock's Table for Male Annuitants.	Actual Deaths.
10–39 inclusive.....	3.5	2
40–44 "	5.9	4
45–49 "	13.8	19
50–54 "	31.9	34
55–59 "	69.1	73
60–64 "	137.1	134
65–69 "	225.6	222
70–74 "	305.1	318
75–79 "	288.4	295
80–84 "	211.8	216
85–89 "	86.4	81
90–99 "	30.1	17
Totals.....	1,408.7	1,415

TABLE II.—FEMALES.

EXPERIENCE, 15 AMERICAN COMPANIES; EXPECTED DEATHS, McCLINTOCK'S STANDARD, COMPARED WITH ACTUAL DEATHS.

Age Groups.	Expected Deaths McClintock's Table for Female Annuitants.	Actual Deaths.
10–39 inclusive.....	2.6	3
40–44 "	4.5	9
45–49 "	11.0	7
50–54 "	27.6	33
55–59 "	56.8	50
60–64 "	113.2	111
65–69 "	178.7	175
70–74 "	229.1	218
75–79 "	228.7	224
80–84 "	159.3	168
85–89 "	80.1	81
90–99 "	31.9	25
Totals.....	1,123.5	1,104

As will be seen, the parallelism is as close as could be expected between ungraduated and finished mortality rates ; it is remarkably close, indeed, when we consider that Mr. McClintock's Male and Female Tables have a common exponential constant. Exception should be made as regards the ages over 89, where Mr. McClintock's rates call for a much higher mortality than that actually experienced. Combining the male and female observations, at ages 90 and above, the expected deaths by Mr. McClintock's standard are 62, while the actual are only 42. The number of cases involved is sufficient to suggest mortality rates lower at those ages than the formulæ Mr. McClintock has used produce.

These new mortality tables come at a very convenient time for use as a measuring rule in making deductions from the new "Combined Experience of Life Annuitants ' (1863-1893) just published by the British Institute of Actuaries and the Faculty of Actuaries in Scotland. This experience is divided between what are termed "old annuities" and "new annuities," the portion under the head of "old annuities" being composed of observations during the period of 1863 to 1893 on annuitants entering before that period, while the portion under the head of "new annuities" includes only those issued within that period. It is the latter which should correspond most closely with the experience of the 15 American Companies ; in fact, a part of the material of the experience of the 15 American Companies, about one-fourth, actually enters into the "new annuities experience" on British annuitants. The average number of years in force of the "new annuities" does not differ greatly from that of the 15 American Companies' annuities. To test this I took the male group entering at age 65, and found the average number of years in force of the British annuities of that group to be 5.9 ; of the corresponding group of the 15 American Companies' annuities, 5.1. Therefore, so far as the effect of selection is concerned, these two experiences are nearly homogeneous.

The following two tables give a résumé, in 5-year age groups, of the "new annuities" experience of British annuitants tested by the expected mortality according to Mr. McClintock's standard :

TABLE III.—MALES.

BEING A RÉSUMÉ OF THE EXPERIENCE ON "NEW ANNUITIES," BRITISH ANNUITANTS.

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Male Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–49 inclusive.	2,942	34.6	35	101 per cent.
50–54 "	2,440	42.3	44	104 "
55–59 "	3,980	91.7	91	99 "
60–64 "	6,297	201.7	199	99 "
65–69 "	8,233	378.5	332	88 "
70–74 "	8,302	557.2	525	94 "
75–79 "	6,100	604.3	576	95 "
80–84 "	2,992	438.8	432	98 "
85–89 "	986	213.5	207	97 "
90–98 "	206	66.7	57	85 "
Totals.....	42,478	2,629.3	2,498	95 "

A similar calculation to the above, applied to the Combined Experience on "old" and "new annuities," males, brings out a ratio of aggregate actual to expected deaths of 98½ per cent.; the excess of 3½ per cent. over the ratio on the "new" annuities only being probably due to the greater average duration of the annuities. Indeed, applying the test to the groups of male annuitants entering at age 65, I found, as against the average number of years in force of the "new" annuities of 5.8, the average number of years in force of the "old" and "new" combined to be 7.2.

TABLE IV.—FEMALES.

BEING A RÉSUMÉ OF THE EXPERIENCE ON "NEW ANNUITIES," BRITISH ANNUITANTS.

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Female Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–49 inclusive.	9,033	61.8	84	136 per cent.
50–54 "	7,928	87.7	96	109 "
55–59 "	13,564	209.7	186	89 "
60–64 "	21,515	480.5	413	86 "
65–69 "	26,904	884.6	865	98 "
70–74 "	25,198	1,240.2	1,203	97 "
75–79 "	17,489	1,297.8	1,360	105 "
80–84 "	8,812	985.5	1,116	113 "
85–89 "	2,832	472.0	546	116 "
90–100 "	631	162.6	167	103 "
Totals....	133,906	5,882.4	6,036	103 per cent.

A similar calculation to the above applied to the Combined Experience on "old" and "new" annuities, females, brings out a ratio of aggregate actual to expected deaths of 106 per cent., being three per cent. in excess of the ratio on "new" annuities only as shown above. Here again, the greater mortality among the combined annuitants probably results from their greater average time in force; the respective average number of years in force (as calculated from the group of female annuitants entering at 65) being, on "new" annuities 6.8 years, on "old" and "new" combined 8.1 years.

An application of Mr. McClintock's mortality rates to that immense mass of material published in 1895 by the committee of the French Companies as experience on French annuitants' lives next suggested itself, and is embodied in the following two tables:

TABLE V.—MALES.

BEING A RÉSUMÉ OF THE EXPERIENCE ON LIVES OF FRENCH ANNUITANTS.

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Male Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–44 inclusive.	3,951	40.0	55	137 per cent.
45–49 "	4,257	58.3	62	106 "
50–54 "	10,725	186.7	220	118 "
55–59 "	21,889	506.6	514	101 "
60–64 "	39,595	1,272.4	1,305	103 "
65–69 "	52,151	2,392.9	2,388	100 "
70–74 "	51,088	3,426.0	3,674	107 "
75–79 "	35,826	3,552.5	3,773	106 "
80–84 "	17,629	2,590.9	2,829	109 "
85–89 "	5,571	1,202.4	1,245	104 "
90–94 "	1,072	335.6	294	88 "
95–101 "	143	67.4	46	68 "
Totals....	243,897	15,631.7	16,405	105 per cent.

TABLE VI.—FEMALES.

BEING A RÉSUMÉ OF THE EXPERIENCE ON LIVES OF FRENCH ANNUITANTS.

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Female Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–44 inclusive.	8,120	46.7	83	178 per cent.
45–49 "	9,019	74.6	99	133 "
50–54 "	20,621	229.3	276	120 "
55–59 "	39,564	612.3	614	100 "
60–64 "	66,153	1,478.1	1,508	102 "
65–69 "	82,469	2,710.2	2,889	107 "
70–74 "	75,721	3,724.7	4,303	116 "
75–79 "	52,538	3,900.6	4,704	121 "
80–84 "	26,103	2,916.0	3,726	128 "
85–89 "	8,861	1,484.5	1,743	117 "
90–94 "	1,842	452.6	488	108 "
95–106 "	250	96.4	71	74 "
Totals....	391,261	17,726.0	20,504	116 per cent.

A comparison of Tables III. and IV. with Tables V. and VI., respectively, shows at once a higher rate of mortality among French than among British annuitants. The data published by the French Committee do not show the duration of the annuities observed, but it is reasonable to suppose that the average duration is at least as high as that of the British annuitants observed 1863–1893, "old" and "new" combined, which seems to be about eight years. Comparing the experience on the French annuitants with that on the combined British annuitants we have, on males, a ratio of aggregate actual deaths to expected by Mr. McClintock's standard, of, among British annuitants 98½ per cent., among French annuitants 105 per cent., showing an excess of 6½ per cent. mortality among the French males. The corresponding ratios on females are: British 106 per cent., French 116 per cent., showing an excess of 10 per cent. mortality among the French females.

Referring again to Mr. McClintock's rates at ages 90 and above and to the suggestion derived from the 15 American Companies' experience that these rates are too high, we find that that suggestion is not borne out by the British experience, which on the contrary fully confirms Mr. McClintock's rates at those high ages, and by a number of deaths five times as large as those in the 15 American Companies' experience.

When it comes to the choice of mortality tables for calculating net premiums and reserves, whether on annuities or on life insurances, it seems to me that the controlling requirement should be that of an ample margin of safety. In handling life insurances, this principle demands that we should assume a mortality materially higher than that which we expect will actually occur, and when we come to annuities the same principle calls upon us to assume a mortality materially lower than that which we expect to occur. This view is strengthened by the fact referred to by Mr. McClintock that the duration of human life, at least among the comfortable classes, has been increasing during the last fifty years, and is apparently still increasing. Such improvement increases the margin of safety in the case of life insurances, but it equally decreases the margin of safety in the case of annuities, and from this it would seem to follow that the same degree of conservatism would require a wider margin of safety in the mortality allowed for in the case of annuities than in the case of life insurances.

If this view is correct, we must, in choosing a standard for net

premiums and reserves on annuities, look for tables which show much lower rates of mortality than those heretofore experienced on any body of annuitants observed through a period of years. As a safe mortality table for insured lives may be derived from the experience on a body of insured lives by excluding the earlier years of the existence of the insurances, so the reverse process, it would seem, would afford a safe mortality table for annuitants, *i. e.*, to derive such a table from the experience upon the lives of annuitants, including only the earlier years of the existence of the annuities. The extreme application of this principle would consist in making up a mortality table based upon the experience on the lives of annuitants in the first year only of the duration of the annuities.

I have examined the Select Tables embodying the British Companies' Combined Experience of Life Annuitants (1863–1893) to see whether that portion of the experience which is derived from the first year only of the existence of the annuities, furnishes material enough for constructing such mortality tables, and for this purpose have drawn off the numbers exposed during the first annuity year for the successive ages at entry, calculating the expected deaths according to Mr. McClintock's new Mortality Tables, and comparing them with the actual deaths. The results for males and females respectively, are shown in the following two tables, in five-year age groups.

TABLE VII.—MALES.

SHOWING THE FIRST ANNUITY-YEAR EXPERIENCE OF THE BRITISH COMPANIES (1863–1893).

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Male Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–49 inclusive.	684	8.0	4	50 per cent.
50–54 "	636	11.0	6	55 "
55–59 "	885	20.2	13	64 "
60–64 "	1,413	44.9	34	76 "
65–69 "	1,527	69.5	49	71 "
70–74 "	1,177	78.6	53	67 "
75–79 "	629	61.2	47	77 "
80–84 "	235	33.8	25	74 "
85–89 "	56	12.1	5	41 "
90–93 "	5	1.6	1	62 "
Totals....	7,247	340.9	237	70 per cent.

TABLE VIII.—FEMALES.

SHOWING THE FIRST ANNUITY-YEAR EXPERIENCE OF THE BRITISH COMPANIES (1863–1893).

Age Groups.	Number of Lives Exposed.	Expected Deaths per McClintock's Table for Female Annuitants.	Actual Deaths.	Ratio of Actual to Expected.
15–49 inclusive.	1,965	13.3	15	113 per cent.
50–54 "	1,862	20.6	20	97 "
55–59 "	2,904	44.8	23	51 "
60–64 "	4,214	93.3	60	64 "
65–69 "	3,857	125.1	82	66 "
70–74 "	2,522	121.8	78	64 "
75–79 "	1,118	81.4	63	77 "
80–84 "	355	39.3	26	66 "
85–89 "	104	17.0	15	88 "
90–97 "	17	4.3	4	93 "
Totals....	18,918	560.9	386	69 per cent.

The remarkable parallelism of the expected and the actual deaths in both the above tables shows that mortality tables could easily be constructed from the first year's material, tables which would be safe for a century to come. If it be thought that such tables would err on the side of excessive caution (an error which would be unfair to the annuitants if annuities were issued on such tables without participation in profits), the experience might be made to include the second as well as the first annuity-year. I find that the second annuity-year shows a mortality equal in the aggregate on males to 88 per cent. of the expected by Mr. McClintock's standard, and on females equal to 89 per cent.; and that the first and second annuity-years combined would show on males, 78 per cent., and on females, 79 per cent. The practical identity of the respective ratios on male and female lives is very striking, as will be seen.

I append three per cent. D and N Commutation columns and Annuity values, calculated on the mortality tables devised by Mr. McClintock for male and female annuitants respectively.

TABLE A.

MALE ANNUITANTS' MORTALITY, 3%.

(McClintock's Mortality Table "A," contained in Vol. VI., pp. 13–23; derived from Fifteen American Companies' Experience, Vol. IV., pp. 275–307.)

Age.	q_x	D_x	N_x	a_x
10	.00755	744,094.0	18,041,251.4	23.246
11	.00757	716,970.7	17,297,157.4	23.125
12	.00759	690,822.7	16,580,186.7	23.001
13	.00761	665,613.6	15,889,364.0	22.872
14	.00764	641,309.4	15,223,750.4	22.739
15	.00766	617,876.2	14,582,441.0	22.601
16	.00768	595,282.0	13,964,564.8	22.459
17	.00773	573,496.3	13,369 282.8	22.312
18	.00778	552,489.2	12,795,786.5	22.160
19	.00780	532,230.8	12,243,297.3	22.004
20	.00784	512,694.9	11,711,066.5	21.842
21	.00791	493,853.0	11,198,371.6	21.676
22	.00796	475,680.0	10,704,518.6	21.504
23	.00803	458,150.6	10,228,838.6	21.326
24	.00807	441,241.4	9,770,688.0	21.144
25	.00814	424,928.2	9,329,446.6	20.955
26	.00823	409,188.3	8,904,518.4	20.761
27	.00832	394,000.9	8,495,330.1	20.562
28	.00841	379,344.2	8,101,329.2	20.356
29	.00851	365,198.0	7,721,985.0	20.145
30	.00862	351,543.1	7,356,787.0	19.927
31	.00876	338,359.9	7,005,243.9	19.704
32	.00889	325,630.4	6,666,884.0	19.474
33	.00903	313,337.1	6,341,253.6	19.238
34	.00919	301,462.4	6,027,916.5	18.996
35	.00937	289.990.0	5,726,454.1	18.747
36	.00958	278,903.9	5,436,464.1	18.492
37	.00978	268,188.3	5,157,560.2	18.231
38	.01001	257.828.9	4,889,371.9	17.964
39	.01028	247,810.4	4,631,543.0	17.690
40	.01056	238,119.0	4,383.732.6	17.410
41	.01088	228,741.1	4,145,613.6	17.124
42	.01122	219,663.6	3,916,872.5	16.831
43	.01158	210,873.7	3,697,208.9	16.533
44	.01199	202,359.4	3,486,335.2	16.228
45	.01245	194,108.5	3,283,975.8	15.918
46	.01293	186,109.5	3,089,867.3	15.602
47	.01347	178,351.3	2,903,757.8	15.281
48	.01406	170,823.5	2,725.406.5	14.955
49	.01472	163,515.7	2,554,583.0	14.623

TABLE A.—Continued.

Age.	q_x	D_x	N_x	a_x
50	.01542	156,417.8	2,391,067.3	14.286
51	.01619	149,520.6	2,234,649.5	13.945
52	.01705	142,814.7	2,085,128.9	13.600
53	.01798	136,292.1	1,942,314.2	13.251
54	.01900	129,943.8	1,806,022.1	12.898
55	.02010	123,762.7	1,676,078.3	12.543
56	.02134	117,741.4	1,552,315.6	12.184
57	.02267	111,873.1	1,434,574.2	11.823
58	.02413	106,151.9	1,322,701.1	11.460
59	.02575	100,572.2	1,216,549.2	11.096
60	.02750	95,128.78	1,115,976.96	10.731
61	.02942	89,817.80	1,020,848.18	10.366
62	.03154	84,635.51	931,030.38	10.000
63	.03384	79,579.35	846,394.87	9.636
64	.03635	74,647.28	766,815.52	9.273
65	.03912	69,838.40	692,168.24	8.911
66	.04212	65,152.63	622,329.84	8.552
67	.04540	60,590.70	557,177.21	8.196
68	.04900	56,154.74	496,586.51	7.843
69	.05293	51,847.58	440,431.77	7.495
70	.05722	47,673.21	388,584.19	7.151
71	.06190	43,636.48	340,910.98	6.813
72	.06700	39,743.37	297,274.50	6.480
73	.07255	36,000.85	257,531.13	6.153
74	.07862	32,416.26	221,530.28	5.834
75	.08521	28,997.94	189,114.02	5.522
76	.09239	25,754.22	160,116.08	5.217
77	.10021	22,693.75	134,361.86	4.921
78	.10869	19,824.91	111,668.11	4.633
79	.11792	17,155.42	91,843.20	4.354
80	.12789	14,691.91	74,687.78	4.084
81	.13873	12,439.56	59,995.87	3.823
82	.15045	10,401.80	47,556.31	3.572
83	.16311	8,579.51	37,154.51	3.331
84	.17677	6,971.03	28,575.00	3.099
85	.19152	5,571.55	21,603.97	2.878
86	.20735	4,373.37	16,032.42	2.666
87	.22438	3,365.49	11,659.05	2.464
88	.24263	2,534.36	8,293.56	2.272
89	.26213	1,863.52	5,759.20	2.090
90	.28295	1,334.99	3,895.68	1.918
91	.30510	929.36	2,560.69	1.755
92	.32860	626.97	1,631.33	1.602
93	.35347	408.73	1,004.36	1.457
94	.37966	256.53	595.63	1.322

TABLE A.—Continued.

Age.	q_x	D_x	N_x	a_x
95	.40717	154.48	339.10	1.195
96	.43593	88.90	184.62	1.077
97	.46587	48.73	95.72	.964
98	.49687	25.23	46.99	.862
99	.52879	12.33	21.76	.765
100		5.62	9.43	.678
101		2.42	3.81	
102		.93	1.39	
103		.33	.46	
104		.09	.13	
105		.04	.04	

TABLE B.

FEMALE ANNUITANTS' MORTALITY, 3%.

(McClintock's Mortality Table "A," contained in Vol. VI., pp. 13–23; derived from Fifteen American Companies' Experience, Vol. IV., pp. 275–307.)

Age.	q_x	D_x	N_x	a_x
10	.00361	744,094.0	19,813,410.8	25.628
11	.00361	719,818.5	19,069,316.8	25.492
12	.00363	696,324.4	18,349,498.3	25.352
13	.00365	673,586.3	17,653,173.9	25.208
14	.00368	651,578.6	16,979,587.6	25.059
15	.00370	630,277.7	16,328,009.0	24.906
16	.00372	609,659.8	15,697,731.3	24.748
17	.00375	589,702.3	15,088,071.5	24.586
18	.00377	570,383.0	14,498,369.2	24.419
19	.00379	551,681.0	13,927,986.2	24.246
20	.00384	533,575.8	13,376,305.2	24.069
21	.00388	516,046.8	12,842,729.4	23.887
22	.00391	499,074.9	12,326,682.6	23.699
23	.00395	482,641.1	11,827,607.7	23.506
24	.00402	466,727.1	11,344 966.6	23.307
25	.00407	451,315.7	10,878,239.5	23.103
26	.00414	436,389.1	10,426,923.8	22.894
27	.00418	421,931.2	9,990,534.7	22.678
28	.00425	407,925.5	9,568,603.5	22.457
29	.00434	394,356.1	9,160,678.0	22.229
30	.00443	381,208.5	8,766,321.9	21.996
31	.00453	368,466.9	8,385,113.4	21.757
32	.00462	356,117.9	8,016,646.5	21.511
33	.00473	344,146.5	7,660,528.6	21.259
34	.00487	332,540.0	7,316,382.1	21.002
35	.00501	321,284.8	6,983,842.1	20.737
36	.00514	310,368.0	6,662,557.3	20.467
37	.00530	299,777.0	6,352,189.3	20.190
38	.00549	289,499.7	6,052,412.3	19.906
39	.00569	279,524.7	5,762,912.6	19.617
40	.00590	269,839.3	5,483,387.9	19.321
41	.00613	260,433.4	5,213,548.6	19.019
42	.00640	251,295.4	4,953,115.2	18.710
43	.00668	242,415.0	4,701,819.8	18.396
44	.00700	233,781.0	4 459,404.8	18.075
45	.00734	225,384.2	4,225,623.8	17.749
46	.00771	217,214.0	4,000,239.6	17.416
47	.00812	209,261.2	3,783,025.6	17.078
48	.00857	201,516.0	3,573,764.4	16.734
49	.00905	193,969.5	3,372,248.4	16.385

TABLE B.—Continued.

Age.	q_x	D_x	N_x	a_x
50	.00960	186,613.1	3,178,278 9	16.031
51	.01019	179,437.4	2,991,665.8	15.672
52	.01085	172,434.7	2,812,228 4	15.309
53	.01156	165,596.6	2,639,793.7	14 941
54	.01233	158.915.6	2,474,197.1	14.569
55	.01320	152,383 8	2,315,281.5	14.194
56	.01413	145,994.2	2,162,897.7	13.815
57	.01515	139,740.2	2,016,903.5	13.433
58	.01626	133,614.9	1,877,163.3	13.049
59	.01751	127,612.9	1,743,548.4	12.663
60	.01884	121,728.2	1,615,935.5	12.275
61	.02031	115,956.0	1,494.207.3	11.886
62	.02193	110,291.8	1,378,251.3	11.496
63	.02368	104,731.4	1,267,959.5	11.107
64	.02562	99,272.19	1,163,228.09	10.718
65	.02775	93,911.35	1,063,955.90	10.329
66	.03005	88,647.26	970,044 55	9.943
67	.03257	83,479.41	881,397.29	9.558
68	.03535	78,407.88	797,917.88	9.177
69	.03837	73,433.96	719.510.00	8.798
70	.04166	68,560.14	646,076.04	8.423
71	.04527	63,789.87	577,515.90	8.053
72	.04922	59,128.10	513,726 03	7.688
73	.05352	54.580.69	454,597.93	7.329
74	.05822	50,155.10	400,017.24	6.976
75	.06334	45,859.61	349,862.14	6.629
76	.06891	41,704.01	304,002.53	6.290
77	.07500	37,698.90	262,298.52	5.958
78	.08163	33,855.73	224,599.62	5.634
79	.08883	30,186.54	190,743.89	5.319
80	.09668	26,703.69	160,557.35	5.013
81	.10519	23,419.46	133,853.66	4.715
82	.11446	20,345.35	110,434.20	4.428
83	.12447	17,492.10	90,088.85	4.150
84	.13535	14,868.64	72,596.75	3.883
85	.14712	12,481.73	57.728.11	3.625
86	.15982	10,335.45	45,246.38	3.378
87	.17354	8,430.61	34,910.93	3.141
88	.18835	6,764.54	26,480 32	2.915
89	.20424	5,330.62	19,715.78	2.699
90	.22134	4,118.26	14,385.16	2.493
91	.23966	3,113.35	10,266 90	2.298
92	.25924	2,298.28	7,153.55	2.113
93	.28014	1,652.90	4,855.27	1.937
94	.30238	1,155.18	3,202.37	1.772

TABLE B.—Continued.

Age.	q_x	D_x	N_x	a_x
95	.32597	782.41	2,047.19	1.617
96	.35093	512.02	1,264.78	1.470
97	.37723	322.67	752.76	1.333
98	.40485	195.08	430.09	1.205
99	.43373	112.71	235.01	1.085
100		61.97	122.30	.974
101		32.28	60.33	
102		15.84	28.05	
103		7.29	12.21	
104		3.10	4.92	
105		1.21	1.82	
106		.44	.61	
107		.13	.17	
108		.04	.04	

NOTE.

AT the request of Mr. M'Clintock the values of annuities at 3½ per cent. are reprinted on the next page from pages 22 and 23, with certain corrections at the most advanced ages, as an appendix to Mr. Weeks's paper. Mr. M'Clintock expresses his regret that by a clerical misunderstanding the correct columns were not copied for publication, but that instead were copied columns previously calculated according to his first assumption that the life table was to stop at age 100. This misadventure is the more annoying to him, because the numbers living published above 100 were added expressly to justify the correct annuity figures, which by an accident in copying failed to appear.—ED.

Annuities at 3½ per Cent.

Age.	Males. Table A.	Females. Table B.	Age.	Males. Table A.	Females. Table B.
10	21.143	23.139	55	11.956	13.464
11	21.049	23.036	56	11.628	13.122
12	20.952	22.929	57	11.298	12.776
13	20.851	22.818	58	10.965	12.427
14	20.746	22.703	59	10.629	12.075
15	20.637	22.584	60	10.292	11.720
16	20.524	22.461	61	9.954	11.363
17	20.407	22.334	62	9.615	11.004
18	20 286	22.202	63	9.276	10.643
19	20.160	22.066	64	8.937	10.285
20	20.030	21.925	65	8.598	9.925
21	19.895	21.780	66	8.262	9.565
22	19.756	21.630	67	7.927	9.206
23	19.611	21.475	68	7.594	8.850
24	19.462	21.315	69	7.265	8.495
25	19.307	21.150	70	6.940	8.143
26	19.147	20.980	71	6.619	7.795
27	18.981	20 804	72	6.302	7.450
28	18.810	20.622	73	5.991	7.110
29	18.634	20.435	74	5.686	6.675
30	18.452	20.242	75	5.387	6.445
31	18.264	20.044	76	5.095	6.122
32	18 070	19.840	77	4.810	5.805
33	17.871	19.630	78	4.533	5.496
34	17.665	19.414	79	4.264	5.194
35	17.453	19.192	80	4.003	4.899
36	17.235	18 964	81	3.751	4.614
37	17.011	18.729	82	3.508	4.336
38	16.780	18.488	83	3.274	4.068
39	16.543	18.241	84	3.049	3.809
40	16.300	17.988	85	2.833	3,560
41	16.051	17.728	86	2.627	3.320
42	15.795	17.461	87	2 430	3.089
43	15.533	17.189	88	2.242	2.869
44	15.265	16.910	89	2.064	2.658
45	14.991	16.625	90	1.895	2.458
46	14.711	16.334	91	1.735	2.267
47	14.425	16.037	92	1.584	2.085
48	14.134	15.734	93	1.442	1.914
49	13.837	15.425	94	1.309	1.752
50	13.535	15.111	95	1.184	1.599
51	13.228	14.791	96	1.067	1.455
52	12.916	14.466	97	.958	1.320
53	12.600	14.137	98	.857	1.193
54	12.280	13.803	99	.763	1.075

A Second Note on Premium Loading.

WALTER C. WRIGHT.

IN a note on premium loading which I read to this Society in 1897, page 129, Vol. V., I endeavored to show by the aid of a specific illustrative case, the equity and advantage of the mode of "loading" for expense, and accumulating or determining reserve, discussed in my paper read to the Society in October, 1890; and especially as to affording an appropriate annual allowance for expense throughout its duration in the case of a policy issued for the payment of a single premium. Such policies are not at present very numerously issued. But continued term insurance, dependent on or paid for by the reserve or surrender value of a policy demanding the payment of a continuous or fixed number of annual premiums at its issue, now embraces a considerable and increasing portion of the paid-up insurance risk being carried by many companies, and materially adds to the large amount which these and other companies have outstanding on their books under policies on which the stipulated number of annual premiums have been fully met.

I now desire to draw attention to the equity and practical advantage of the above-mentioned mode of loading and computing reserve, as applicable to all such paid-up insurance, and to illustrate the point with particular reference to the first-named sort.

This is insurance undertaken without reëxamination upon the lapse of payment of premiums bearing annual expense loadings under the original status or terms of every policy to which it may apply. It is a sort of risk under which the company's mortuary experience is not unlikely to be more unfavorable than the general average, and it is also a risk which, while it lasts, may be increasing and greater in annual amount than that under the same policy, supposing no lapse of payment of premium had occurred, considering in each case the limitation of risk by premium held in reserve. If, as is customary, no discount or allowance is given to the individual freshly applying for a policy of life insurance, who saves the company the expense of the intervention of an agent by presenting himself at the office for the purpose, why should a person granted insurance of the description I am considering be given

better terms as respects future charge for expense than when continuing payments of premium? Such a thing would neither be equitable nor calculated to encourage persistence in payment of premiums, and I think I need not argue the point. But what is actual practice in this regard, and how would the method I favor remedy its defect?

I learn from the table of extended insurance of one of our leading companies that the holder of an annual premium whole life policy, taken at the age of 32, will be entitled to five years' extension of insurance upon lapse after the payment of five annual premiums. The net reserve of this policy, Am. Ex. 4 per cent., at the end of five years is $50.32, and the single premium for five years' ensuing term insurance is $41.92, showing an allowance for expenses (to say nothing of surrender charge which would seem to be due, in this case, in some measure) of $8.40 for the entire five years' insurance, or of $1.68 per $1,000 per annum. The only drawback to the liberality of this agreement is that the term insurance is made non-participating, the value of which feature to the company is, of course, uncertain and indeterminate in advance. The initial terms of the policy, according to the same table of mortality and rate of interest, allowed $6.52 on $1,000 annually for expenses in excess of the net premium. These are small figures, but represent exactly the conditions of a large and growing volume of insurance, and remembering that few companies now have any considerable portion of "loading" left after meeting their annual expenses, they show a sufficiently decided contrast to confirm my claim of their gross and injurious, if not dangerous, violation of equity.

Following the method recommended by me, and waving all lapse charge, the reserve at lapse of the above policy would be $57.14, and the term of temporary insurance purchasable by it four years and ninety-four days. The yearly provision for expenses, death cost of insurance and risk carried during the four full years of the period of extension, but under the terms of the policy as sustained by uninterrupted payments of premium, and also as a term insurance, would be as follows:

In Case of Continued Payments.

Policy Year.	Death Cost.	Expense.	Risk.
6th.	$8.58	$5.15	$930.10
7th.	8.63	5.17	916.90
8th.	8.66	5.19	903.21
9th.	8.71	5.22	889.06

In Case of Lapse.

Year of Extension.	Death Cost.	Expense.	Risk.
1st.	$8.81	$5.29	$954.68
2d.	9.10	5.46	967.43
3d.	9.41	5.64	982.17
4th.	9.75	5.86	996.03
94 days.	2.51	1.50	1,000.00

In producing these figures I have reckoned according to the Am. Ex. table plus 60 per cent. for expenses, and 4 per cent. interest, which gives a level annual premium of $23.28, or $6.10, due to loading more than the Am. Ex. 4 per cent. net premium. Slightly different assumptions would have given a premium affording an excess of precisely $6.52, but would not have been so convenient for my present use, and the above figures, of course, serve exactly as well to show how equably and consistently the method employed provides for yearly expenses, both under the initial terms of the policy and under the terms after lapse. Were it not for making this note too lengthy I would like to give an illustration also showing how the inconsistency between the term of extension after nine payments of premium on a ten-premium policy, etc., and the extension for the full policy term which the payment of one additional premium effects, when using the present modes of loading, entirely disappears when using that I am discussing.

As a preface to the remarks with which I desire to close, let me quote a paragraph from the paper of Mr. Van Cise, and one also from the address of President Miller, read at the last meeting of this Society.

The first is :

"In the light of this simple statement of facts, which shows that

the profits from nearly all sources are much less now than they were in 1868, has not the time arrived when the actuaries of America should stand like a stone wall against the reduction of rates of loading below what common prudence would seem to require? If 28 4-7 per cent. was not thought too much thirty years ago, should we not insist that at least 25 per cent. of the gross ordinary life premiums charged to-day should consist of loading, without going into the question of what rate of interest should be used in the calculation of net premiums and reserves? I, of course, refer to policies on which profits are promised, whether these profits are to be paid annually or at the end of a term of years; but even on policies without profits, should we not insist that at least 10 per cent. of the gross premiums should be loading? It is not likely that this low rate would cover expenses, but probably the deficiency could be made up from other sources."

The second is:

"In the construction of new premium rates I should recommend a uniform and quite high loading on Ordinary Life Policies and the same loading in dollars and cents for higher premiums, with an additional and comparatively small uniform loading on the excess of such premiums over the net premiums for Ordinary Life Policies. I have found that very satisfactory and well-graded loadings can be provided for by this simple rule. Arbitrary loadings, such as 30 per cent. on Ordinary Life Policies and 25 per cent. on Twenty-Payment Life Policies, are objectionable, because at age 70, for instance, the two classes of policies are substantially identical, and the gross office premium for the Ordinary Life Policy at advanced ages might thus exceed the Twenty-Payment Life rate. A great deal of trouble and inconvenience can be saved, and all inconsistency avoided by the adoption of a uniform rule which will produce a well-graded system of loadings for all classes of policies."

The fundamental purport of these carefully considered remarks is in no respect divergent, and is directly in line with my own conclusions. You will find that, considered as 3 per cent. premiums, those for whole life policies shown, by way of example, in my "Plea for Scientific Life Insurance Premiums," almost precisely satisfy the "25 per cent." minimum limit of loading set by Mr. Van Cise; and if Mr. Miller's closing sentence is worthy of strict regard, as I think it is, I fail to find a method of operation more perfectly adapted to satisfy it, and to simplify and harmonize all the features of life insurance accounting coming after the construction of com-

mutation columns. If the method outlined by Mr. Miller or any one else is more so, and is worthy of general acceptance, I insist that whoever may be its sponsor owes it to his profession to expand and illustrate it at least as persistently and consistently as I have this one for the past nine years. How, for instance, does Mr. Miller's plan apply to single premium and paid-up insurance? I believe I am as open to the conviction of new truth as any man, but I can unqualifiedly declare that I have met no criticism of the method I am urging in all these years which has raised the slightest shade of doubt in my mind as to its satisfactoriness. I was glad to note in Mr. Miller's address the affirmation that commissions must be made practically consistent with the values of the policies to which they pertain, instead of the reverse, as I think the contrary belief is the principal cause of doubt and difficulty in most minds on this subject.

The Normal Rate of Interest.

CHARLTON T. LEWIS.

IN computing premiums and reserves, the rate of interest assumed is at least as important as the rate of mortality. In fact, the probable error in the rate of interest determined by our present data and methods, is indefinitely greater than in the rate of mortality. While the latter has already been ascertained, for civilized society and the present age, accurately enough for practical purposes, we are still spending enormous labor in perfecting its details and the methods of investigating them. But the former is little understood, and nearly all study of it is guess work. Yet the difference between the Carlisle Table and the H^M 5 vanishes in comparison with the difference between 5 per cent. and 3 per cent. interest in our assumptions. Here, then, is the most attractive and promising field for the actuary, that which calls for his best exertions.

To discuss such a problem profitably, its terms must be accurately defined. Without reviewing the long economical controversies concerning the definition of capital and the nature of interest, I must begin by making clear and precise the sense in which I shall use these terms. Capital, then, is the aggregate of products of labor having exchangeable value. Its measure and the chief instrument of its exchanges is money. Money thus becomes the flux of capital, by which it is melted into a mass. Without money each kind of capital would have its own market, under special laws. All kinds are fused together by money, which makes them commensurable, so that the exchangeable value of each appears as purchasing power.

Interest is the difference in the exchangeable value of capital arising from difference of time.* The office of all capital is to

* Another side of economic interest affords an equally just view of its nature. If a given amount of labor alone in a year produces $1,000 in value, and the same labor working with a given capital for the same time produces $2,000 in value, it is evident that the increased product of $1,000 is due to the capital. In general if a desired product is produced in the same time by less labor with the addition of a certain capital than without it, the capital takes the place of the labor dispensed with, and may be said to earn or produce the same amount as the displaced labor, whose wages are now represented by interest. But it must not be inferred that because capital increases product, it therefore necessarily increases value. See esp. Böhm-Bawerk, *Capital and Interest*, Book II., Ch. 1.

satisfy future wants, but some parts of it will be consumed sooner than others. Immediate need is more pressing than that of a remote future, and this pressure makes the capital which satisfies the former more valued. This is the sentimental factor. But capital in hand can be consumed so that larger amounts of capital shall be produced. This is the industrial factor. Both make capital at hand or at an earlier date commonly exchangeable for larger capital at a later date.

This is a general fact, but not a necessary truth. For example, wheat deliverable in May is worth more to-day than wheat deliverable in December. The difference in quoted price is four cents; that is, a bushel of May wheat sells in May money for six per cent. more than a bushel of December wheat in December money. But May money sells in the same market for two and one-half per cent. less than December money. Hence May wheat is worth about three and three-tenths per cent. more than December or present wheat. This premium on May wheat is easily explicable. While this country produces more wheat than it needs, it will import none, and the approaching harvest in the other hemisphere does not concern us. The wheat already harvested will last till next summer, will not increase meanwhile, but costs for storage, insurance, etc. Hence May wheat bears a premium. On the other hand, wheat deliverable after the next harvest is often materially lower than cash wheat; anticipated abundance making the wheat cheaper in present money than present wheat. Thus in the case of great staples whose supply comes once a year interest is often negative.

Exchange agio is the difference in exchangeable value of capital arising from difference of place. The par of exchange between two places, A and B, is the precise ratio between the standards of measurement for capital in the two places. The money price of exchange fluctuates about this par; when money will purchase more at A than at B, exchange on A rises at B above par and the difference is premium; exchange on B falls at A below par and the difference is discount. Actual transfer of money takes place, of course, when the premium at A or the discount at B is enough to cover the cost of transfer, allowing current interest for the time occupied by the transfer. Thus, exchange at each place tends to adjustment so that the premium at A shall cover the cost of transfer plus the current interest at A for the time required, and the discount at B shall cover the cost of transfer plus the current interest at B for the same time.

Now the fluctuations in the rate of exchange between places are parallel to the fluctuations in the rate of exchange between times,* but with one fundamental difference. While all capital is provision for a nearer or more remote future, capital in general, made fluent and Protean in character by the use of money, has the property of productiveness. That is, a given capital can be so applied by employing labor and utilizing natural forces as to produce in the future a larger capital. This general property of exchangeable capital makes time a uniform factor in its value. It makes capital which is available earlier more valuable than the same amount at a later day. Since enterprise always stands ready to use capital productively, it is always willing to exchange for the capital available at the earlier date a larger capital of a later date, the addition being limited only by the expected increase of value. Hence, under the conditions of our industrial life, exchange on the future is always at a discount; that is, capital commands interest. The only important exception is found in staples, the preservation of which is a burden. The general law may be formulated thus: the normal level of local exchange is par, or the precise ratio of the local standards of valuation; the normal level of time exchange is par plus the average rate of productiveness of capital during the interval.

This normal level, like the true mean level of the ocean, is an ideal, about which the actual level perpetually oscillates, under countless disturbing forces. The limits of fluctuation are obviously not equidistant from the normal level. The ocean of capital has a bed, the absolute par of time exchange, below which the rate in an industrial society, and except for capital in a few special forms or under peculiar conditions,† cannot fail. But no upper

* Since this paragraph was written, I have found in *Beccaria, Economia Pubblica* Pt. 4, § 36 (referred to by Böhm-Bawerk, *Capital and Interest*, p. 51), these words: "L'interesse del danaro è l'interesse, ossia l'utile del tempo. Il cambio, ossia un valore ceduto in un luogo per avere un equivalente nell 'altro, ha il suo interesse specifico e particolare; cosi l'interesse del cambio sarà l'utile del luogo. *Da questa sola considerazione bene sviluppata nasce la teoria del cambio.*" But he makes no attempt to develop it himself, and his detailed analysis of exchange excludes this consideration entirely.

† Not rare in number, but small in proportion to the aggregate of industrial capital, such as houses with paid care-keepers, goods and even money in storage, etc. Other cases of an actual discount in time exchange, as in the transfer of capital from a beleaguered city, etc., are merely apparent, the discount being really insurance.

limit can be fixed. All fluctuations are governed by the familiar law of marginal utility ; so that, as soon as an actual deficiency of capital is revealed, extreme needs begin to assert themselves in violent competition, and the rate may rise indefinitely. This is especially the case with loans "on call," and largely with short discounts, when the marginal demand, from enterprises which must bridge a crisis or utterly fail, determines the local and temporary rate for all capital. But where a surplus supply of capital appears, more and more of it is withdrawn from the market as the rate falls towards par and the inducement to lend is lessened, until supply and demand are balanced at some level above par.

Thus every influence which disturbs the equilibrium between the supply of capital and the demand in any market will affect the rate of interest there. These disturbing forces may be grouped in two orders, which I shall call figuratively the wave forces and the tidal forces ; that is, first, those which act within definite and often narrow limits of space or time ; and secondly, those which act for long periods and upon the markets of the world. The two orders cannot be sharply distinguished, but grade insensibly together, yet the general character of each is clear. In the first order are preëminent the various disturbing forces which act through money itself.

Governments generally coin money, and thus do a great service to society. The perfect work of government in facilitating exchanges is done when money is coined in such amounts as come to the mints under the laws of production and trade. But governments rarely understand this. They go on to meddle with the amount and the value of the money supplied ; to manufacture substitutes for money ; to enact rates of exchange between different forms of real or fictitious money, or rates of interest, and in many ways to disturb the natural course of the markets. In former ages unscrupulous rulers would debase the coin ; modern legal-tender laws are a more disguised method of robbery. Each large issue of such currency causes violent fluctuations, first for a very short time in rates of interest on temporary loans, and then more lastingly in the nominal prices of goods ; and when, if ever, the paper is redeemed, or approaches redemption, there follows another series of spasmodic fluctuations.* These disturbances are aggravated by distrust, apprehension and uncertainty.

* See especially *Appreciation and Interest*, by Professor Irving Fisher, New York, 1896.

Further, in a highly developed commercial community, coin gives place for the largest use of money to credit. Payments are mostly made by clearances of bank deposits. The vast economy of the system justifies and necessitates it. But the available amount of such credits is sensitive to many influences; being disturbed by undue speculation and its subsidence, by technical legal restrictions, by special large enterprises, by individual or corporate failures. Every shade of change in confidence affects the immediate ratio of supply to demand of capital and thus the current rate of interest. The strength or weakness of banking reserves stands next in importance to public confidence itself in producing passing waves in the actual rate, and its effect is intensified by laws requiring a minimum reserve to be maintained.

Other wave forces which act with great intensity at certain times and places are wars and rumors of wars, exceptional and irregular expenditures by governments, special demands for money and credit at certain seasons, legislation upon currency, banking and taxation, or still oftener, the apprehension of questionable or disturbing legislation, and the activity of speculation in particular markets or branches of trade.

It will be observed that nearly all the disturbing forces named in this class have for their common and necessary effect, when their influence is at all apparent, an increase in the rate of interest. For the most part they either diminish the fluent capital within reach, without lessening the demand for it, or else increase the supposed hazard of the lender, and add to true interest an insurance premium. Hence it has become common to suppose that the actual current rate of interest regularly consists of the normal rate plus insurance; and to seek the normal rate for any period in the very lowest rate obtained upon the best investments.* This is a serious error, since certain disturbing forces depress the rate as surely, if not so largely, as others raise it.

The two most obvious groups of these forces are first, those which affect particular investments. For instance, the bonds issued by stable governments are eagerly purchased by special classes of investors, in preference to other securities yielding higher rates. This is sometimes in part due to a blind popular confidence in government; as in 1864, when the financial world was in dread of national bankruptcy, many men sold property and hoarded greenbacks. But more generally it results from the ready conver-

* Even Böhm-Bawerk follows the multitude of economists in this error.

tibility of these bonds in a wide market, and from the legal privileges conferred on them, as exemption from taxes, as a basis for bank notes, and as in all cases an investment acquitting trustees of responsibility. Many municipal issues, and even the best railroad bonds share some of these advantages. The rate expected from many investments, such as land and the shares of banks, trust companies, railroads and many industrial corporations, may also be largely depressed, that is to say, the market price raised above the capitalized value of the income from them, by the anticipation of increased value hereafter.

The other obvious group of forces often tending to depress the rate below the normal consists of those which concentrate waiting capital in the great markets. There is always a vast though variable mass of capital realized or withdrawn from former enterprises and awaiting investment in new ones. During the interval, it seeks employment in the great loan markets, and is a main source of supply. The amount varies with the seasons and the activity of trade, but often exceeds the effective demand for daily loans and short discounts in these markets, and rates of interest fall. The true character of the depression is evident from the wide difference between rates at such centres and rates elsewhere, especially for "call loans" and bank discounts. In New York and Boston such rates have usually been on the average about two per cent. lower, and have often remained for considerable periods from three to five per cent. lower, than the average bank rates for similar loans throughout the country.*

The disturbing forces hitherto discussed belong to the first order, the wave forces. The second order is for our purpose far more important. For the comparatively temporary and special effects of the former reveal their character, as modifiers, not determining factors, of normal interest. But there are certain forces which produce across the normal level of interest not so much local or transitory waves as secular tides over vast areas, and these may have every appearance of permanent changes in that level.

* These facts are a perpetual exposure on a vast scale of the fallacy of the doctrine so often taught even by leading economists, that money always flows to the market where interest is high. "Money flows thither, where its purchasing power is greatest, and that is not where interest is high. Capital flows thither, where interest is highest; but not all capital is money, and whether capital will flow whither it is needed in the form of money depends on whether money is there the form of capital most in demand."—Th. Hertzka, *Die Gesetze der Handels-u. Socialpolitik*, 1880.

Strong support for this assertion may be found in the study of the fragmentary notices of rates of interest left us from the earlier centuries of the Roman Empire, and again from the commercial history of Europe in the sixteenth and seventeenth centuries. But as trade and intercourse among nations are developed, and the relations of distant markets grow closer, the principle becomes clearer. The nineteenth century has witnessed a succession of tidal movements in interest, approaching periodicity, and covering the commercial world; only thinly disguised by the perpetual and irregular wave movements within narrower limits. Under all mediate fluctuations, there was a fairly continuous and progressive fall of average rates throughout Christendom for thirty years after 1815; a similarly constant rise for more than twenty-five years after 1845; and then a still more regular and universal decline for nearly as long after 1872. These movements did not begin or culminate everywhere at the same time. The rise about the middle of the century began in Germany and America earlier than in England, and the subsequent fall was more regular and uninterrupted in France and Germany than in England or here. Each movement was much disturbed by local and temporary influences, especially by wars, legislation and commercial crisis. Yet each can be traced with distinctness, on large averages, as an event in universal history.

This series of facts negatives many current theories. It utterly destroys the fancy that abundance of money in itself makes interest low. The most marked and general rise of rates ever known was in progress for the twenty years after the discoveries in California and Australia, which suddenly multiplied by twelve the world's yearly supply of new gold. In London, from April, 1852, to January, 1853, the bank rate was two per cent., and a common topic of conversation, in view of these discoveries, was "the utter hopelessness of there ever being again a high rate of interest in this country."* Yet the minimum bank rate rose steadily to five per cent. in 1853, and higher in succeeding years, and did not stand at the low average maintained from 1850 to 1854 during any subsequent lustrum until after 1893. The same set of facts refutes with equal conclusiveness the theorem taught by many economists, that increased wealth and economic progress of themselves lower interest rates.† Since the rate at any time and place must be im-

* Tooke and Newmarch, *History of Prices*, V., 288.

† Turgot, Dühring, Thorold Rogers, Baudrillart, Bastiat, Jevons, etc.

mediately determined by the relation between the supply of available capital and the demand, an increase of such capital must tend to lower the rate, if the demand remains the same. But to show that advanced civilization and growing wealth lower the rate, it must be proved that these changes do not increase the demand, or do not increase it so rapidly as the supply. No such proof has been or can be found.

On the contrary, all experience proves that the demand for capital finds its supreme stimulus in the expectation of productiveness. This expectation is excited chiefly by discovery and invention. To explore and bring under cultivation new tracts of land is to find new and profitable employment for capital; and interest has always been notably high in newly-opened countries. Now, the discovery of a new process, the invention of a new machine, the development of a new natural force, is in this respect like the opening of a new island or continent. The awakened spirit of enterprise promises to itself the richest return from every adventure, and sanguine undertakers compete for the capital it requires. The years following 1845 were the great period of railroad construction and of wonderful progress in industrial production. The rapid extension of the use of steam for transportation by sea and land, and of manufacture by machinery, absorbed wealth which had accumulated through many years of comparative stagnation; transferred vast masses of floating into fixed capital, multiplying the prospective product, and opening the promise of immeasurable wealth. The process continued until reversed by its overdoing; that is, until the consumption of floating capital and the demand for more had on the large average forced interest rates higher than its productiveness would support. The crisis of 1857 was a serious check, followed by rapid recovery. The crisis of 1873 marked the culmination of the tidal movement for the world.

That all nations were incomparably richer, that capital was more abundant, civilization more advanced, economic progress at a far higher stage in 1871 than in 1845 who can doubt? That in each of these respects the commercial world in the years 1850 to 1870 had moved immeasurably forward since the years 1750 to 1770 is beyond dispute. Yet the rates of interest in each case were generally and materially higher at the later than at the earlier date. Great as the increase of capital had been, the increase of demand for it had been greater. The obvious explanation is that the prevailing spirit of enterprise pushing into new fields had

forced the rate of interest upward, and kept pressing it ever to, or even above, the normal level of average productiveness. If it be asked what causes the alternations of enterprise and stagnation in the world of industry and trade, signalizing one period by inventions and discoveries which change the aspect of human society, and leaving another period to drowse in the quiet and uneventful accumulation of timid capital, no complete answer can perhaps be given. The physical and the moral universe alike are full of rhythmical movements whose causes are obscure. Explain it as we may, the astounding revolution in human affairs which took place during the great railway construction period was substantially ended about 1873; a long period of comparative quiescence followed, until within a few years past a new era of invention, discovery and aggressive enterprise has begun. No epoch in history has witnessed such revelations of the resources of nature for the service of man. None has excited such keen and eager search for new productive uses of capital. The evidence is strong that the tide has turned, and that the industrial and commercial experiences of half a century ago are about to recur on a vaster scale.

It seems probable that history will date the turn of the great tide in the year 1897. In that year the public funds of the great commercial nations of Europe reached their highest prices, and from that time till now every one of them has slowly fallen. In the years 1895 to 1897 the rates of discount in every commercial centre of the world reached a minimum, after a long, irregular decline; and from 1897 till now these rates on the average have everywhere, progressively though irregularly, advanced. The last class of investments to respond to changed conditions by a change in the rate of interest is always that of mortgages upon realty. But these, too, have begun to advance, so that the mortgage bonds of the great loaning banks of Central Europe are now issued at a higher rate than two years ago.* It is especially noteworthy that the advance of rates in Europe has been greatest in Germany, where industrial enterprise has been most active and industrial progress most rapid.

In this country the rise in rates of interest since 1897 has been large, but most conspicuous in call loans and discounts, which are

* The facts summarized in this paragraph are given in minute detail by Mr. Raphael Georges Lévy in two valuable papers in the *Journal des Economistes*, Nos 2 and 3, for 1899.

largely controlled by obvious wave forces. On the other hand, the price of government bonds has been sustained by special influences already described. But that a marked change is taking place in the average yield expected from first-class investments is indicated by the recent course of prices for gilt-edged bonds. For example, twenty-three such securities actually sold on the New York Stock Exchange, November 18th, when the markets were strong and confident under the rapid fall in the rate of call loans brought prices on the average of 3.14% lower than the average prices of the same securities in June of this year.* Meanwhile, interest had been paid upon them all for six months, the average rate being 2.13% of the par value. Thus, the decline has been more than the income, and the investors who purchased gilt-edged bonds six months ago and sell them now are on the average worse off by more than one per cent. of their whole investment than if they had hoarded their money, or loaned it without interest. The only and sufficient reason for this fact is that the real rate of interest on capital, which was for many years falling, is now rising again.

If further proof is asked that this explanation is complete, it is only necessary to point out that the securities in question are the debts of corporations, whose capital stock in almost every case has increased in market price while its bonds were falling. Suppose that two investors had employed a million dollars each in June in purchasing, the one in equal amounts, the standard bonds of these twenty-three companies, the other, in equal amounts, the stocks of the same companies (when the stocks are classified, assume that the investment is divided equally among the several classes), and that each makes up his profit and loss account by the prices of November 18th; the result will be as follows:

Total bond purchase	$1,000,000
6 mos. interest received	19,480
Market value Nov. 18th	969,550
Net loss	10,970

* The investment bonds sold on that day were Atchison 4s: Baltimore and Ohio 4s and 3½s: Central Pacific 3½s: Chesapeake and Ohio Consol. 5s: C. B. and Q. Extension 4s. St. Paul (Minn.) 6s: N. W. deb., 1909: Chicago Terminal 4s: Colorado South. 4s: D. and R. G. 4s: Ill. Cent. 4s. 1953: Manhattan 4s: Norf. and W. Consol. 4s: N. Pacific prior lien 4s: Reading gen. 4s: Rio G. West. 1st 4s: St. L. S. W. 1st 4s: So. R'way 1st 5s: U. P. 1st 4s: Wab. 1st 5s: West Shore (N. Y. C.) 1st 4s, and Wis. Cent. new 1st 4s.

Total stock purchase	$1,000,000
6 mos. dividend	15,867
Market value Nov. 18th	1,062,200
Net profit	78,067

Profits and interest are rising. The present valuation, therefore, of a fixed income from interest is falling, while the present valuation of future profits is rising. A movement has begun, the reverse of that which has been general and continuous for nearly a generation.

The forces which have turned the great tidal movement are obvious, and are as wide as the civilized world. Invention and enterprise have taken new life everywhere. The strong impulse to political expansion which has recently inspired the leading governments of Christendom corresponds with a similar impulse in every branch of mechanical activity, of production and trade. The rapid development of steam navigation, of railway improvement, of ship canals, of electrical art in a thousand forms, the increase of buildings, furnaces, mills, machinery, the opening of new colonies, in short, the conversion of floating into fixed capital, goes on at an accelerating pace. The indications are strong that the new era has but begun. Whenever hitherto such an epoch of invention and enterprise has checked a long-continued accumulation of idle capital and turned the great tidal wave of interest from ebb to flow, the process has been progressive for many years, and has continued to gain force and rapidity long after it had first become conspicuous. The apparent reasons for expecting this to be the case are far stronger now than in 1750 or in 1850.

How far will the movement extend? It is impossible to foresee to what extent disturbing forces will affect it. But it is not difficult to show that no such force is likely to reverse it or to diminish its impetus. Every influence which can be expected to act upon it for years to come will but intensify its effects. The forces which have so long depressed rates of interest could be effective only in an epoch of comparative stagnation of enterprise and timid accumulation. They are already largely overcome, and are rapidly weakening. On the other hand, a great war is always possible, and would consume existing capital and strengthen the demand for it, though without increasing its productiveness. But if the world's peace is maintained, there is not in prospect any check to the gradual rise of interest, at least until the average rate shall fully reflect the average yield of productive capital.

The question what that average yield is demands the actuary's methods applied to the data of the economist. My object is to stimulate inquiry, not to dogmatize on its results. But having for the last five years continuously avowed a strong conviction that the extreme depression in rates of interest was but an ebb tide which must soon turn, and having in 1898 ventured the confident assertion that the turn had come, I may now point out some of the leading and general facts on which these beliefs were founded. More than twenty years ago the late David A. Wells made a careful survey of the industrial activities of this country, and announced as the result of his study that the average annual yield of productive capital was full six per cent. and could not be expected to fall lower for at least a generation to come. Reviewing with him his data and his inferences, I was convinced that the assertion was correct. But rates of interest were already falling and continued to fall. Mr. Wells reëxamined the question, and published his conclusion ;* that the enormous increase of production, and competition for profitable investment, tend inevitably to reduce interest, without reference to the productivity of capital, to the minimum at which the motive for accumulation will disappear. In answer to my inquiry how he would reconcile this belief with the unquestioned fact that the yield of capital had not diminished, he admitted the inconsistency, but insisted that the facts were too strong for him. Our agreement to discuss the subject anew, in the light of the principles above unfolded, was frustrated by his sickness and death. But the facts as collected by him clearly indicate that the rate of interest for several years was below the average rate of increase of productive capital. When this is once established the inference is strong that temporary disturbing forces have depressed the rate of interest, and that the tide, which has fallen below the normal level, must rise again.

There could be no better measure of the true normal yield of invested capital than the average percentage of interest realized by life insurance companies upon their invested assets. But the determination of this percentage for any year or series of years is not a simple problem, and the answers to it which are commonly accepted are not wholly trustworthy. The materials exist for ascertaining the true answer, and the thorough analysis of them would repay the labor. For the present a brief suggestion of probable limits for the result must suffice. A little more than twenty years

*Recent Economic Changes, New York, 1891, pp. 418–422.

ago the rate obtained was full 6 per cent., and by some of the largest companies materially more. It fell by almost continuous stages till 1897, when it was apparently 4.92 per cent., and in 1898 had risen but a small fraction, to 4.98 per cent.* This may possibly be some further indication that the turn of the tide was reached in 1897, but of itself has little weight. It must be remembered, however, that the investments of these companies form a vast and gradually growing aggregate, so that the amount of interest received upon much the greater part of them has been determined in former years, and is determined during the year prior to the calculation only for new or renewed investments. If the assets are each year valued severally at or near the market price at the end of the year, the resulting rate is influenced by temporary market conditions. On the other hand, just to the extent to which purchase price or slowly modified book value is regarded in valuation, to that extent does the rate reflect the normal percentage of yield in former years rather than in the current year. These considerations must be taken into account, and the effect of each estimated in detail to reach the true average rate of interest. Such an examination will probably prove that the true rate in 1897 was considerably below the apparent rate of 4.92 per cent., and possibly somewhat below 4.5 per cent., but that the average rate for the whole period of declining interest from 1872 to 1897 was above 5.4 per cent.

It seems reasonable to believe that this last-named average, taken through a period of declining rates, fairly represents the permanent average income from safe investments, and that the minimum rate reached in 1897 by this great body of capital is the

* These rates are given by the Insurance Year Book for 1899, p. 449, and since the method by which the rates for all years are determined is the same, may be accepted as showing the movement of rates. But it does not show the actual rate. The reported increase includes additions to "assets" (positive or negative), from sales of investments. But if interest falls, securities yielding fixed rates rise in market value. "Profits" from sales of them, therefore, make the apparent rate of interest upon them the higher, the greater the fall in real interest. Again, the principal upon which interest is computed is the "mean assets" of each year; a sum which differs materially, though not always uniformly, from the sum which would purchase the securities at the average price of the year. On these and some minor features of the method an actuarial investigation is desirable. But it is clearly the general effect of the method to represent interest as much too high when it has been falling, and much too low when it has been rising, and thus to disguise and minimize the effect of tidal fluctuations.

minimum likely to be reached while the general industrial conditions and tendencies of our time continue. Several other lines of examination on which I have tentatively entered seem to converge towards the same conclusion. But while accepting this as a working hypothesis, it is of importance to remember that the average interest upon similar investments in some other countries, though subject to similar fluctuations, has uniformly been lower than in the United States, and that the facilities for the investment of capital and the interchange of market influences are increasing. On the whole, therefore, while it seems to be beyond reasonable doubt that the assumption of four per cent. as a rate of interest upon reserves is safe for at least a generation to come, a conservative provisional estimate of the permanent average yield to be expected hereafter from invested capital for many years would perhaps fix it at about five per cent.

Should a Three per Cent. Valuation be made Compulsory?

WALTER S. NICHOLS.

NEARLY all American financiers will agree that a lower rate of interest in this country should be anticipated in the future than has ruled in the past. The testimony so recently gathered by President James W. Alexander shows this fact, and the experience of the last thirty years amounts almost to a demonstration. Whether the time has come when a maximum rate of 3 per cent. should hereafter be assumed for life insurance purposes is a different question, upon which opinions will be divided. If it was a question de novo of computing premiums and reserves, I have little doubt that a 3 per cent. standard would be generally adopted. But the change of standard, with the complications which it involves, raises the query whether such change is called for, and if so, when. According to the last report of the Massachusetts Commissioner, the interest earnings of the companies reporting were 5½ per cent. on their mortgage investments, and about 4½ on other securities. Protected as they are by other profit margins, I presume that the safety of the companies will not be seriously disputed unless the obtainable rate should fall below 4 per cent. and, as the Commissioner says, the companies should have some justification for raising their rates from 10 to 20 per cent. The justification must be sought in the imminence of the apprehended fall. The voluntary action of some of our leading companies, however, shows that, in the judgment of their officers, the time has come when, as a matter of precaution, 3 per cent. should be assumed, at least for future contracts.

The factors which control the interest rate have, I think, never yet been fully comprehended. One economist will emphasize the demand for, and supply of, movable capital; another, the facility for its transfer; another, its profitable employment. Doubtless they are all interacting elements. More than sixty years ago De Morgan, writing on this very subject, pointed to the rate of 2 per cent. which had ruled in Holland in the eighteenth century, and the reduction from 10 to 6 per cent. in England since the time of

James I., and added: "The rate of interest has been halved within the memory of man, and a heavy war might double it again." To me the problem of the future rate seems as insoluble as in the days of De Morgan.

Our views are naturally influenced most by the facts most immediately before us. We have been eyewitnesses of its reduction through causes still in active operation, and likely, for a time, at any rate, to continue so. We have seen that capital is rapidly increasing and the country growing older, and that money can no longer be profitably used at former rates. We know that production, through the invention of new machinery, and new sources of power and new industrial plants, is increasing in excess of population, and that the concentration of capital and industry in trusts and corporations is tending to reduce profit margins, while checking the demand for loans from independent borrowers. All these things that point unmistakably to a continued falling rate are taking place right before us.

But there are other factors affecting the future which are not so obvious. Were inventions at an end, and all the world in the shape of industrial England and America, there might seem little hope for the future of the interest rate. But to view this question in all its breadth we should transport ourselves in thought to the twentieth century. During that century England's coal and, with it, apparently, her present industrial mechanism, will practically disappear. There is every reason to expect that before that time arrives new sources of power, calling for changed methods and places of production, will have superseded the old. In theory every wave beating on her shores, and every tidal current, as well as the sunlight falling on her plains, can be harnessed, and the power of distant coal fields condensed and transported to her factories, and the economic revolution forced on England is likely to be a type of what will be going on elsewhere. All this, if it be true, means the wholesale destruction of old, and the absorption of new, capital. The field yet open to the inventor is apparently limitless, and the costly experiment of to-day becomes the commercial process of to-morrow. A single new process or important improvement may wipe out millions of invested capital, and call for fresh loanable funds to replace them. The canals of our fathers, dug at such an enormous outlay, have lost their usefulness. Thousands of country mills are rotting, while the steam plants of Minneapolis are doing their work. Electric light and power are crowding gas and steam

in the field where, a few years ago, they held a monopoly. The short haul passenger traffic system of the country has been revolutionized by the trolley, and it is a debatable question among mechanical engineers whether the long haul may not eventually follow. This would mean the transformation of our entire steam transportation system. A small percentage of Niagara is already in harness. The great milling centres of New England are engaged in a desperate struggle to save their invested capital against the growing competition of cheap labor and newer machinery in the South. The automobile and the bicycle have despoiled the stock farms of the West. The telephone has come in to partially displace the telegraph and the mail for local service. Such are only a few signal instances of the march of invention and improvement which is wiping out millions of fixed capital and calling for new to take its place: a march so rapid that new processes and methods are among the chief risks of invested capital. And no end is in sight, nor reason apparent, why the march should not go on for a century to come.

If we may trust the vague evidence obtainable, the many interest rates of the Middle Ages in England ranged around 10 per cent. They had fallen with the gradual accumulation and increased security of capital to something like 3 per cent. a century ago. Then followed the great continental wars of Europe, the rise of standing armies, and of a national debt, and the rate again rose with increased absorption of capital. With the return of peace it again fell, until the development of steam transportation and the colonization of new countries since 1840 created another rise. Again the accumulation of wealth overtook the demand, and from 1870 onward we have witnessed a steadily declining rate. What most strongly impresses us is our personal observations since 1870, which is but a single fluctuation of the historic rate.

The 2 per cent. of Holland a hundred years ago was its inferior limit. New doors began to open and absorb the surplus funds. May we, too, not be near our inferior limit? The industrial nations of Europe and America comprise a small percentage of the world's people. What changes are to follow the development of Russia, the opening doors of Asia, and the colonizing of Africa, not to speak of other corners of the globe, we cannot even dream. When the tombs of the Pharaohs, and the haunts of the Bedouin, and the borders of Tartary are being invaded by the steam road, and the buzz of the trolley can be heard in the streets of Seoul, and Mongo-

lian Japan throws aside her seclusion and dons the cloak of Western civilization, what adequate picture can be framed of the probable demands for capital, or its productive power during the century to come? I feel, I say, that we are as much in the dark regarding the future of the interest rate of to-day as was De Morgan sixty years ago. If the security of capital, as well as its profitable employment, is a controlling factor in determining its earning power, there is some reason to think we may have reached an inferior limit, in view of the prospective fields before it.

A company may voluntarily assume a lower rate as a matter of precaution or expediency. But it does not follow that such a change on the part of all companies should be compulsory. The change might be advantageous to one and damaging, or even destructive, to another.

It might be easy for a company with a large surplus and difficult for one with a small surplus. It might be easy for an old company and ruinous for a young one. It might close the doors to legitimate competition, and create a virtual monopoly. These are all matters to be considered in a change of the legal standard, and if they are found to exist, then the change should be determined by its seeming necessity, and be made, if at all, along lines that will best conserve its interests. It seems to me that legislative regulations for business interests, shaped by speculation regarding the uncertain contingencies of the future, is a dangerous principle, at least in the States. When the four per cent. standard was generally adopted there was an implied agreement that companies might safely organize and conduct their business according to that standard, and any subsequent change which would jeopardize their business would be a breach of public faith unless manifestly necessary. A few years ago, when a 3 per cent. valuation was seriously agitated in the States, I believe its compulsory application to existing contracts would have wrecked a number of our companies. Under such circumstances it would seem to me that any change of standard should be so shaped as not to injure vested interests unless the danger was imminent. I think the interest question is apt to be safer in the hands of the insurance managers who are responsible for their companies and know their needs, than with lawmakers who make no pretensions to expert knowledge.

The indications now before us point unmistakably towards a falling rate; this it would be idle to dispute. But we may assume that capital will never lose more of its earning power than will

furnish an incentive to save, and fix an inferior limit. The historic story of the interest rate is that of a vibrating pendulum, and we have simply been living while the pendulum was swinging downward.

Europe is an armed camp in which the tension is constantly growing. The exploitation of Asia and Africa, with their spheres of influences calling for delimitations, may be but the prelude to a more Titanic war than De Morgan dreamed of, whose destruction of capital will mark the turning point of the pendulum.

Address of the President, Mr. Thomas B. Macaulay, in opening the Fall Meeting, October 19th, 1899.

Gentlemen of the Actuarial Society of America:

In attempting to fulfill the duties of the office to which you have done me the honor of electing me, I desire to express in the warmest tones my deep appreciation of the sentiment of good-fellowship and Anglo-Saxon brotherhood which prompted our American fellow-members to honor Canada by electing a Canadian to the presidency of this Society. I could have wished that the mantle of this international compliment had fallen on worthier shoulders, but it nevertheless serves to remind us that in the Actuarial Society of America Americans and Britons are not merely cousins, but brethren. As Kipling says, we are

> "Sons of the selfsame race, and blood of the selfsame clan,
> As we speak to each other face to face,
> And answer as man to man,
> And loyally love and trust each other,
> As only freemen can."

When considering what subject would be most appropriate for an inaugural address, it occurred to me that it might not be a waste of time if we were to glance briefly at the early history of our profession, with which we on this side of the Atlantic are perhaps not as well acquainted as we might be. Some very interesting facts regarding this are given by Mr. A. H. Bailey, Dr. T. B. Sprague and Mr. T. E. Young in the pages of the Journal of the Institute. From these gentlemen we learn that in the old days of Rome an Actuarius, or, as it was sometimes called, an "Actarius," was a person who kept the acta, or minutes, of the Roman Senate or of the courts of law. In modern language he was a registrar or recording secretary. This meaning continued to attach to the word, when, in course of time, it became Anglicized. An officer with the title of actuary has always been and still is attached to the lower House of Convocation of the Church of England. When the first British life assurance company, the "Amicable," was founded, the chief officer was called the "registrar." When, however, the old Equitable was established in 1762, the chief officer was, with equal appropriateness according to the ideas of that time, called the "actuary," and the modern use of the word may

be said to date from the perhaps almost accidental selection of the title by the founders of that company. Gradually but surely the word acquired a new and more limited significance. It was necessary that the chief officer of a life company should not only keep the records of contracts entered into, but that he should also be able to quote the terms on which the company would be willing to enter into such contracts. He naturally therefore, had to give attention to statistics regarding human life, and to their bearing on the financial interests entrusted to him; and thus it has at last come about that the name is applied only to those persons who are authorities in regard to the financial side of the science of life contingencies. The profession itself may almost be said to have had its beginning with the establishment of the Institute of Actuaries of Great Britain in 1848. Prior to that time there had been a few valuable contributions to the development of the subject, but in regard to most points, matters were in a very crude condition. Many of the managers of the companies were ignorant of the very basal principles on which their institutions rested; and worse than all, we are told that those who did understand the matter were actuated to a large extent by a spirit of selfishness, which impelled them to treat their knowledge as so much private capital, which they were unwilling to impart to others. There was no common ground for meetings and discussions, and until the "Assurance Magazine," now known as the "Journal of the Institute," was established, there was no publication devoted exclusively to such questions. The nearest approach to an Actuarial Society was the Managers' Association of the Scotch offices, which had been founded a few years previously, and in which each of the Scotch companies were represented by its chief official. This Society resembled closely the "Chamber of Life Insurance," which existed for a while among the American companies. It had done, and is still doing a good work, but its aims were and are more practical than scientific, and its membership is one of companies rather than of individuals. When the formation of the Institute on a purely scientific basis was proposed, the managers of a number of the leading companies opposed the innovation, and when they were overruled by the younger members of the profession, they withdrew and founded an opposition society working along lines somewhat similar to those of the Managers' Association. This society was known as the "Actuaries' Club," and continued in existence until the granting of the Royal Charter to the Institute of Actuaries,

when the two societies were happily merged into one. The club is, however, still kept alive as a social organization, admission to its ranks being, I believe, confined to members and ex-members of the council of the united society.

The Institute entered upon its career of usefulness in October, 1848, with a membership of 94 Fellows and 37 Associates, making a total of 131. Very early in its history it decided to establish examinations for the benefit of its junior members, and the first one was held in 1850. From this time on these examinations have taken place regularly, and I may say, in a parenthesis, that there has been a gradually strengthening opposition to the admission of members in any other way, and that now it is practically impossible to become a Fellow of the Institute except by passing the four examinations which are laid down. But resuming the thread of our history, the Institute by 1853, when it was five years old, had increased its membership to the splendid total of 263. It was not, however, destined to have the easy and continuous growth with which our own Society has been favored. Although the first suggestion regarding the formation of a Scientific Society would appear to have come from a Scotchman, Mr. W. T. Thomson, then manager of the Standard Life of Edinburgh, the Scotch members now became dissatisfied. They comprised, as Dr. Sprague tells us, more than one-third of the membership, and appear to have felt that they had not their proper weight in the deliberations of the Society, and derived but little benefit from it. There were also differences of opinion as to the principle on which the officers should be elected. In the year 1855 they withdrew in a body. A number of other members took the same course about the same time, partly, no doubt, for the same reason and partly from other causes. As a consequence, when the Institute completed its ten years of existence, in 1858, its membership had fallen back until it had only 45 Fellows, 18 "Official Associates" and 81 Associates, a total of 144. From this time, however, its progress has been steadily upward and onward to its present proud position.

If for a moment we pause to compare our own record with that of the Institute, we must remember that although we began with but 38 Charter Members and no Associates, the number of Life Offices existing fifty years ago in Great Britian was much in excess of the number located even now on this continent. When we further remember that our first ten years have been marked by nothing but harmony and prosperity, and that we have now 106 full mem-

bers and 6 Associates, a total of 112, being almost exactly three times the number with which we started, I think it will be agreed that we have reason for self-gratulation and thankfulness. And I venture to predict that we are but at the beginning of our growth so far as membership is concerned. Our Associate grade can hardly be said to be more than properly established, and with the opportunities and stimulus which our examinations now hold out, it is not unreasonable to expect that the number of our Associates will very rapidly increase. I note with great satisfaction that even since our last meeting sixteen gentlemen have applied for permission to take the first of our examinations. And it is with equal pleasure that I also observe that for the first time we have gentlemen coming up for the final examination. From this time on we are likely to have the satisfaction of welcoming each year full members to our ranks "by examination." The outlook for our Society, both as regards extension of membership and increased usefulness in every direction, was never so bright as at the present moment. The days of our childhood are past, and the period of our vigorous manhood has arrived.

The early defection from the ranks of the Institute was, perhaps, a blessing in disguise, for it resulted in the formation of the Faculty of Actuaries in Scotland. This body merely examines students and grants them its certificates. It has a valuable library, but holds no meetings. Its founders supposed that the presence of seniors would have a deterring effect on the younger men, and contented themselves with establishing, under the auspices of the Faculty, the Actuarial Society of Edinburgh, where the juniors could meet together and discuss subjects of interest. It is only of late years that this Society has printed its more important papers.

I will not take time to refer to the later history of the profession, with which we are all more or less familiar. I will merely point out that the best evidence of the present development of the science, and of the public recognition now given to the profession, is to be found in the facts that the parent Society is now incorporated in the old land by Royal Charter, and that in addition to it, the Scotch Faculty and The "Insurance Institute of New South Wales," there are now in existence, naming them in the order of their establishment: our own Society, The "Institut des Actuaires Français," The "Association des Actuaires Belges," The Dutch Actuarial Society and the Italian Association of Actuaries. There is also, of course, the permanent Committee for International Congresses of

Actuaries, whose headquarters are at Brussels, and under whose auspices two such Congresses have already been held, a third having been arranged for next year in Paris.

But while we look with satisfaction upon the record of successes already achieved, we live not in the past, but in the present, and we look not backward, but forward. Much has already been accomplished, but the justification for the existence of our Society is to be found not so much in the work already done as in that yet undone, and which we hope our Society will be able to do in the years to come. Our mission is to extend the knowledge and to aid the intellectual life of all the members of our profession. It is often said that advice costs little, but if I could I would say a few words which would reach the ears not only of our own members, but also of the great body of young men who are brought more or less into touch with actuarial matters at the head offices of our companies. I would remind them of the need, which is felt in all large Institutions, for men on whom responsibility can safely be placed, and who are endowed by nature and by education with the capacity for leadership. The world is full of persons who are only able to do what they are told, and who must be kept continually under the eye of a superior officer. Those who wish to rise above this lower level must show that they are worthy of promotion. Self-reliance, sound judgment, accuracy and thoroughness are largely, although not entirely, matters of temperament and early training, but education is quite as necessary, and it can only be acquired. The natural and the acquired qualifications often go hand in hand, however, for the man who is thorough, and who wishes to come to correct and well-balanced decisions, will desire to get to the bottom of things, and to understand for himself the why and the wherefore of what he does. If he be a clerk in some actuarial department, he will not allow his life to be dwarfed by the narrowing influence of his routine tasks. He will get out of the ruts into which one so easily falls, especially in a large office, and he will endeavor to make himself the master rather than the servant of his work. In bygone times it was difficult on this Continent for such a person to properly follow up the studies necessary to make him an all-round and thoroughly qualified actuary. There are few things more discouraging than isolated study, without companionship, without any special course laid down, and without even a special object in view. Reading under such circumstances is apt to become desultory and unsystematic,

only such points being taken up as happen for the moment to be interesting. This condition of affairs, is, however, now past. Our examinations supply both the course of study and the incentive that were formerly lacking. They give an opportunity to the young men in our offices which they never had before. I do not mean, however, that the mere passing of our examinations is proof that a person is able to fill the best positions that a company has to offer. There are many other qualifications besides education required in leaders of men, such as the principal officers our large life companies must be. Many of the greatest mathematicians have been pronounced failures in practical life. Yet, on the other hand, a thorough knowledge of the principles of the business would be of inestimable value to the executive officers of a life company, from the president down, and the possession of this knowledge is the possession of a very important qualification.

This brings us face to face with the question of the scope of the studies laid down by us for the students who enter for our examinations. Shall we limit ourselves to the purely mathematical side of life assurance, or shall we take a broader view, and include those aspects of finance and insurance law with which the manager of a life company should be familiar? Personally I strongly favor the wider scope. I believe that we have to a large extent the making of our own position in the insurance world. If we decide that we are to be mere mathematicians, then mere mathematicians will we be; but if we make ourselves authorities on investments, law, the drafting of policies, and, in short, those matters in general which arise at a head office, then we will assuredly be given in time that weight in the councils of our companies to which our knowledge entitles us. Such a course as this will do more than anything else to improve the prospects of our junior members, for if our graduates, if I may so call them, are qualified to take other positions than those demanding mere mathematical knowledge, many of them will certainly from time to time be promoted to positions outside of the actuarial departments proper, with advantage both to themselves and to those who remain.

But we will assume that the ordeals are safely past, and that our young friend has attained the membership in the Society for which he has been striving; is he then entitled to rest and to consider that his work is done? On the contrary, he should consider that the passing of the examinations is but the preliminary step, and that his real work is merely beginning. He has now the opportu-

nity to win his spurs. There are hosts of subjects to which he can with profit apply himself, and in regard to which the result of his thought and investigations will be warmly welcomed. I will venture to enumerate a few of the more important: The mortality among those engaged in various occupations, such as firemen, sailors, hotelkeepers, lumbermen and miners; among special classes of lives, such as the North American Indians, half-breeds, colored lives assured in the ordinary companies, both within and without the tropics; among lives assured in industrial companies; among those who took part in the Spanish War; among assured lives in the Southern States, distinguishing by counties, as was done in the experience of the thirty American offices; among lives assured in assessment companies, having special reference to the effect of heavy withdrawals; among unvaccinated policyholders; among policies kept in force after the tontine or semi-tontine periods; among policies kept up under the extended term provision, or other non-forfeiture regulation; among persons assured under different forms of contract; among persons having defective family histories, or who are otherwise more or less under the average; lessons to be drawn from the new British annuity experience; the birth, marriage and death rates experienced in the superannuation and pension funds of banks and other institutions, with suggestions as to the best bases for establishing same; benevolent societies, such as the Odd Fellows, Free Masons, etc.; sickness assurance; statistics of accidents in various occupations; popular delusions regarding actuarial matters, such as that shown by the recent publication of the Chicago Board of Health; the application of the theory of errors to life assurance matters; the various systems of loading premiums; methods of calculating surrender values; ratios of lapses and cancellations; legal questions, such as those arising out of policies in favor of wives and children; and, in short, the whole range of the science of probabilities and of vital statistics.

I would like to emphasize to all our members, old and young, that they owe a duty to the Society to prepare papers on such subjects as they themselves have been specially interested in. Such contributions need not be lengthy or elaborate; in fact many of the most interesting we have had have been mere notes of a few pages. Bacon well said: "I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavor themselves by way of amends to be a help and ornament thereunto." Our

Society is necessary to the profession. To a certain extent it is the result, but to a far greater extent it has been the cause of the growth and development of actuarial thought on this continent. The whole profession rises as it rises. Anything that extends the scope and interest of its meetings benefits us all. Every member who presents a thoughtful paper, or makes remarks of value in the discussions, contributes something. If any good ideas occur to you, or if you have any special statistics which you feel at liberty to communicate, give them to us. And it is not altogether a question of giving, for those who give the most are also those who receive the most, for study brings its own reward.

Just one other word in closing. What would we think of a student at a university who never came in personal contact with the professors, but had to be satisfied with the text-books, or with the notes of some other student? We would at once admit that such a person was handicapped, and could hardly be expected to keep pace with those who were able to attend the lectures. I take the liberty of saying seriously, that in just the same way those who fail to attend our meetings place themselves at a great disadvantage. The whole profession shares in the prosperity of the Society, but the rewards are not evenly distributed, and the best are given only to those who are present.

Abstract of the Discussion of Papers Read at Previous Meetings.

Note on Interest Rates. H. W. St. John.

Page 9, Vol. VI.

Mr. Messenger: Mr. St. John's thoughtful and carefully written article on interest rates shows very plainly the difficulties which we all come across in the matter of definition, when reading up on some subject in the line of finance or economics. These difficulties are generally due to one of three causes. First, improper definition; second, disregard of definition after once made; and third, total neglect to define at the beginning the important terms used in an article.

In addition I find there is also more or less misconception on the part of many in regard to the nature of a definition. I am strongly inclined to take the position that one has the right to define the terms which he is to use in an investigation in the manner which best suits his purposes. There is nothing in the nature of things which makes a particular definition the only correct one. I regard a definition as being, for the most part, simply a tool to work with. The best workman uses the best tools, and if the tool which he happens to be using does not answer his purposes, he puts it aside and takes a better one, provided a better one can be obtained.

We have illustrations even in the exact sciences of two essentially different definitions of the same term being used, and under such circumstances that we can hardly say that one is right and the other wrong. For instance, in nearly all arithmetics, we find a general agreement that a circle is a portion of a plane surface, bounded by a curved line, every point of which is equally distant from a point within called the centre, while the bounding line is called the circumference. On the other hand, when we come to study the higher mathematics, we find as a general rule that by a circle is meant the curve, which in arithmetic was defined as the circumference.

It hardly seems reasonable, therefore, under the most favorable circumstances, and in the case of the highest scholarship, to expect complete uniformity and general agreement in regard to the definition of scientific terms. From the very nature of things, we should look for more or less difference in the definitions given, just as we should expect more or less difference in the way in which certain words are pronounced, even by men of recognized education. Readers of Kidd's "Social Evolution" will doubtless remember his collection of ten definitions of the word "religion" given by men of acknowledged authority. It will certainly be admitted that about as many able men have attempted to define

this term as any word in the English language, and it certainly would be very desirable to have a general agreement as to its definition. Yet of these ten definitions there are no two alike, and as a rule they differ in essential points.

But while we cannot expect complete agreement in regard to the definition of scientific terms, there are two things which ought to be carefully regarded, but which are frequently neglected, and which lead to more or less misunderstanding. One is that as far as possible, if a term has once been defined, and that definition has been generally accepted and seems to meet the requirements, then that definition should be used. The other point is that when a definition has once been accepted, it should be strictly adhered to throughout the same discussion. If these two points had always been carefully observed by the writers on subjects in finance and political economy, the difficulties to which Mr. St. John refers in his paper would have been greatly lessened.

What Mr. St. John has to say in regard to variation in interest rates is most suggestive and its importance fully appreciated by actuaries. We have reached that condition in actuarial science where we can feel much more confidence in our estimates of the mortality for the next fifty years than in regard to estimates of the interest rate for the same period. A difference of 1 per cent. in interest may mean the difference between success and failure for a life insurance company. Fluctuations have been very great in the past, and there is no reason to expect that there will not be considerable fluctuation in the future. Cicero states in one of his letters that Brutus took 48 per cent.; Adam Smith gives the rate of interest in Bengal during the eighteenth century as 40, 50 and 60 per cent. On the other hand, about the middle of the eighteenth century the English Government borrowed money at 3 per cent., and France passed a law making all interest above 2 per cent. illegal. At the present time United States Extended 2 per cent. Bonds are quoted at a little above par.

In regard to conditions causing decline in interest rates, while I believe the statements made in the paper under discussion are, in the main, correct, as far as they go, yet I consider the question a much more comprehensive and complex one. The subject is such a difficult one that it is with some hesitation that I even suggest an explanation, but I am inclined to believe that a gradual decline in interest rates usually accompanies an increase throughout civilized countries in the extent and degree of what in evolution is called homogeneity.

Hand in hand with advancing civilization, notwithstanding its ever-growing complexity, goes an increasing homogeneity, one of the greatest characteristics of progressive evolution in enlightened countries of the present age. The great commercial nations are becoming daily more closely bound together; are gradually becoming subject to the same conditions. The same business methods are being adopted throughout the whole world of trade,

and nearly every country has the full benefit of any valuable invention a short time after its success is demonstrated.

Exploration, colonization and wonderful progress in methods of communication and transportation, within quite modern times, practically have doubled the territorial extent of the civilized world, and at the same time brought all parts closer together, and made them better places to live in and safer places for the transaction of business. As a result, commercial activity has been greatly increased and competition become greatly extended, more general and much closer. Profit in trade has become surer, but the chance for abnormal profits greatly diminished.

Clearly this great social and commercial evolution will affect future interest rates—rates on long term loans—in at least two important respects: first, it will diminish the extent of fluctuation; second, it will have a constant tendency to bring about a decline. Of course, there are certain conditions which are liable to be confronted at any time which, temporarily at least, may change the whole situation; for instance, a state of war between some of the great commercial nations, the passage of unwise laws, and a general failure of crops sufficient to disorganize the whole commercial world. But unless we are unduly pessimistic, we ought to feel safe in assuming that the laws of the future will be, on the whole, wiser than those of the past; that the wars of the future will be less frequent and shorter than those of the past, and that with every succeeding decade, the workings of nature will be more and more under the control of civilized man. Consequently it is with some degree of confidence that I repeat by belief that we can look forward to interest rates becoming gradually lower and more stable.

This conclusion, however, is not to be interpreted as meaning that the decline is to be uniform and continuous. Variation in interest rates is subject to all of those forms of rhythmic motion which characterize nearly all kinds of activity, whether manifested in the case of an individual life, or entering into some complex sociological question. The wave is often rising while the tide is going out.

SPECIAL TABLES FOR THE ESTIMATION OF MORTALITY AMONG ANNUITANTS.

EMORY M'CLINTOCK.

PAGE 13, VOL. VI.

MR. W. C. WRIGHT: We are greatly indebted to Mr. McClintock for having produced an excellently graduated table of mortality from the annuitant data collected by Mr. Weeks. Below the age of 40 it is almost entirely theoretical, but this is unavoidable owing to the scantiness of the data at those ages, and there can certainly be no reason to object to this as long as the accumulation of new data may not discredit it. Conservative as the new table is for valuing annuities issued on female lives, it is still not so much so

but that it gives a decrement exceeding the actual facts by $8,994, or 10.2 per cent., as applied to the exceptional female life experience of the Massachusetts Hospital Life Company, which I presented to this Society at our last meeting. As applied to the male life experience of that company, it exceeds the actual decrement by $7,689, or 17.6 per cent.; and it is not the most conservative table now available for the valuation of such annuities, though I think it is perhaps the most perfectly graduated.

What its rank is among the standard tables which we now possess for valuing annuities may be readily noted by reference to the accompanying table, in which I have arranged the values of q_x at stated ages, taken from several tables, in ascending order from left to right, and which, with the addition of the English Life Healthy Districts table, I have adapted from pages 210 and 211 of the volume of the "Combined Experience of Life Annuitants (1863-1893)," recently published by the British actuaries. It should be borne in mind that the graduation of the table of mortality deduced from this experience, as shown in the volume, is only provisional, which may account for the rank in my abstract of the values of q_x at different ages according to this table not being more fixed.

My conclusion, based on the figures of the table I have prepared, is that the Healthy Districts table is apparently the most conservative and best table we have for valuing male annuities, and Mr. McClintock's is so for female annuities.

The thought has occurred to me that with some critical adjustment an excellent table for both males and females could be composed from the least values of q_x at all ages, taken from the several available standard tables. If, as some think, the demand for annuities may greatly increase in future, and many may be written to take effect in disposal of the proceeds of life insurance policies, a very conservative table might not be suitable for the valuation of all sorts. There is no reason why annuities should not be written on participating terms as well as life policies; and to use an unquestionably conservative table with such a provision would be much better, I think, than to use one less so and be in doubt as to its safety. I am inclined to think that if the business of writing annuities does grow much in future without artificial stimulus, which I doubt, the general rate of mortality among annuitants, female as well as male, will be found to approximate to the rates of the Healthy Districts table.

TABLE 5.—W. C. Wright.

COMPARATIVE TABLE OF q_x ACCORDING TO SIX STANDARD TABLES OF ANNUITANT MORTALITY, ONE OF INSURED LIFE AND TWO OF LOCAL EXPERIENCE.

Division 1.—Males.

Ages.	E. L. H. D.	McC.	H^M	Gov., 1829.	French.	Gov., 1860.	Com. Ex., 1899.	Gov., 1883.
50	.01301	.01542	.01595	.01610	.01634	.01742	.01982	.01990
			Com. Ex., 1899	H^M			Gov., 1883.	Gov., 1829.
55	.01655	.02011	.02040	.02103	.02104	.02300	.02374	.02500
		Gov., 1883.	McC.	Com. Ex., 1899.	Gov., 1860.	French.	H^M	
60	.02367	.02695	.02750	.02810	.02846	.02962	.02968	.03150
	Gov., 1883.	E. L. H. D.	Com. Ex., 1899.	McC.	French.	Gov., 1860.	Gov., 1829.	H^M
65	.03661	.03680	.03859	.03911	.03917	.04065	.04070	.04343
	Com. Ex., 1899.		McC.	French.	Gov., 1829.		Gov., 1883.	
70	.05525	.05556	.05721	.05956	.06110	.06145	06163	.06219
	Gov., 1829.	Com. Ex., 1899.	E. L. H. D.	McC.	Gov., 1860.	Gov., 1883.	French.	
75	.07980	.08246	.08398	.08522	.08600	.08748	.09064	.09836
		E. L. H. D.	McC.	Gov., 1860.	Com. Ex., 1899.			
80	.12050	.12493	.12790	.13052	.13136	.13250	.14189	.14465
	E. L. H. D.	Com. Ex., 1899.	Gov., 1860.	McC.	Gov., 1883.	Gov., 1829.	H^M	French.
85	.18020	.18098	.18343	.19151	.19550	.20620	.20989	.21412
	Gov. 1860.	E. L. H. D.	Gov., 1883.	French.	H^M	McC.	Com. Ex., 1899.	Gov., 1829.
90	.25038	.25073	.27357	.27795	.27945	.28296	.28909	.37710
	Com. Ex., 1899.		French.	Gov., 1883.	Gov., 1860.		Gov., 1829.	H^M
94	.28322	.31445	.32584	.35374	.36648	.37966	.49800	.50730

TABLE 5—(Continued).

Division 2.—Females.

Ages.	French.	McC.	Gov., 1883.	Gov., 1829.	Com. Ex., 1899.	E. L. H. D.	Carlisle.	Gov., 1860.
50	.00938	.00960	.01123	.01200	.01220	.01221	.01342	.01365
		Com. Ex., 1899.	McC.	E. L. H. D.	Gov., 1829.	Gov., 1883.	Gov , 1860.	Carlisle.
55	.01192	.01279	.01319	.01410	.01430	.01585	.01639	.01792
	Com. Ex., 1899.	Gov., 1829.		French.	Gov., 1883.	Gov., 1860.	E. L. H. D.	
60	.01692	.01860	.01884	.01947	.02050	.02090	.02227	.03349
	French.	Gov., 1829.	Com. Ex., 1899.	McC.				
65	.02633	.02670	.02707	.02773	.02944	.03139	.03387	.04109
	Com. Ex., 1899.		McC.	Gov., 1883.	Gov., 1860.	French.		
70	.04117	.04140	.04166	.04457	.04781	.04815	.05085	.05165
	McC.	Gov , 1829.	Com. Ex., 1899.	Gov , 1860.	French.	Gov., 1883.		
75	.06334	.06390	.06682	.07182	.07388	.07548	.07719	.09552
					Gov., 1883.	E. L. H. D.	Carlisle.	French.
80	.09668	.10420	.11073	.11303	.11534	.11624	.12172	.12581
	Gov., 1829.	McC.	Gov., 1860.	E. L. H. D.	French.	Com. Ex., 1899.		Gov., 1883.
85	.12150	.14711	.16760	.17035	.17169	.17187	.17528	.17983
					Com. Ex , 1899.	French.		
90	.22010	.22134	.23693	.24031	.24361	.24502	.26056	.26632
	Carlisle.		E. L. H. D.	Com. Ex., 1899.	French.	Gov., 1860.	Gov , 1883.	Gov., 1829.
94	.25000	.30238	.30777	.33359	.33566	.34072	.34211	.39040

MR. MACAULAY: The granting of annuities dependent on human life was probably the very first beginning of the science of life contingencies. Traces of this practice are to be found as far back as B. C. 40, if not earlier. From that time onwards the system would appear to have gradually extended, and by the middle of the sixteenth century the buying and selling of annuities by corporations was comparatively common. The English Government in 1692 introduced legislation for the raising of money by the issue of life annuities, and in 1699 the first British company was founded for the express purpose of granting annuities. It might reasonably be expected that after an experience extending thus over several hundred years, the most complete and satisfactory tables would be in existence for the calculation of the premiums for such contracts. It is, however, a strange fact that this is not the case. There is probably no department of our business in which the want of thoroughly applicable tables is more felt. Every table we possess is more or less open to criticism. We have, for example, no proof that the class of people who purchase annuities from the British Government are the same as those who purchase annuities from the life companies, some supposing that the Government may receive a larger proportion of the class who have their annuities purchased for them rather than buy them. Then the valuable experience of the Fifteen American Companies, contributed a few years ago to our transactions by Mr. Weeks, may be objected to on the ground that it is hardly as extensive as is desirable, and that it moreover includes an undue proportion of lately admitted lives. The very extensive experience of the French companies has not had the results worked out for male and female lives separately, so that comparatively little use can be made of it. Moreover, we can not be sure that the mortality among Anglo-Saxon lives will be the same as that among Continental lives. There is little doubt that the combined experience of the British companies, now being prepared by the Institute of Actuaries and the Faculty of Actuaries, will, when completed, be the best guide which we will have in this matter. At present, however, we have nothing but the preliminary data, and we can hardly as yet adopt it as a basis. I had at first hoped that Mr. McClintock's tables might supply a suitable temporary basis for premiums, but examination shows that this is not the case. It was no doubt unreasonable to expect that the tables could be put to such a use, in view of the fact that the author himself makes no such claim for them; but on the contrary distinctly sets out that they are intended merely for the valuation of existing policies taken in the aggregate. It was quite proper, under such circumstances, to assume a rather heavier rate of mortality at the higher ages than that expected to prevail among annuitants entering at those ages; but that assumption unfits the table for use in calculating premiums. To ascertain how Mr. McClintock's tables compare with the new experience of the British companies, I have had the expected deaths worked out and compared with the actual. I have also grouped the figures of actual and expected for

the Fifteen American Companies, as given by Mr. McClintock, and included them in the comparison.

It will be seen that for male lives, at ages under 40, Mr. McClintock's table predicts a lighter mortality than that actually experienced by the assurance companies; from 40 onwards, the mortality predicted agrees very closely with the actual experience. On turning to the table for female lives, we find that Mr. McClintock's table predicts for ages under 50, in the same way, a lighter mortality than that really found to prevail; but from age 50 onwards it agrees very closely with the actual experience. The total deaths in the combined British and American experience were 99 per cent. of that predicted by Mr. McClintock's table in the case of male lives, and 106 per cent. in the case of female lives. These variations are on the side of safety. It is evident that Mr. McClintock's table may be put down as a suitable standard for the valuation of a body of current life annuities, and that of course is all that is claimed for it. To obtain a basis for annuity premiums we require "select" tables.

Mr. McClintock draws attention to the fact, which almost every annuity experience shows, that the mortality among male annuitants in middle life is actually greater than that found to prevail amongst assured lives. He states that it is reasonable to suppose that the mortality below age 60 among male annuitants will continue to be rather heavier in the future as it has been in the past, and that if so, there is no reason why rates should not be made accordingly, having due regard to the danger of combining annuities according to one table with assurance contracts according to another. There is, of course, much to be said in favor of this position, and yet my own opinion is that it is not wise for any company to issue annuities on lives which it does not know to be under the average, on any other supposition than that they are at least as good as the average assured life of the same age. To grant annuities to men who are in first-class health and belong to long-lived families, at rates so low that the company cannot reasonably hope to make any profit out of the transactions, but on the contrary may fairly expect to lose, is, I hardly think, justifiable, merely because the company hopes to obtain other lives sufficiently below the standard to enable the company to make enough profit on them to offset the loss it will make on the good lives. Such a line of reasoning would justify the acceptance of a poor life at ordinary rates, because of the hopes that enough good lives will be secured to obtain an average. The poor annuity lives may be tempted to go elsewhere, just as the good assurance lives may be. And in any case I think that every contract into which a company enters should be self-sustaining, and that no company should issue a policy of any kind which is likely to result in a loss to it. It is not unreasonable to assume that every person who applies for an annuity is as good a life as the average policy-holder in a life company, and that no concession below the rate thus required should be made unless actual proof of impairment of health or habits be shown.

The company that I am connected with has on several occasions taken into account the state of health of an applicant for an annuity, and as a consequence quoted premiums under the tabular rates. Only a few days ago we had a case submitted to us of a man who was partially paralyzed. It is, I know, rather risky to cut the rate for an annuity, and on this account our custom has been to make the net premium by the Government table a minimum, below which under no circumstances will we go. In other words, in the case of a decidedly under-average annuitant, we are willing to throw off all the loading, but not to encroach upon the net premium. Our experience with such cases has been very limited, but so far nothing has arisen to cause us to doubt the wisdom of this course. By it the first-class lives pay a premium commensurate with the benefits they receive, while concessions are limited to those really entitled to them.

What premiums should fairly be charged for annuities does not, properly speaking, arise out of Mr. McClintock's paper, and yet it is so closely associated with it that I take the liberty of stating here what my views on the subject are, and perhaps the briefest way will be to give the method by which my own company has recently computed a new set of these premiums, if I may be excused for doing so. The Government 3½ per cent. premiums for male and female lives respectively was taken as the basis. A proviso, however, was made to the effect that in no case should the net premium be allowed to fall below that required by the H^M Table with the same rate of interest. This proviso does not in any way interfere with the table for female lives, which is higher throughout; but it results in the substitution of the H^M annuity premiums for all male lives under age 51. And here I would point out that if we take an assurance experience, such as the H^M Table, as a minimum for annuity premiums, as suggested, we are not assuming that such annuitants are equal to lives who have just passed the doctor, but merely that they are equal to the entire body of lives in a company, most of whom have been on the books for many years. Moreover, this table predicts an even heavier mortality than companies on this continent expect to experience. It is therefore as reasonable a minimum as can well be named, if we are to have any minimum at all, or if we admit the principle that an annuitant no more than an assurant should be accepted at a premium below that fairly called for by his own personal health and expectation of life. To the minimum net premiums thus found a loading of 5 per cent. was added. The new experience of the British companies, however, shows that the Government net premiums are insufficient at ages from about 50 onwards. A special padding was accordingly added to the net premiums, beginning at ¼ of 1 per cent. at age 51, ½ of 1 per cent. at age 52, and increasing by ¼ of 1 per cent. for every age thereafter. This, of course, is purely arbitrary, and such an addition will probably not be necessary when we can base our rates upon the new experience. It, however, I think serves fairly well as an interim arrangement. Even apart from the infor-

mation furnished in the new experience, there is another reason which should, I think, influence us in the direction of loading somewhat the annuity premiums at the higher ages. It is possible for a man to foresee with comparative accuracy what his state of health will probably be for a year hence for example, excluding of course accidents and zymotic diseases. He can form a more or less correct opinion as to his prospect of living, say, five years; less accurate as to his prospect of living ten years, and still less as to fifteen, twenty or twenty-five years. It is thus evident that a person aged 75 or 80, with a total expectation of but a few years, has the power of exercising selection against the company to an extent which is quite impossible to one aged only 45 or 50. When we remember that the loadings usually charged —the margins for safety and profit—on annuity premiums at these higher ages, would be wiped out if the annuitants were to live even six months longer than the tabular expectations, I think most of us will agree that it is desirable to specially protect the company at these ages. It must also be remembered that in the case of young lives we have the use of the money for a long term of years, and have the opportunity therefore of making a profit from surplus interest, while there is but little time to make such a profit in the case of elderly annuitants.

In closing I may say that for my own information I have had the ages of the entrants in the government experience and in the new British company experience analyzed, and the result is shown in the following table:

Comparison between Government Annuity Experience and Institute New Experience (Combined Old and New) Entrants in Decennial periods of life.

Ages.	Males.				Females.			
	Government.	Percentage of Total.	Institute.	Percentage of Total.	Government.	Percentage of Total.	Institute.	Percentage of Total.
Under 20	6	.05	7	.10	34	.18	19	.10
20–29	110	1.01	47	.70	264	1.32	133	.70
30–39	403	3.69	144	2.14	880	4.43	402	2.12
40–49	1,188	10 87	468	6.96	2,638	13 28	1,440	7 60
50–59	3.032	27.74	1,370	20.36	6.135	30.89	4 608	24 32
60–69	3,899	35 68	2,615	38.87	6.955	35.02	7,677	40.51
70–79	1,952	17 86	1,732	25.74	2,702	13.61	3,922	20.70
80–89	329	3.01	335	4.98	248	1.25	717	3 78
90–102	10	.09	10	.15	3	.02	32	.17
Total	10,929	100 00	6 728	100.00	19,859	100.00	18,950	100.00

AVERAGE AGE AT ENTRY.

Government Males, - - - -	60.2
" Females, - - -	58.3
" Total, - - - -	59.0
Institute Males, - - - -	63.6
" Females, - - - -	62.4
" Total, - - - - -	62.7

From this it is evident that the government receives a much larger proportion of entrants at the younger ages than the assurance companies do, and as those entrants who are not self-selected enter chiefly at these younger ages, it is not unreasonable to assume that the government receives a larger proportion of such non-selected cases than the companies do, and that a table based on the government experience is therefore hardly suitable for use by assurance companies.

Annuity Experience.

Male Lives.

Comparison of Actual Deaths with Expected—by Mr. McClintock's Table.

Age	"Combined Experience of British Life Companies." (1863–1893.) Aggregate Table.				15 American Companies. (Weeks'.)			Experience of British Companies and 15 American Companies Combined.		
	Exposed to Risk.	Expected Deaths. (McClintock.)	Actual Deaths.	% Actual of Expected.	Expected Deaths. (McClintock.)	Actual Deaths.	% Actual of Expected.	Expected Deaths. (McClintock.)	Actual Deaths.	% Actual of Expected.
15 to 29	233	1.910	3	157.1				1.910	3	157.1
30 " 39	773	7.373	12	162.7	3.5	2	57.1	10.873	14	128.7
40 " 49	2,414	30.956	29	93 6	19.7	23	116.7	50.656	52	102.6
50 " 59	7,670	160.235	174	108.6	101.0	107	105 9	261.235	281	107.6
60 " 69	17 471	698.480	666	95.3	362.7	356	98.1	1,061.180	1,022	96.3
70 " 79	18,522	1,507.281	1,496	99.2	593.5	613	103.3	2,100.781	2,109	100 4
80 " 89	6,078	1,014 099	1,002	98 8	298.2	297	99 6	1,312.299	1,299	98 9
90 " 99	407	134 180	119	88.7	30.1	17	56.5	164.280	136	82.8
Total	53,568	3,554.514	3,501	98.5	1,408.7	1,415	100.4	4,963.214	4,916	99.0
15 to 39	1,006	9 283	15	161.6	3.5	2	57.1	12.783	17	133.0
40 " 99	52,562	3,545.231	3,486	98.3	1,405.2	1,413	100.5	4,950.431	4,899	98.9
Total	53,568	3,554 514	3,501	98.5	1,408.7	1,415	100 4	4,963.214	4,916	99 0

Female Lives.

Age	"Combined Experience of British Life Companies." (1863–1893.) Aggregate Table.				15 American Companies. (Weeks'.)			Experience of British Companies and 15 American Companies Combined.		
	Exposed to Risk.	Expected Deaths. (McClintock.)	Actual Deaths.	% Actual of Expected.	Expected Deaths. (McClintock.)	Actual Deaths.	% Actual of Expected.	Expected Deaths (McClintock.)	Actual Deaths.	% Actual of Expected.
15 to 29	705	2.890	5	173.0				2.890	5	173 0
30 " 39	2,230	11.394	23	201.8	2 6	3	115.4	13 994	26	185.8
40 " 49	7 481	57.288	78	136.1	15.5	16	103 2	72 788	94	129.1
50 " 59	25,112	347.783	338	97.2	84.4	83	98 3	432.183	421	97.4
60 " 69	58,528	1,657.148	1,610	97.2	291.9	286	98.0	1,949.048	1,896	97·3
70 " 79	58,067	3.497 331	3,662	10.47	457.8	442	96 5	3 955.131	4,104	103.7
80 " 89	19,772	2,527.930	2,951	116 7	239.4	249	104.0	2,767.330	3,200	115.6
90 " 99	1,546	400.298	434	108.4	31.9	25	78.4	432.198	459	106.2
Total	173.441	8,502.062	9,101	107.0	1,123.5	1,104	98.2	9,625 562	10,205	106.0
15 to 39	2,935	14 284	28	196.0	2.6	3	115 4	16 884	31	183.6
40 " 99	170,506	8,487.778	9,073	106.9	1,120.9	1,101	98.2	9 608.678	10,174	105.9
Total	173,441	8,502.062	9,101	107 0	1,123.5	1,104	98.2	9 625.562	10,205	106.0
15 to 49	10,416	71.572	106	148.1	18 1	19	104.9	89 672	125	139.4
50 " 99	163,025	8,430.490	8,995	106.7	1,105.4	1,085	98.1	9.535 890	10,080	105.7
Total	173,441	8,502.062	9,101	107.0	1,123.5	1,104	98.2	9,625.562	10,205	106.0

MR. LEWIS: These figures are taken on the number of lives entering, and not at all on the amounts. Wouldn't it be modified very essentially if the percentage was taken of the amounts? The amounts entering at the lower ages are very much more nearly in correspondence.

MR. MACAULAY: The British Government does not publish the figures by amounts.

MR. LEWIS: Inasmuch as the amounts taken by the Government are ordinarily very small, and even those taken by the companies on young lives are very much smaller than those taken on older lives, it seems it would be very desirable to get at it by amounts.

MR. MACAULAY: I am not in a position to give that information.

MR. LEWIS: Permit me now to call your attention to one feature of this paper which seems to me to be of absorbing interest and which gives a broader aspect to the whole discussion. In the author's prefatory remarks and, indeed, throughout the paper, he shows that his views are influenced by the strong probability that there is a progressive improvement in the lives of that portion of the community at least which is taking out annuities. Now it happened to me a year or two ago to go into a somewhat extensive examination of vital statistics, as far as they could be obtained, with reference to the question whether there is a progressive improvement in the average life and vitality of the community. Of course, some of the most important elements of that inquiry are the data upon which mortality tables and annuity tables have been constructed. Without going into details I wish to point out to you one fact which I found very startling, and I should be exceedingly glad if any one who has studied it more carefully would explain. While the data on which mortality tables, taken in the order of time, have been constructed, so far as I can ascertain from examining them, give no clear indication of a progressive increase in vitality, the successive tables of annuities, on the contrary, do give such an indication. I think you will find it to be unmistakable in examining the tables of annuities. But while it is possible to arrange the data on which mortality tables are constructed so as to indicate such a result, yet the proof is by no means satisfactory. That is, the data on which life tables are founded, taken alone, give no clear and satisfactory indication of a progressive improvement in mortality.

Another question I would like to put to Mr. McClintock, and which, I have no doubt, in the preparation of this paper has occupied his thoughts, and that is whether or not there is not a progressive improvement in the average life of the community? Most statisticians seem to agree that during the last seventy-five years, in fact, during the last century, there has been a marked increase in the average life, yet there seems to be a very considerable number, the importance of whose names and opinions none of us will gainsay, who think that while the average life has been increased, while the expectation of life at ages under

forty has been very materially increased, there has not yet been shown any addition whatever to the expectation of life at ages above fifty. Now, if this is found to be sound, it would tend somewhat to counterbalance the conclusions which have been suggested with regard to the probabilities of the continuance of annuities.

MR. MCCLINTOCK: In response to Mr. Lewis's question, the improvement in medical science, which we know has taken place, has tended to keep people alive who otherwise would have died, yet I agree that medical skill has really done little for old people who have learned to take care of themselves. They are apt to die of such diseases as indicate a breaking down of the constitution. I agree with him (Mr. Wright), as a matter of course, that these tables of mine do not give as low a mortality on the average as the Healthy English Table. In constructing these tables I had rather the view of striking the mean of annuitants' experience, so far as we have had any large body of annuitants' experience to go by. As Mr. Macaulay has indicated, my tables were constructed for the purpose of forming a basis for reserve calculations on a body of annuities for which it is natural to prefer to make the reserve on some other basis than the Actuaries' Table or the American Table, and the lack of some such standard for annuity experience caused me to look into this matter to see what I could do, tentatively and temporarily only, for I have no idea that these tables can supply a permanent standard. They are designed for present use, as far as they may be found useful as a fair basis for reserves. As stated repeatedly in my paper, they were intended for reserve calculations. I did not intend that this table should be used for premium calculations, unless indeed with just this allowance that Mr. Macaulay makes in the way of special loadings at advanced ages. The course he has stated as having been pursued by his company has been repeatedly pursued by the larger New York companies, as regards loadings at the older ages. The paper of Mr Weeks, read this morning, does indicate, I think, that for practical purposes, as annuities run, these tables are a fair guide to the probable mortality. On one point referred to by Mr. Macaulay I do not quite understand whether he agreed with me in this statement I made, namely: "It is reasonable to suppose that the mortality below sixty, among male annuitants, will continue to be rather heavy." I do not quite know whether he thinks that is correct or not.

MR. MACAULAY: My idea is that it is possible, but that we are not justified in assuming that it will certainly be so.

MR. MCCLINTOCK: It seems to me reasonable to think that the males who take annuities below sixty will continue in the future as in the past to include a large proportion of persons whom parents or caretakers find it desirable to help by the purchase of annuities, and indeed I doubt whether many men having control of their own money do purchase annuities below sixty. Unless some attention were paid to the rather heavy mortality of male annuitants it would be difficult to make rates that would compete with those which are prevalent in the business.

MR. CRAWFORD: From my observations, I find myself in entire harmony with Mr. McClintock's last remark. My attention has frequently been called to the number of inquiries that we have had from guardians and trustees, etc., for annuities for dependants and orphans, on lives that were acknowledged to be under the average. We might expect upon the part of annuities taken out by guardians, trustees, etc., that the mortality would be heavier than if the annuities were bought by the parties themselves.

MR. NICHOLS: These remarks suggest a mental experience of my own. I don't know whether it has any application at all to the facts before us. I have often thought that if I had no family dependent upon me I would invest my means in an annuity and then go out and have a good time while I lived. Now I was wondering whether this disposition on the part of some members of the male persuasion might not influence the character and the mortality of these annuities of which Mr. McClintock spoke.

MR. WRIGHT: In regard to what has been said in reference to the increased longevity of the community I might remark that Dr. Abbott, Secretary of the State Board of Health of Massachusetts, who has given much attention to the subject and has prepared statistical tables from the Registration Reports of the State that appear to show that there is such a distinct improvement, and that it results from checking the spread of contagious diseases. Of course if you examine these reports you will for yourselves decide how reliable his conclusions are. It may be observed that they consist with what has been said that the improvement has appeared greater among the younger than among the older lives, for the reason that it is the younger that are more exposed to the influence of contagious diseases.

As I listened to what was said in regard to the evidence in this line of the Government tables, I examined my own table prepared to illustrate my comment on Mr. McClintock's paper to see if it showed any evidence of what was said here, but it does not appear to. As far as it indicates, the rates of mortality of the Government tables of 1829, 1860 and 1883 are inextricably mixed, and the later shows no more tendency than the earlier ones to retain the first rank in my table.

MR. MACAULAY: I would like to make a brief explanation. I do not take the position that all persons who apply for annuities at the younger ages are first-class lives, but merely that we are not justified in assuming, without clear evidence in each particular case, that they are not as good lives as the average persons now assured in our companies; or, in other words, that we are not justified in assuming that they are not equal to the average of the community. I need hardly say that any suggestion from Mr. McClintock carries great weight with me, whether his views agree with my own or not, but in this instance I do not think that we are justified in assuming that the companies will always receive as large a proportion of non-self-selected lives as the government has been in the habit of receiving, or even as the other companies

have received in the past, and that in any case it is desirable as far as possible to treat each annuitant upon his own merits, just as is done in the case of applicants for life assurance.

MR. LUNGER: Referring briefly to Mr. Lewis's question, I think that it is well to note that the recent improvements in medical treatment and hygiene have had a more appreciable effect upon infantile mortality than upon adult mortality. You will find that the mortality at the very youngest ages of life has been reduced in this country to a remarkable extent in the last ten years, and this saving of infantile life will of course bring forward in the adult ages, as time goes by, a larger percentage of persons than formerly, and this will tend to increase the expectation of life, especially in the earlier years of adult life.

THE EQUITIES OF THE POLICY-HOLDER UNDER TERM EXTENSION WHERE LIENS EXIST ON THE POLICY. JESSE J. BARKER.

PAGE 24, VOL. VI.

MR. E. P. MARSHALL: Mr. Barker calls attention in his very timely paper to a practice which has become almost, if not quite, universal among all the companies of granting an automatic extension of the original amount of a policy upon a failure to pay the premium. In this day of sharp competition, in many instances, this extension provision proves the last convincing argument with the prospective applicant. I think that all will agree that, ordinarily, as a measure of equity to the policy-holder, the plan has proven satisfactory to the companies and all that could be claimed for it by its originator. Still it not infrequently occurs that a policy-holder, after the expiration of the period of extended insurance, claims that he has been unfairly treated by the company.

Nine States have passed laws prohibiting the forfeiture of life insurance policies for the non-payment of premiums. All of these laws, I think, without exception, provide a fixed surrender charge. A difficulty, however, is encountered where a policy has been allowed to lapse by the insured, and where liens exist arising either from unpaid premiums or for cash borrowed of the company. Mr. Barker very clearly shows how, under existing contracts and the laws as they stand, it would be possible for a company to pay, in case of a policy becoming a claim by death, during the period of term extension, a sum in excess of the original face of the policy. In order to prevent this injustice being done, his suggestion to consider the policy as a sum of its parts, and not as an entirety, where a lien exists, is certainly worthy of serious consideration, as furnishing a possible equitable method of solving the difficulties of the situation. This plan, if adopted, could certainly work no hardship to the insured, and would be only just to the company.

The second plan suggested by Mr. Barker is to deduct the amount of the liens from the face of the policy and grant extended insur-

ance for such time as the balance of the cash value would purchase. While it is true the company might not be called upon to pay under such a contract more than the face of the policy, yet I imagine it would lead to such demands upon the company for still further concessions that it would be practically nullified.

It seems to be quite evident that some protection greater than what is now in vogue should be considered by the companies. Experience has proven in too many instances that the insured at the time of obtaining a cash loan upon his policy had no intention of ever repaying the loan or paying any future premiums, but simply adopted this method of securing as much cash as possible upon his policy. To meet this tendency some companies have in making loans adopted a form of contract, or inserted conditions in the note, by which it is endeavored to cover such cases.

The following extract is taken from a form of note used by one company making cash loans upon its policies, and I think covers the point above referred to :

"If any instalment of interest on this note or any premium or premium note on the aforesaid policy shall become due and be unpaid, then forthwith the principal of this note, with interest thereon to the date of such default, shall be immediately due and payable. If the said policy shall, at any time, lapse for non-payment of premium, or premium note, all provisions in said policy providing for the issue of a paid-up, or term policy, shall thereupon, and by reason thereof, forthwith become null and void."

It is also the practice with some companies to provide for notice and sale of the policy in case there is a failure to pay the premium or the annual interest accrued upon the loan. Where loans are outstanding against the policy at the time of lapse, or where the policy has been assigned to the company by the policy-holder, or the legal holder thereof, the provision is usually, if not always, embodied, that the policy may be disposed of at public sale or otherwise, due notice to all concerned being given. The opportunity is thus granted, in that event, for any or all interested to come forward and buy in the policy for the insured or the beneficiary, as the case may be. Upon the other hand, if the company itself is the purchaser, the amount loaned practically becomes a cash surrender value paid for the policy.

I think it may be safely assumed that, as a result of the large and liberal loans now granted by the majority of companies upon their policies, in very many cases where these loans are being granted, it will result practically in a purchase of the policy by the company. In such cases neither equity nor justice demands a continuation of the original policy, and where such a policy is forfeited by reason of the non-payment of either the premium or interest, the best interests of all will be better served by a provision in the policy or note providing for the cancellation of the policy outright and the payment to the insured, or the legal holder, of the balance of whatever cash may be due as a surrender value at the date of lapse.

It is a question whether in bringing so prominently to the attention of the insured the ease whereby a policy can be surrendered or a cash loan obtained, the companies are not thereby encouraging the policy-holder to hypothecate his policy, and in too many instances to use the money so obtained in speculation or questionable investments. While I believe every facility should be extended to the insured to keep his insurance in force, and every assistance possible, consistent with safety, rendered whereby he may be enabled to tide over a temporary financial embarrassment, yet on the other hand the policy-holder should be discouraged from endangering that security which has so often proven the only support of old age, and in event of early death the mainstay of the widow and orphan.

MR. LUNGER: If I were to confine myself to the subject selected by Mr. Barker for his paper, my remarks would necessarily be very brief. Since the subject of Term Extension in general is one of more than passing interest, I shall take advantage of the latitude which is permitted in discussions before the Society to lead up to my views of his recommendations by stating some reasons why Term Extensions are helpful to the companies and why they are in keeping with the best principles of life insurance. I will also endeavor to analyze briefly the part that "selection" plays under Term Extension.

Undoubtedly Mr. Wright's object in proposing the Massachusetts Non-forfeiture Law of 1861, by which Term Extensions were first introduced, was to compel the companies of that time to recognize, without the formality of complying with technical requirements, the right of the insured to an equitable interest in the reserve on his policy. The first company of any size to voluntarily introduce Term Extensions in its policy was the Mutual Benefit of Newark. Its decision may have been influenced by the example of the Massachusetts law, but, knowing as we do, the sincerity which marks every official action of the Mutual Benefit, it is reasonable to believe that their chief motive was to extend to their policy-holders the benefit of a safe and prudent concession, which would protect policy-holders from the result of their ignorance or neglect.

Since these initial steps in the introduction of Term Extension, one company after another has adopted the plan until at the present time forty-two of the companies whose policies are printed in Flitcraft's Manual, including the Massachusetts companies, grant Term Extensions. Some of the companies which have voluntarily adopted the plan have no doubt been led to do so for the same reasons as the Mutual Benefit. Other companies have found the plan necessary to meet competition, and still other companies have adopted the plan merely to make their contracts more attractive; but whatever the reason, the result, in my judgment, has been beneficial to the business. The equity of the policy-holder in the reserve has been recognized by a plan which enables the insured, under stress of circumstances, to secure the full protection of his insurance for a stated period. The attractiveness of policy con-

tracts has been added to, thus making it easier for agents to sell them, and also enabling managers to offer lower commissions; while the office routine and records of the companies have been simplified by largely doing away with small paid-up policies which are difficult to trace to their termination and expensive to continue on the books. Therefore, it may be asserted that equity, economy and the recognition of the best principles of life insurance all call for Term Extensions.

Of the forty-two companies giving Term Extensions twenty-six grant the benefit without application from the insured (automatically). This practice, I trust, will be followed by all the companies, either with regard to Term Extension or Paid-up Insurance, as it utterly destroys any ground for complaint that the companies benefit by a non-compliance with technical requirements. Moreover, in a mutual company surrender values granted automatically fittingly fulfill the obligation of a trusteeship, virtually accepted by such a company when it issues a policy.

Several companies have been deterred from granting Term Extensions by the fear that it would encourage an adverse selection against the company that would prove over-expensive in practice. In his paper Mr. Barker expresses the fear referred to, and states that, "if all healthy lives upon retiring selected paid-up insurance, while the unhealthy ones are permitted to take extensions, then the selection will surely work against the company by increasing the average amount of its policies expiring by death." He also expresses the belief that "a selection against a company will be shown in the death losses." Now the facts do not justify the fear or encourage the belief. The printed experience of the Mutual Benefit indicates that the mortality in that company under Term Extensions has been slightly below the general mortality of the company, and about a year ago Mr. Miller stated before us that this experience brought down to date gave the same favorable indications. In my own company, while our experience under Term Extensions began in 1885, our records still show, both as regards mortality and losses, ratios several points below the general figures of the company. Evidently the insured do not argue or act as Mr. Barker intimates they may.

As Mr. Wright said this morning, we can best test the strength of a proposition by taking an extreme example. To apply this test to my statement, let us suppose the case of a man who is aware that he has but a short time to live. If he should take Term Extension, how much would the company lose, assuming the foreborne premiums to be a loss? Possibly one year's premium, possibly two. But these cases would be so few and far between that the losses brought about by such deliberate selection would not amount to a considerable sum, and would more than be offset by the favorable selection. To go a little further, let us take the case of a man whose health is impaired. Would it be possible for him to assume that he has before him an expectancy of six years and three months, or eleven years and ten months, or of any other period?

Not at all. And even if he felt confident that the period of Term Extension in his policy was longer than his expectancy, he would not dare in his condition to assume that he would die before his policy had expired. There is another consideration which bears upon this subject. My experience in trying to convince persons who have been declined that there were good reasons for the company's action, has taught me that those whose health is impaired are exceedingly hopeful of life, and this conviction has been strengthened during the past three years by an experience in dealing with under-average lives. The most difficult cases to deal with are those which show the shortest expectancy. The philosophers would have us believe that we approach life's last milestone with resignation, but practical work-a-day experience in life insurance seems to indicate that as we near the end of our journey the hope of life increases in intensity.

In my judgment, therefore, whatever "selection" may be applied to Term Extensions, except in rare cases, will not be the result of a calm, deliberate attempt to get the better of the company, but rather the expression of a desire to secure what to the insured seems to be the largest benefit. In my company only about five per cent. of the persons whose policies are carried on Term Extension actually apply for the benefit, although under certain forms of policies which are entitled to Term Extension by retroactive measures, it has been our custom to write the insured in event of the lapse of his policy and give him the choice between Paid-up Insurance and Term Extension. The letter states that if Paid-up Insurance is not applied for within thirty days the policy will be carried for Term Extension. In these cases, and also under policies which contain tables of Term Extensions, under our system of investigating the cause of lapse, the benefit is likely to be brought to the attention of the insured by our local cashier or agent, or through our renewing department at the home office. We have reason to believe, therefore, that in my company every person entitled to Term Extension is actually advised of his right thereto, and as the benefit is given automatically in all cases, the fact that so few apply therefor must be taken as a tacit acceptance of Term Extension for a large amount in preference to Paid-up Insurance for a small amount. If any indifference to the benefit exists it would emphasize, and very emphatically, the great value of Term Extension to those who are indifferent to or ignorant of the equities to which they are entitled.

Mr. Barker very wisely points out that if liens are cancelled and the net equity applied to extend the full amount of the policy for a reduced period, it is necessary that the insured shall be in good health, and that, therefore, "no condition shall be permitted to arise or exist whereby the contract should ever be permitted to cost the company more than the original amount of the policy." Actuarially it is safe to assume that the remainder of the cash equity in the policy, after deducting the amount of the lien, may be applied to carrying the original amount of the policy for a reduced period of

extension. Mr. Barker cites the case of $100,000 on which a loan of $20,000 has been made, and the original policy is then placed on Term Extension for $100,000 for a reduced period. Now, if the insured should die during the period of extension, it might be claimed that this contract had cost the company in all $120,000; but, on the other hand, it would have to be admitted that the loan had been paid in full and the proper premium received both for the original insurance and the Term Extension, and that in a thousand such cases the shorter period of Term Extension would offset the amount of insurance represented by the loans. From a practical standpoint, however, I think it is good business policy not to carry on Term Extension an amount exceeding the net obligation of the company as a death claim at the date of lapse. By following this rule the temptation to the very form of selection that Mr. Barker and others most dread is quite effectually removed.

Mr. Barker goes on to say that, to prevent such selection, it was formerly the practice with companies granting Term Extensions in event of death, within a stated period, to deduct the foreborne premiums and any liens outstanding to the date of lapse. The agreement to carry the net obligation only on Term Extension covers, of course, the deducting of liens, and so far as foreborne premiums are concerned there seems to be little necessity for such deduction, even when the Term Extension is carried for the amount of the original insurance. Until recently it has been the practice in my own company not to grant Term Extension on policies subject to liens at time of lapse, but we now extend the contract for the net amount payable as a death claim at time of lapse. We have never deducted foreborne premiums, for the reason that we believe that we have been amply paid for the Term Extensions granted from the very inception of such extensions, and that the deductions which we might have made under the rules of some other companies would have been a mere bagatelle compared with the gross sum applied to the purchase of Term Extensions.

Mr. Barker presents two suggestions for the treatment of policies subject to liens. The first is to extend the policy for a reduced amount for the full period of Term Extension granted in the contract. My objection to this plan is that it would be very difficult to express verbally in the policy contract, and also difficult to explain to the insured, who are accustomed to deal in round amounts, and not in insurance equivalents. The second plan, however, is to extend the policy for an amount equal to the original sum insured, less the indebtedness, but for a reduced period. In other words, if the policy is for $10,000, and there is a loan of $3,000 on the contract outstanding at date of lapse, the sum of $7,000 is extended for such period as the net cash equity will pay the premium as Term Extension. This plan is the method adopted by my company and contained in its policy contracts. It is easy to express, easy to explain, and in practice gives satisfaction. In the case of companies doing business in States which prescribe minimum surrender values it is necessary to be careful, however,

not to adopt a table of single premiums for the Term Extension, which would give values at any point below the minimum of the law.

MR. CRAWFORD: I will just touch upon one or two points not hitherto touched upon. I note in the second paragraph that Mr. Barker, in the third line, after speaking of this system, states: "That it has not increased the mortality experience, even where it is automatic." The implication seems to me to be that automatically extended insurance is more dangerous than that applied for. Yet below we have the statement, "that if given to all alike there can be no such adverse selection as is referred to." Now, it seems to me, if automatically given, it must be given to all alike, and hence, I am at a little difficulty in reconciling these two statements. Mr. Barker, in the last paragraph upon the first page, in stating that it is more likely that the selection against the company will be shown in the death losses, conditions this upon the case of options being given to the retiring policy-holder of taking either a paid-up policy or an extension. It seems to me that there is a possibility there of adverse selection, and if that be admitted, it seems that it would be the part of wisdom to guard against that possibility.

Mr. Barker seems to have assumed that the term extension provided for in the contract was to be continued after charging off the lien. Of course if the lien is charged off and the balance used to purchase term extension, the extent of that term is greatly shortened, and the company gets its compensation there because of the lessened term for which the risk is carried. So that I find myself unable to agree with Mr Barker in his statement, that under no consideration should the man ultimately, or possibly, be paid one hundred and twenty thousand dollars under an original hundred thousand dollar contract. I find myself unable to agree with Mr. Barker in that statement, because we have a certain amount of money with which to pay for a certain amount of insurance for a certain time, as is illustrated in the case of single premium term insurance. We will give for a five-year term a larger amount of insurance for the same money than we will for a ten-year term. We have simply the old status cancelled. We have in hand a fund which we admit is the policy-holder's equity. How shall we secure him that equity?

I think the possibility of a very serious selection against the company is here, and therefore I think that we ought to provide some sort of hedge for the purpose of protecting the companies. The companies are not now to so great an extent as formerly protecting themselves against that possibility, but I have a feeling that in letting down these safeguards there has been perhaps a little lack of wisdom, and it would be better to be more conservative, and to guard what I think we might regard as an open door to a fresh selection, at least as far as possible, as we do against other adverse contingencies that are liable to arrive.

MR. VAN CISE: Mr. Lunger stated that he found it was only five per cent. who exercised their option. That fact proved, I think, that there is no great demand for extended term insurance on the

part of the policy-holder. In the next place it seems to me that one effect of granting extended insurance is to make policy-holders careless in the payment of their premiums. After going on for a certain term of years, the insured say, "I have got my life insured, why shouldn't I drop it? If I need, I can take out a new policy if I am in good health." In regard to the question of selection at the end of Tontine periods, as has been stated by Mr. Lunger, and as I have found in my own experience, there is a selection against the company. We find that the policies kept in force represent a deteriorated class of lives, a class in which the mortality is larger than the general mortality of the company. It seems to me the same principle must apply to some extent in the question of automatically extended insurance. The motive for taking this kind of value in preference to cash value or paid-up insurance must appeal to bad risks, to impaired lives, who have not a prospect of living longer than the term for which they get this extended insurance. It seems that must be inevitable, that there must be some selection against the company in that respect. It is simply human nature, if a man has been pronounced to have Bright's disease, or some other incurable disease, he will want extended insurance.

MR. WRIGHT: I believe that when proper accommodation is given to policy-holders in the payment of premiums the feature of extended term insurance becomes useless, and that if a conference were had with the policy-holder at the time of the need arising, it would be found that in the great majority of cases he would prefer his equity in some other form. And I further believe that if the general experience under extended term insurance were tabulated in the same thorough way that the annuity experience in the British companies has been, it would be found to be an experience distinctly inferior in vitality to the general average of the companies contributing.

A NOTABLE ANNUITY EXPERIENCE, MASS. HOSPITAL. W. C. WRIGHT.

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MR. MCCLINTOCK: I wish to express in my own behalf and for the rest of the Society the feeling of pleasure that we all experience in getting an additional fund of information on an important subject. The experience of the Massachusetts Hospital Life Insurance Company is not large, but so far as it goes it is extraordinarily favorable to the investors and unfavorable to the company. It differs widely from all other known annuitants' experience. It appears to me most likely that the annuitants of this Boston company were an unusually select class of lives. Mr. Wright has presented the facts with a good deal of pains.

There is one point on which I have some doubt, and that is the presentation of annuity experience by reference to the amounts of the annuities. I have confidence in the presentation of

insurance experience by amounts insured. The effect of that is to give due weight to the amount of the actual risk year by year. The object of a table of insurance premiums is to enable the company to pay losses, and for that reason it is desirable to get the probable loss, rather than the probable number of deaths; and there does not seem any better way to get the probable loss by death than to take the actual losses by death. But when it comes to annuities it is not so easy. Now it appears at first sight, and it seems to have been carried out here by Mr. Wright, that annuities at different ages should be weighted according to the size of the annual payment, but in reality it appears rather that they should be weighted according to the amount at risk of death for the year, namely, the reserve, and not the amount of annual payment.

MR. WRIGHT: The materials are furnished for comparing actual and expected losses by numbers as well as by amounts; and perhaps some one will take the trouble to do this later on, and also give us from this material a graduated table of mortality.

NOTES ON THE RATE OF INTEREST FOR PREMIUMS AND RESERVES OF LIFE COMPANIES. WM. M'CABE.

PAGE 28, VOL. VI.

MR. MACDONALD: The paper read by Mr. McCabe at the last meeting of the Actuarial Society, entitled "Notes on the rate of interest for premiums and reserves in Life Companies," suggests for discussion a most important and almost inexhaustive subject. I regret that I am only able to offer, owing to press of business matters, a passing remark.

The fundamental data in determining the basis for computation of premiums and reserves in life companies are an authentic table of mortality and a proper and safe rate of interest.

In so far as the former is concerned, it will be generally conceded that less difficulty obtains than in regard to the latter. The experience of the past, properly tabulated, may be accepted as a safe guide for the future. Variations which may occur in the future rate of mortality, as compared with past experience, will, in all probability, be comparatively trifling, and not such as to prejudice the best interest of a company if properly managed.

In so far as the rate of interest is concerned, the circumstances are entirely different, and while it is necessary to study the past, yet its experience cannot be accepted as an infallible guide, but merely as a finger-post, indicating the direction in which we should tend.

In determining the rate of interest to be employed, some considerations which naturally present themselves are, first, the adoption of a rate which will be a guarantee of safety, having due

regard to the exceptional nature of a contract of life insurance, and the fact that it may continue in existence for half or even three-quarters of a century, and, secondly, the adoption of a rate which will avoid the necessity of change in the near future, owing to a possible further decline in the earning power of investments.

Heretofore it has been generally regarded as expedient and necessary that a difference of probably not less than from one to one and one-half per cent. in the rate of interest earned, as mentioned by Mr. McCabe, should obtain over the theoretical rate employed in the computation of premium rates and reserves. In comparatively young countries, such as Canada and the United States, where affairs have not yet assumed the settled aspect, as in the case of older countries, and there are liable to be greater fluctuations in the values of securities, and where the law of supply and demand in regard to money operates more actively, all of which tends to the obtaining of higher rates of interest, it is no doubt not only important but necessary, for the purpose of security, that the difference between the rate earned and the theoretical rate should be greater than under the conditions which obtain in older countries. At the same time, prudence would seem to demand that a minimum of three-quarters to one per cent. should under any circumstances be maintained as between the actual and theoretical rates. The possible fall even in the future in the rate of interest, and possible losses upon investments, etc., render the preservation of such a margin necessary.

Another important feature which demands consideration in determining the rate of interest, and argues in favor of the adoption of a low rate, is the necessity of guarding against a decline in the rate of dividends allowable to policy-holders due to a change in the basis of valuation. It is important that dividends should, as far as possible, be in an increasing ratio from term to term as declared. Smaller dividends in the earlier years with fairer prospects of an increase from time to time thereafter, will afford a much larger measure of satisfaction to policy-holders and be conducive to the better interests of the company than would even much more favorable results where the policy had been subjected to a reduction of dividend during its currency.

The following is a table showing the rates of interest earned for the years 1878 to 1897, inclusive, of nine American Companies, reporting to the State of New York, of the Canadian Life Companies, and of British Companies, as prepared by Mr. A.G. McKenzie, and published in an article bearing upon this subject in the Journal of the Institute, Part 5, Vol. 34, which I have taken the liberty of making use of, as I have not had time to prepare it myself. In the case of the British Companies, however, the years under observation are 1877 to 1896, inclusive.

RATES OF INTEREST EARNED BY						
British Companies.			American Companies.		Canadian Companies.	
Year.	Yearly Rate.	Average, 5 Years.	Yearly Rate	Average, 5 Years.	Yearly Rate.	Average, 5 Years.
1877	4.34					
1878	4.33		6.45		6.95	
1879	4.34		5.61		6.81	
1880	4.31		5.33		6.71	
1881	4.22	4.31	5.27		6.70	
1882	4.23		5.55	5.64	6.19	6.67
1883	4.17		5.45		6.23	
1884	4.20		5.50		6.09	
1885	4.15		5.07		6.08	
1886	4.12	4.17	5.30		6.07	
1887	4.12		5.34	5.33	5.77	6.05
1888	4.09		5.14		5.77	
1889	4.12		4.90		5.61	
1890	4.14		4.84		5.50	
1891	4.12	4.12	4.93		5.54	
1892	4.10		4.86	4.93	5.37	5.56
1893	4.05		4.77		5.35	
1894	3.96		4.69		5.24	
1895	3.88		4.82		4.68	
1896	3.84	3.96	4.65		4.76	
1897			4.86	4.76	4.65	4.93

Reference to this table will show that the average earnings of the companies during the four quinquenniums were as follows :

	British.	American.	Canadian.
1st quinquennium.....	4.31	5.64	6.67
2d "	4.17	5.33	6.05
3d "	4.12	4.93	5.56
4th "	3.96	4.76	4.93

It will thus be observed that the reduction in the rate of interest in the Canadian Companies fell as between the first and fourth quinquenniums 1.74 per cent., the American Companies $\frac{86}{100}$ of one per cent., and in the case of the British Companies $\frac{35}{100}$ of one per cent.

As might naturally be expected, the fall in the rate of interest in the Canadian Companies, owing to the operation of the conditions alluded to above, is greater than either that of the American or British Companies, the rate in the American Companies showing a greater decrease than that in the British Companies. It is only fair,

however, to assume that the rate of interest in the British Companies would have shown a more marked decrease were it not for the fact that several of the companies have been investing in recent years more largely in foreign securities than formerly, or in other words, had the proportion of foreign investments held by British Companies during the recent years of the period under observation not been greater than in the earlier years, a more marked decrease would have been observable in the earning power of these companies

Less than twenty years ago four per cent. was regarded in England as a safe rate to be employed, and even such an eminent authority as Dr. Sprague, as late as 1884, stated that "for the next thirty years the present average rate may be maintained by those offices which are ready to move with the times," the average rate referred to being four per cent. Yet within ten years and before one-third of the period had elapsed, the rate earned by the British Companies had fallen below this figure.

In so far as the earning power of Canadian Companies is concerned, it should be mentioned that heretofore, with one or two exceptions, they have been much hampered in regard to the nature of the securities in which they could invest, being restricted almost entirely to loans on real estate, municipal or government bonds, loans on policies or stocks. At the recent session of the Parliament of Canada a general Act regulating and greatly extending the class of securities in which Canadian Life Companies might invest was passed, the effect of which will no doubt be to enable the companies to invest their funds somewhat more advantageously than heretofore.

While it is not to be accepted, inferentially or otherwise, from the figures in the foregoing table, that the fall in the rate of interest will continue in the same ratio in the future as in the past, yet everything indicates that even lower rates will obtain than at present. For a period of years, prior to 1896, there had been a general and marked commercial depression during which time money was largely diverted from its natural channels, and instead of being employed in manufacturing and other industrial pursuits, safe fields for investments were sought in such securities as those in which life insurance companies can properly invest their funds. This naturally accentuated the lowering of the interest rate. The general improvement in trade which has ensued within the last year or two will, without doubt, to some extent, relieve this, and it is possible that securities which have hitherto been unobtainable except at rates which would yield a very low return, will be procurable on better terms. This relief, however, can only be regarded as of a temporary nature. The great expansion in trade and manufacturing industries will but result in the accumulation of greater wealth, which will mean a greater plethora of money for investment, and, unless the demand increases correspondingly, which is not by any means probable, will result in a further reduction in the rate of interest. Evidence of this will be found if we regard for a moment the tremendous accumulation of wealth which has taken place in all

the principal countries of the world, or say, particularly, in two of them, Great Britain and the United States.

For example—according to Mulhall, we find the estimated wealth of Great Britain to be as follows :

1840—£4,100,000,000 stg., or per inhabitant, £111 stg.
1860—£5,560,000,000 stg., or per inhabitant, £190 stg.
1888—£9,400,000,000 stg., or per inhabitant, £247 stg.

Of the United States, in

1850—£1,486,000,000 stg., or per inhabitant, £64 stg.
1860—£3,366,000,000 stg., or per inhabitant, £107 stg.
1870—£5,413,000,000 stg., or per inhabitant, £140 stg.
1880—£9,077,000,000 stg., or per inhabitant, £180 stg.
1888 (estimated)—£12,800,000,000 stg., or per inhabitant, £210 stg.

That this marvellous accumulation of wealth will continue in the same ratio in the future as in the past can only be a matter of conjecture, but in view of the great industrial development of recent years, not only in the United States but in Great Britain and other countries of Europe, we can fairly assume that the accumulation of wealth will continue in the future as in the past, if not in the same, at least in a diminished ratio.

In view of the considerations which have been barely alluded to, it appears to me that we must look for a still further lowering of the interest rate. Assuming that during the next fifteen years the rate falls as much in Canada and the United States as it has fallen in England during the past fifteen years (and in the foregoing table it must be borne in mind that the fall in the rate of interest of the British Companies of $\frac{35}{100}$ of one per cent. represents, in reality, only the experience of fifteen years, as the difference in the average rates for the first and fourth quinquenniums only are taken into consideration) it will mean that at the end of that time, the rate of interest which can be earned will not exceed, in the case of Canadian or American Companies, four and one-quarter per cent., and I think that the general consensus of opinion to-day is that it will be even nearer four per cent. The best evidences of public opinion that we can have on this point are contained in the opinions of many eminent financiers as indicated through the Equitable Life about a year and a half ago.

In the United States, I understand, that in no case has the State basis of valuation been fixed at a lower rate than four per cent., though many of the companies recognizing the changed conditions, and the necessity for making provision for the same, have voluntarily adopted three and one-half per cent., and some three per cent., as a basis for valuation purposes.

In Great Britain fifteen to twenty years ago, the rate generally employed was three and one-half per cent. or four per cent. To-day the rates in general use are three and three and one-half per cent., and a few companies have even adopted two and one-half per cent.

Taking therefore the experience of the past, combined with the

present indications as to the possibilities of the future, it would appear that, in so far as new contracts are concerned, the rate of interest to be assumed should not exceed three and one-half per cent., and prudence would even suggest the possibility or expediency of the adoption of a still lower rate.

So far as present contracts are concerned, it would appear, however, that four per cent. might be considered as a safe basis to adopt. Existing investments will probably enable the companies to earn for the next fifteen or twenty years an average rate of not less than four per cent. at least, by which time a large proportion of the present contracts will have fallen in; and to compel by legislation, or otherwise, the adoption of a lower rate might tend to disturb the business and result prejudicially to the interests of policy-holders generally.

Mr. Macaulay: The question of the rate of interest which should be laid down as the Government basis for the calculation of reserves was, as is well known, the subject of legislation at the last session of the Canadian Parliament. Great diversity of views was found to exist. As a final result the following legislation was enacted: (a) Policies issued after 1st January, 1900, shall be valued by the H^M Table with 3½ per cent. interest, and (b) Policies issued before that date shall be valued at the present rate of 4½ per cent. until January, 1910, at 4 per cent. from that date till 1st January, 1915, and thereafter at 3½ per cent. This was a compromise which gave neither side what they asked, but which has been accepted by most of the companies as fairly satisfactory. There is nothing to be gained by reopening what is now a dead issue so far as Canada is concerned, and it is in every way desirable that a question of this kind should be discussed on its general merits apart from its local application. While, therefore, the subject of Mr. McCabe's paper is of deep interest to me, and I desire to put on record some thoughts or jottings with regard to it, I yet wish it to be understood that I cast not the very slightest reflection upon any person who may not agree with me. I will endeavor to look at the subject as a whole, and will say as little as possible about the merits or demerits of the Canadian law in particular. I am afraid that this is one of the things about which we must be content to differ.

(1.) When considering this matter one cardinal point must always be borne in mind, and that is that the object and effect of Government legislation is merely to define what rate of interest shall be used in determining whether or not a company is solvent. The question of what basis the companies shall themselves voluntarily use in setting aside their reserves is not involved at all, except to the extent that this latter rate must of course be as high as that laid down by the legislature for solvency purposes. Unless a very clear distinction is drawn between these two entirely separate standards—that of solvency and that to be voluntarily adopted—the whole force of the objections of those who are opposed to such Government legislation will be missed.

(2.) Any test of solvency which is higher than that actually required is artificial, and I believe injurious. I do not think it is wise for any legislature to enact a law by which a company may be declared to be technically insolvent when it is not actually so. For this reason, in my opinion, any unnecessarily severe test of solvency should be opposed. At the same time I am equally convinced that it is desirable for the companies themselves to voluntarily set aside as high reserves as in the judgment of their actuaries is desirable, in order that they may thus establish a wide margin of safety, over and above the requirements of solvency.

(3.) Referring to the case of Canada for one moment, I am inclined to think that an almost ideal arrangement would have been to allow the old legal standard to remain as a test of solvency, but on the other hand, to encourage the companies to themselves set aside more stringent reserves than those called for by the law. This would practically mean the establishment of the British system of "Freedom and Publicity," with the modification that the accuracy of the published accounts of the companies would be vouched for by the Government Superintendent of Insurance, and that there would be a minimum below which none of the companies could allow their reserves to fall. That the companies could have been trusted to strengthen their reserves without compulsion from the Government is shown by the fact that thirteen of the fifteen companies had already taken steps in this direction.

(4.) When considering the question of the sufficiency or otherwise of any standard as a test of solvency, we must look not at one of its factors alone, but at all. Even if we grant, for the sake of argument, that 4½ per cent. is possibly a somewhat high rate of interest to assume, we must not forget that there are two other factors which go to make up the standard, and which must be taken into account. In the first place, the valuations are made by the net premium method, so that the entire loading on the premiums remains untouched as a special margin. In the second place it is very doubtful whether any company assuring Northern lives in Canada and the United States will ever experience a mortality as heavy as that predicted by the H^M Table or by the Combined Table. Where, therefore, either of these Mortality Tables is employed, there is another margin. Probably the American Table of Mortality, with only 4 per cent. interest, but taking account of gross premiums, deducting of course a fair allowance for expenses and eliminating negative values, would be a more scientific test of solvency, but as the Combined Table, or the H^M Table with 4½ per cent. interest and net premiums gives for active companies even higher reserves than those which would be thus found, I think we can fairly claim that the latter test is severe enough. In any case I do not think there is any justification for lowering the rate below 4 per cent. in connection with a net premium valuation, as a basis of solvency.

(5.) It is often supposed that the law of Great Britain lays down no definite rules for determining the solvency or otherwise of a

company. Theoretically that may be so, but for practical purposes I think it is hardly true. The Act of 1870 (Section 21) settles that the valuation shall be a gross premium one, but does not state what Mortality Table or rate of interest shall be employed, nor what deduction shall be made from the premiums for expenses. The schedule attached to the Act of 1872 lays down, however, that for the purpose of distributing the assets of an insolvent company the reserves shall be calculated by the Seventeen Office Table and 4 per cent. interest. It is very unlikely that the Court would depart much from the basis thus laid down in the Act of 1872. I think we may therefore fairly assume that the British standard of solvency is a gross premium valuation by the Seventeen Offices' Table, or its successor, the H^M Table, and 4 per cent. interest, with such deduction from the premiums as expert testimony may decide to be necessary. The results brought out by this method of valuation, deducting say 12½ per cent. from the gross premiums for expenses, would in most of our companies be less than 90 per cent. of those brought out by the Seventeen Offices or the H^M Table, with 4½ per cent. interest and net premiums.

(6.) In the United States no legislature in the entire Union has exacted a lower rate of interest than 4 per cent.; and when this rate is used in association with the American Mortality Table, it gives reserves, which in the case of most companies are about equal to those brought out by the H^M Table with 4¼ per cent. interest.

(7.) The State of New York, which formerly had a 4 per cent. standard, abolished it a few years ago.

(8.) The new Canadian standard of solvency, is as regards new business, more severe, I believe, that that in existence in any other part of the world.

(9.) The evils of unduly severe standards are well known. Competent authorities consider that they are responsible for the failure of many American companies which might otherwise today have been strong and prosperous. It will probably also not be denied that some British offices which are now among the largest and most powerful would at certain periods of their history have been declared insolvent if tested by a severe standard.

(10.) Every prudent company should endeavor to maintain a wide margin between it and the line of insolvency. If that line be placed unduly high it becomes necessary for the company to raise its own standard still higher. I think no company should be satisfied unless it has a difference of at least one-half of one per cent. between the rate which it employs in its own calculations and the rate laid down by the Government. According to this ideal, if the Government standard be 4 per cent., the companies should make their calculations at not more than 3½ per cent.; and if the Government standard be 3½ per cent., the companies should aim at getting their premiums and their reserves down to a 3 per cent. basis. The line of insolvency is such a terrible thing that every company must keep away from it, and the actuaries, as the pilots

of our life companies, must see that the ships entrusted to their charge keep at a safe distance. If any legislature builds artificial rocks away out from the real shore line, these artificial rocks become just as dangerous as the real rocks, and must be avoided in like manner.

(11.) A company is made strong by increasing its surplus, or, in other words, its margin over and above the requirements of solvency, but it is not made strong by artificially increasing its liabilities.

(12.) Retroactive legislation is especially objectionable. By this I mean legislation which changes the basis under which policy contracts were issued, as is done whenever the rate of interest in regard to old policies is altered. Nothing but the most serious considerations can justify such a measure. The very admission of such a right involves great dangers. If a legislature can now change the basis of contracts issued under a four per cent. or a four-and-a-half per cent. law, we have no assurance that the same thing will not be done five, ten or fifteen years hence in regard to policies now being issued under the three-and-a-half per cent. law. The companies are entitled to stable legislation, as otherwise they can never be sure just where they stand.

(13.) The reserve on a policy is not the accumulation of one year, but is the result of instalments dating back to the very commencement of the assurance. If the rate of interest employed in calculating the reserve on such a policy be lowered, the effect is to compel the company to increase those instalments for every year of the past just as if the legal rate of interest had been established at the lower rate from the beginning. The effect is, therefore, practically to nullify the old law even as regards the past, and to override the rights accrued under the previous legislation.

(14.) There is no source from which the companies can make good the extra reserves called for by a retroactive measure, except by reducing the profits of policy-holders below what could otherwise fairly be given. When we remember that the extra reserves which such a law compels a company to set aside are not to be measured by the standard of bare solvency laid down by the law itself, but by that still more severe standard which the companies will feel themselves obliged to adopt as a measure of common prudence, the magnitude of this unnecessary addition thus made to the reserves, and of the consequent unnecessary reduction in profits, becomes clear. In this connection I would point out that the Tontine companies are not so much affected by such legislation as are those which divide yearly or quinquennially. The first have still in hand undivided practically the entire surplus earned by the policies still on their books, while the second have already paid away the profits of past years, and, of course, can never get them back.

(15.) While any unnecessarily severe standard is unwise, the objections are not so serious to one that applies to new policies only. Yet there are serious objections even to this. It is

argued that companies can raise their premiums for new assurances sufficiently to cover all contingencies, but in practice this is not always possible on account of competition. If, for example, the Canadian Legislature had been unwise enough to make the standard for new policies the H^M Table with three per cent. interest, the Canadian companies would have been placed in a most unenviable position. They would have had but two alternatives open: (1) They might raise their premiums sufficiently to give them a proper margin over the requirements of solvency; but if they took this course their premiums, on account of the severity of the H^M Table, would be so high that they could hardly compete successfully with their American rivals; (2) If they felt compelled as a result of competition to charge only such premiums as charged by American companies, they would not have proper margins of safety over the Government standard. The premiums might be ample but for the artificial line of insolvency, but that line could not be ignored even though it were artificial. The companies, in short, would have had no option but to either do business with perilously small margins or be ruled out of the race altogether. Fortunately Parliament did not go below three-and-one-half per cent., but as this rate is coupled with the H^M Table, it may be roughly taken as equal to the American Table with three-and-one-quarter per cent. interest. There are already indications that this part of the law may bear evil fruit in the Dominion, for rumor has it that some of the smaller companies are unwilling to charge as high premiums as the larger ones think are absolutely necessary because of the law. One result, therefore, is likely to be that some of these companies will do business with very small margins.

(16.) My own position, in brief, is simply that when new legislation is introduced on this subject, it should not apply at all to existing policies, and that even as regards new policies the greatest care should be taken to not establish a standard higher than that actually required. On the other hand, every company ought to voluntarily set aside substantial extra reserves beyond the bare exactions of the law.

Mr. Hann: Mr. William McCabe, in his notes, calls attention to the opinion expressed in the English Bankers' Magazine for November, 1898, that 3 per cent. interest is the highest rate a well-managed company will be justified in assuming it will earn. He then gives in a concise form, good reasons in the main for the adoption of a lower rate than those rates generally in use. His concluding paragraph is axiomatic and, therefore, incontrovertible. What the *safe* rate is, however, is a matter of opinion.

In the notes perhaps too much weight is given to the earnings of Government securities and too little to the distinct functions of the Legislatures and the companies in a matter of such moment. It is self-evident that the latter are in far better position than the former to judge of the sufficiency, or otherwise, of their premiums to provide for the payment of their future obligations. The companies, surely, should be left to take the initiative, because they

only are in a position to know the best remedy to apply in strengthening their reserves, without undue contraction or injury to the business. There are sources of gain besides the loading as an off-set to deficiency in interest; for example, interest on foreborne claims due to favorable mortality, to say nothing of other sources. Profits under contracts, based on the old rates, will, of course, be less than in the past, at least for a period after the change. Suppose a company's premiums are based on 4 per cent. interest in the United States, if its risks are numerous and well selected, and its assets securely invested, there can be no question that, with prudent management, a company could pass from a 4 to a 3 per cent. basis in about ten years, provided, of course, that no further business was based on the higher rate, and that the reserve on the new business was based on the rate assumed in the new premium scale. Interest has fallen in the past, is falling and may continue to fall, yet who will say we are not pretty near the time when its decline will be greatly retarded. At present capital appears to be accummulating faster than it can be profitably used. While this is so there must be keen competition for such securities as life assurance companies need. On the other hand, there is a large class of persons who will have to put out their money in more profitable investments than heretofore. They cannot live on their present income. They will either go into business or place their money in more productive channels. This would be perfectly consistent and justifiable. A low rate of interest goes hand in hand with a prosperous condition of trade. Many new avenues of investment may yet be opened up. This has occurred in the past and may again. From the nature of the case Government securities yield less than other first-class securities. No life assurance company can afford the luxury of heavy investments in United States securities or British consols, notwithstanding the sweet simplicity that hedges them. It is very comfortable to have a margin of about 1 per cent. in the average rate earned over that assumed. We may not always be able to attain this; we can aim for it. When the legislatures see that the companies are making provision to strengthen their reserves, then will be the time for them to change their standards in harmony with the above provisions, and ample time should be given so that the weakest can stand. The rate does not appear to be declining very rapidly in Great Britain. Mr. David Deuchar showed in J. I. A., Vol. 28, that the average rate of interest for 20 years from 1865 to 1885 ran from 4½ to 4.2 per cent. on life assurance investments, and some of them come nearly to 4 per cent. at the present. Again it is true that a few English offices do value on a 2½ per cent. basis, not because of the average rate of interest, but with a view to keep up their rate of bonus. This question of rates of interest in premiums and reserves should be regulated by healthy competition, rather than by paternal government. It should be watched and safeguarded by the greatest care, because a deficiency in this respect is more disastrous than an excessive rate of mortality. The former operates

with cumulative force, while the latter may ease up at intervals, and, in the long run, the strain from such may not be severe.

THE ACTUARY'S PLACE IN SCIENCE. W. S. NICHOLS.

PAGE 27, VOL. VI.

PROF. MASON: We have to thank Mr. Nichols for what is, perhaps, the clearest statement in a few pages of the present condition of science that has appeared in periodical literature for a very long time. His statement of the precise difficulties which are now encountered by investigators in the different branches of science is so clear and striking that I think the view we have of the present condition of the scientific world must be clearer after reading that paper than it was before, very much clearer; and his statement of the failure of scientific methods in certain directions is exceedingly forcible. Where I differ from him is in his assumption that the scientific method for the future will have to be changed. It is necessary, as he himself so well sets forth, in the collecting of the phenomena of the different branches of science, that we adopt a working hypothesis which will explain them, and the test of that hypothesis is its application to future phenomena, and its success or failure in their explanation. And it is surely no disgrace to a science to have to change its working hypothesis. The fact that a theory which would explain phenomena fifty years ago will not explain those which have been collected since, and cannot be reconciled perhaps with phenomena in other branches which were then unknown altogether, does not prove it to be unsound. On the contrary, it seems to me that the method of investigation, being the one which has been adopted in modern philosophy, is just as certain of result now as it ever was, and no more. Nor do I think that the application of the doctrine of probabilities and the methods of calculating chances, which we have in our particular profession to use continually, will especially help out the scientific investigator. I mean, no more in the future than in the past. All investigators in science already use all the various methods which have been devised to eliminate possible errors in their observations. The methods which we actuaries have to employ are essentially the same as those in common use in all astronomical observations.

MR. IRELAND: The paper of Mr. Nichols on "The Actuary's Place in Science," has for its inspiration, and its focus, the idea that actuarial methods of scientific investigation are the methods that might wisely and profitably be employed by students of all sciences, mental or physical. "The actuary," he says, "holds the key to the standard by which all truth in its last analysis is measured, and his place in science is that of the conservator of this universal standard of truth." In presenting this idea he has made use of a wealth of material, by way of introduction and illustration,

that suggests a great architectural structure like the statue of Liberty enlightening the world, where the light-giving torch is upheld by a work of art so vast and so elaborate that the support, and not the flambeau, is what catches the eye of the casual observer.

Agassiz, Aristotle, Tycho Brahe, Calvin, Copernicus, Cuvier, Darwin, Davy, Descartes, Faraday, Fourier, Galileo, Gauss, Hamilton, Helmholtz, Huxley, Kant, Kepler, Lamarck, Laplace, Lucretius, Malthus, Newton, Picard, Ptolemy, Adam Smith, Tyndall have all been drawn into the service, some of them as examples of theorists, who were wrong, and some as members of the happier band whose conclusions have not yet been set aside.

The fundamental problem of actuarial science was to construct a firm and durable structure on a shifting and variable foundation, much as the military engineer is called upon to build, over deep and rapid waters, a pontoon bridge that will safely support horse and foot and artillery, crowded as closely upon it as their captains may choose.

This same problem had already come, in a small way, to the attention of the historical gambler, whose bets have helped those who think themselves his betters to illustrate various scientific points connected with the application of the doctrine of chances; but the business of insurance, in its different branches, may safely be credited with having been what led to the development, on a large scale and for humanitarian purposes, of the science of estimating *probabilities*, and making their values of sound practical use.

If, now, we seek to follow up Mr Nichols's suggestion, and reduce to practice his doctrines, we naturally ask for more details, for something more definite that shall tell us how to apply to given subjects and in specified cases the actuarial methods that he has in mind. It is more than likely that, in answer, we should be told that the mental attitude is the important thing; that truthseekers in any domain should recognize the difference between impressions, even though well fortified, and demonstrations; and should study and apply the rules that govern the measurement of probabilities in the work of the most accomplished actuaries. It is in this direction, however, that further writing on the subject would be of interest, and seems almost necessary to the completeness of the essay.

The idea of the general application of actuarial methods is a suggestive one, and full of inspiration to the broad thinker; it may be of real value to those of us who have been inclined to make narrow the range of our vision and consider only the minor and supplementary details of the scientific work in which we are engaged, for it holds up prominently, for our consideration, the pioneers in our work, the intelligent and courageous explorers who laid down the principles and methods that it has been our easier work to adopt and to employ.

WHAT IS THE PROPER LOADING ON LIFE INSURANCE PREMIUMS?
JOEL G. VAN CISE.

PAGE 45, VOL. VI.

MR. WING: The statements made by Mr. Van Cise are to the point and so obvious as to do away with any call for discussion. That the question is pertinent at this time is apparent from the Gain and Loss Exhibits of American Companies. Taking all the companies reporting to the Connecticut Insurance Department, we find that the expenses have been in excess of the provision therefor in loading during each of the three years prior to the close of December last, as follows:

	1896	1897	1898
Loading, . .	$41,810,736	$44,627,277	$46,809,683
Expenses, . .	43 959,538	46,302,929	49,874,064

This result obtains with the loading averaging twenty-five per cent. of the gross premiums received. It is clear that if the present rate of expenses is to be continued the provision for it should not be decreased. There is no margin for a distribution of surplus on account of gains from this source. Therefore, I do not see that Mr. Van Cise's distinction between participating and non-participating policies applies, unless lower commission is to be paid on non-participating business.

The item of "Expenses" is so closely related to the "Interest Account" that we cannot properly consider any ratios of one without the other. Both items may be reduced or increased to considerable extent by perfectly legitimate bookkeeping methods, differing in different offices, and possibly at different times in the same office. But the question of the rate of interest as related to the provision for expenses in the loading seems likely to be vitally important in the near future from another cause. If, instead of reserving at four per cent., it shall be decided by law or common consent as wiser to reserve at three per cent., a large portion of the present provision for expenses will be diverted at once to the requirements of maintaining the reserve, and we shall be compelled to increase the premiums charged, reduce expenses, or provide for them from other sources than the present loading.

During the last few years, while rates of interest on best securities have been declining, the market value of the same class of investments has been advancing, so that companies generally during the last five years have made large gains in the market value of their securities which have more than compensated for the reduced rates of interest earned in their Gain and Loss Exhibits.

We find, therefore, large increases in the surplus of the companies, notwithstanding low rates of interest and narrow margins for expenses. It may be well for us to note how large a proportion of the increase in surplus is derived from gain in market values. All the companies reporting to the State of Connecticut showed an increase of $54,000,000 in surplus in the two years ending Decem-

ber, 1898. Of this amount $23,500,000 is from advance in market values, the direct result of lower rates of interest, or at least a logical accompaniment thereto, for it is difficult to say always which is cause and which is effect.

As showing the trend of affairs in connection with expense ratio and interest, including gains in market values as shown from the Gain and Loss Exhibits published by the Connecticut Insurance Department, I have selected ten of the companies reporting, and give for the three years 1896, 1897 and 1898 for each of the companies and for all the companies doing business in that State :

FIRST : The percentage of loading to Gross Premiums received, not including in Gross Premiums the items Nos. 4, 5, 6 and 7 in the Statement blanks.

SECOND : The percentage of actual expenses to the loading.

THIRD : The percentage of interest earned (counting increase in market values for the year as part of the interest) to mean liabilities.

		Per Cent. of Loading to Gross Premium.	Per Cent. of Expenses to Loading.	Per Cent. of Interest to Liabilities.
1	1896	26	99	5.41
	1897	21	123	6.52
	1898	20	131	6.26
2	1896	27	95	5.15
	1897	27	91	6.35
	1898	26	94	7.79
3	1896	23	86	5.20
	1897	24	86	5.74
	1898	23	90	5.67
4	1896	28	118	5.34
	1897	28	106	6.05
	1898	26	107	7.41
5	1896	22	103	4.50
	1897	22	111	5.62
	1898	25	110	5.88
6	1896	27	99	5.62
	1897	27	99	5.33
	1898	27	99	6.20
7	1896	25	82	6.85
	1897	22	92	6.78
	1898	22	94	6.85
8	1896	23	93	4.52
	1897	25	90	5.04
	1898	25	107	6.27
9	1896	19	97	5.11
	1897	20	98	6.26
	1898	19	98	7.65
10	1896	24	134	4.80
	1897	26	141	4.57
	1898	26	154	5.58
Summary of all Companies.	1896	26	105	5.43
	1897	25	104	6.37
	1898	25	107	6.76

It will be noticed that I have designated the individual companies by number instead of by name, but each company's identity can be ascertained easily if any one chooses to take the trouble to make the calculation.

It is important that our executive officers should not deceive themselves with any thought that rates of interest are advancing, as seems indicated by this table if taken just as it stands. Paradoxical as it may seem, the figures indicate exactly an opposite conclusion, and warn us not to count too confidently on a continuance of such gains. We may be sure that if rates of interest continue as they have been during the last few years, companies will realize no such rates as indicated in this table. We may also be sure that if, for some cause not now foreseen, there should be a return to considerably higher rates of interest than we have had lately, there will be a corresponding reduction in the market value of our assets. So patent has this become that many dealers in first-class bonds quote such securities as yielding such and such a percentum interest instead of at such and such percentum of the par value.

I have assumed that this view of the interest gains is germane to the question discussed by Mr. Van Cise, not only because interest and expense run into each other so easily in bookkeeping, but because it is clear that some companies at least are already poaching upon other sources besides the loading for the payment of expenses. They will do well to spare the interest reserve from any such encroachments.

I do not consider that either Mr. Van Cise or myself has answered satisfactorily or scientifically the question under discussion, but I think we are agreed in the idea that if we cannot say surely what is the proper loading on a premium we can say truly that the proper expense of a company is something less than the loading provided for in the premium.

WM. E. STARR: I fully agree with Mr. Van Cise that the loading should be ample to provide for expenses and a margin of safety beyond any reasonable conception of prospective fluctuations in mortality and interest, and thus to accumulate a surplus for distribution.

His arguments cover the ground so as to leave little room for amplification, and I shall not follow them in detail. I approve and second his appeal to actuaries to resist any near approach to the danger line by attempting to furnish excessive benefits for inadequate premiums.

On one point I fear I shall not be able to agree with Mr. Van Cise. He says he will limit himself to "a consideration of the proper percentage to be added to the net premiums on ordinary life policies," etc. If he means to be understood as advocating the loading by percentage of net premiums, his views and mine are irreconcilably divergent.

I suppose every actuary is an accountant. I was an accountant long before I was an actuary and in the course of my life I

have been more or less conversant with partnership and corporation accounts, and so far as my observation goes the invariable rule is to pay expenses out of profits before distribution, and thus apportion expenses and profits upon the same basis, in all kinds of business except life insurance, that exception obtaining only since the adoption of the contribution plan of dividing surplus.

Thus an important partial reform introduces an anomaly that impairs its partial value. A distribution according to contribution is not the correct thing unless the contribution has been made upon correct principles.

In a pamphlet entitled "Life Insurance Premiums and Reserves," by Sheppard Homans, copyrighted by the Spectator Company, 1888, may be found a demonstration which I will not copy and cannot improve:

> That "Every level premium policy is in reality a contract for a yearly decreasing amount of insurance, and a yearly increasing amount of investment";
>
> That "there is no necessary connection between the two";
>
> That "insurance or indemnity may be purchased without investment, as investment may be purchased without insurance";
>
> That "instead of contracting with a life insurance company for both insurance and investment, which together make up the sum insured, two separate contracts might be made—the one with a life company for the yearly decreasing amounts of insurance only, the other with a savings bank or trust company for accumulating the deposit or investment portions of the premiums; in case of death in such case the insurance company would pay the net amount insured only, while the savings bank would pay the accumulated deposits, the two together making up the amount guaranteed."

Let us now suppose the two contracts to be made with two different companies, in manner and form as suggested by Mr. Homans; and let us further suppose that the two companies, though doing business independently, have an identical membership, and that both do business on the mutual plan.

The pecuniary interest in the insurance contract is measured at any time relatively by the net premium for the insurance of the current year, and the apportionment of the expenses will have the same relative measure and be provided for by a percentage loading on the *net premium* or the *normal tabular death cost*, which is the same thing.

The pecuniary interest in the savings bank contract will be measured at any time by the accumulation of deposits, which must be always equal to the reserve that would be held by a level premium life company. The expenses of savings banks are always paid out of the interest realized in excess of that allowed to the depositor.

I am not aware of any reason why expenses should not be

provided for in a level premium life insurance company, just as the same expenses would be provided for in a savings bank or in a natural premium life insurance company.

I know the difficulty of breaking away from old customs and traditions, but I do not understand why a reform in the matter of expense loading should be more difficult than in the correlative matter of dividends.

I am sure that a general formula for loading the premiums on all kinds of policies equitably would be a credit to the actuarial profession.

MR. CRAIG: Whatever may have been the opinion in the past, it is probable that opinions do now vary, and will continue to vary in the future, as to what is the proper loading on life insurance premiums. It is perhaps an open question whether the loading received constitutes such an important element to the successful transaction of business in a life insurance company as Mr. Van Cise would lead us to believe.

He states that, in February, 1868, all the life insurance companies whose home offices were in New York City, agreed upon a uniform table of premiums, based upon the American Experience of Mortality, and 4 per cent. interest with a loading of 40 per cent. upon the net ordinary life premiums, and that this loading was not deemed excessive, even under the favorable conditions which existed at that time, as compared with those which exist to-day, or such as may reasonably be expected in the future.

In this connection, we beg to quote from the report of the superintendent of the insurance department of the State of New York:

"The calendar year 1867 shows only four new life insurance companies incorporated during the year, one of which disappeared almost at birth, leaving three only, as against five new companies in 1866 and three already organized . . . . in the year 1868. The additional new companies already projected for 1868 will doubtless mark its calendar as incorporating the largest number of life insurance institutions of any year in our State history."

Of all the companies to which the superintendent alluded as having been organized in the years 1866, 1867 and 1868, together with all those which were in process of organization, and which subsequently organized in that year, only one is in existence to-day. It is quite evident that this bare statement carries with it the conviction that an adequate theoretical loading is, *per se*, no guarantee of safety. It may also be said that American actuaries have a wider range of knowledge to-day than they had thirty years ago, and it is quite reasonable to assume, that if the knowledge of thirty years ago were equal to that of to-day, all other conditions remaining the same as they were, many of the companies which became insolvent would be alive and prosperous to-day, so that as a factor of safety, we cannot rely upon the loading alone.

Mr. Van Cise states that in his judgment "two important points ought always to be in the minds of actuaries when called upon by

managers, or when taking it upon themselves to construct tables of premiums : first, the absolute safety and security of the company's contracts; second, the accumulation of a surplus fund, out of which to pay dividends to persistent policy-holders, if the company is conducted upon the mutual plan. The first point, as to safety, will not admit of dispute, but unless I mistake the signs of the times, the second point as to the accumulation of surplus, is fast losing its importance in the eyes of some of the gentlemen who have framed and introduced some of the new forms of policies which have come before the American public during the past few years."

There are some concessions in the contracts of American life insurance companies which have come to stay. Some of these, more particularly the paid-up concession on limited payment life policies, are known by all actuaries to be liberal beyond the limits of equity, and yet, because of the popular favor with which this concession is met, based on its simplicity and apparent fairness, it would be exceedingly difficult for any one company, or group of companies, to undertake to make any change. This feature of the policy contract convinces us that it is not absolutely essential that *every* premium should be so computed that under all possible conditions of surrender a portion of the reserve should remain in the hands of the company as a surrender charge. This statement is made on the assumption that in actual practice there are many cases where policies are surrendered for paid-up contracts before any surplus has accumulated, without any reduction in the liability.

While it may be said that this same argument applies to other concessions made in policy contracts, yet it seems to me that there is some danger in the extended insurance feature granted in the policies of many of the companies. In the first place, many persons are insured under the policies applied for who would not have been accepted on the term plan, and yet without any precaution or safeguard, these same persons have the privilege, after two or three years, of converting their contracts into term insurance. In the second place there are inequalities and inconsistencies found in many of these concessions, in some cases the term insurance, under the automatic extended insurance feature, costing the insured actually less money than if originally insured under a term policy.

Let us take three policies issued in a certain company, at age 41, on the Ten Year Endowment, the Fifteen Year Endowment, and the Twenty Year Endowment plans respectively.

The person insured under the Ten Year Endowment Policy would have paid in two years, $218.56, the person insured under the Fifteen Year Endowment Policy would have paid in three years, $217.20, and the person insured under the Twenty Year Endowment Policy would have paid in four years, $219.20. The total payments made in each case are practically the same, and yet under the Ten Year Endowment Policy there is an extended insurance granted of seven years and ten months, under the Fifteen Year Endowment an extended insurance is granted of nine years and eleven months, and under the Twenty Year Endowment the

extended insurance is nine years and eleven months, so that the total insurance under these respective policies, for practically the same total premium is nine years and ten months—twelve years and eleven months—thirteen years and eleven months.

If these policies are to be construed as term policies by premiums limited to two years, three years, and four years respectively, then we have the anomaly that the smaller the number of premiums the shorter is the term over which the insurance extends.

Taking another case in a different company, age 30 at issue, annual premium $18.25, Whole Life Policy; after four years, extended insurance is granted of two years and ten months, which for the purpose of this illustration may be counted as three years, making seven years' insurance in all. The annual premium, at the same age, for a Seven Year Term Policy is $12.85, seven times which makes $89.95, whereas the insured secures the same result by paying $18.25 four times, or $73.00.

In another case, we will take a life policy issued by a certain company at age 32, annual premium $25.75 for one thousand dollars, with five years' premiums paid. The extended insurance granted at the end of this period is five years, making ten years' insurance in all. The total premiums paid amount to $128.75, which, divided by ten, gives an average annual premium of $12.88 as against the Ordinary Ten Year Term rate of $14.20 in the same company. At age 44, under the Twenty-payment Life plan, the annual premium is $47.25, and after four years the extended insurance is six years and one month, which for our purpose may be called six years, making ten years' insurance in all. The total premiums paid amount to $189.00, or an average of $18.90 per year, while the regular ten year term rate is $22.00.

In the case of a Fifteen Year Endowment Policy in still another company, issued at age 41, the total payments in three years would amount to $212.46, at the end of which time extended insurance would be given for twelve years, making fifteen years in all. This company does not issue a fifteen year term rate, but the ten year term rate is $17.43, while one-fifteenth part of the total premiums paid is $14.16.

The criticism may be made that it is not fair to take the aggregate premium paid in three, four or five years and divide it by ten or fifteen to secure a sum with which to compare the corresponding term rate—that for the purpose of comparison these ought to be construed as premiums paid over a like number of years, for a longer term of insurance. It must be borne in mind, however, that these policies were not taken out as term policies, and that the initial expense incurred was on the assumption that they would be continued as issued, and was higher than if they had originally been issued as term policies ; so this extra expense is assumed to offset the limited payment feature of these premiums as applied to term insurance. On a practical basis, the above conclusions are probably fair, and so it follows that a man may not only obtain term insurance when it would not have been granted if originally applied for,

but can actually obtain it, so far as the company is concerned, at a cheaper rate than if applied for by a first-class risk.

These general remarks are made simply for the purpose of indicating that the loading is a more complex question than appears upon its face, because the insured has the option of changing the character of the contract, and every actuary will probably have to determine for himself where the line of safety lies.

So far as the second point which Mr. Van Cise makes in the paragraph quoted above, in regard to the accumulation of a surplus fund out of which to pay dividends, it may be said that this statement is very general in its terms and may mean little or much, according to our interpretation of it.

In the commonwealth of Massachusetts the surplus funds of a company are limited by law to 10 per cent. of its reserve liabilities. Where no restriction is placed upon the managers of a life insurance company, the amount of surplus funds to be accumulated lies largely with themselves, and as between the accumulation of an immensely large and apparently unnecessary surplus on one hand, and the reduction of loading on the other, it is probable that the advocates of a reduction in loading would gain the day, for this would mean, sooner or later, a reduction in expenses, and this would be hailed with much greater satisfaction by the insuring public than a constantly increasing surplus, which might be used like a bunch of carrots on the end of a pole was by the driver of a balky mule: to dangle in front of his nose as an incentive for him to perform his duty, but not intended to reach his mouth.

In another part of his paper, Mr. Van Cise says: "If $28\frac{4}{7}$ per cent. was not thought too much thirty years ago, should we not insist that at least 25 per cent. of the gross Ordinary Life premiums charged to-day should consist of loading, without going into the question of what rate of interest should be used in the calculations of net premiums and reserve?

"I, of course, refer to policies on which profits are promised, whether these profits are to be paid annually, or at the end of a term of years; but even on policies without profits, should we not insist that at least 10 per cent. of the gross premiums should be loading? It is not likely that this low rate would cover expenses, but probably the deficiency could be made up from other sources."

The logical conclusion from the above would seem to be that, if 10 per cent. of the gross premiums on policies without profits would be sufficient, with profits from extraneous sources, to cover expenses, the relative increase of loading above 10 per cent. of the gross would depend entirely upon the desire for the accumulation of a large or small surplus, because anything above that would produce some surplus, and the higher the rate of loading the greater the surplus. One important fact ought not to be overlooked, viz: that on all deferred dividend policies the mutual premiums charged are converted into stock or non-participating rates to all those who for any cause fail to keep their policies in force to the end of the specified time. As this period in most cases is twenty years, and

as but a small proportion of those who enter continue for so long a time, it is evident that so far as the insured is concerned, the bulk of this business is written on stock rates with a high loading.

As a matter of fact, no general rule can be laid down at the present time for the government of all companies, as their policies vary and also their methods of conducting business.

MR. MCCLINTOCK: On the subject of loadings I agree entirely with Mr. Craig. I cannot agree with one portion of Mr. Wing's remarks. I think he used the phrase, "provision for expenses" as an equivalent and synonym for loading. Now there are a variety of expenses which are properly defrayed from other sources than loading. The expenses and taxes attendant on the care of assets should be paid out of the gross interest and rents received. The surrender charges which are left behind by retiring policy-holders afford a proper provision towards defraying the expense of replacing the risks withdrawn. That is an old maxim of life insurance practice which was originated many years ago by Elizur Wright. Then there is a considerable saving from the claims for death losses which are always less than those expected. Mr. Wright holds that expenses should be charged in proportion to the cost of insurance, and we may go with him as far as regards the balance of the sums which have been provided for the year's death losses. Finally, there are miscellaneous profits, such as from non-participating business, and other kinds which I have not the time to detail, and it is only the balance after all these sources are used up that must necessarily be charged to loading.

THE VALUE OF ACTUARIAL TESTIMONY IN THE COURTS. SYDNEY N. OGDEN.

PAGE 48, VOL. VI.

MR. HARVEY: I have to take issue with our esteemed fellow-member, Mr. Ogden, in the matter of the conclusions he has reached in his paper on "The Value of Actuarial Testimony in the Courts," read at the May meeting.

In the first place, the testimony of the actuary—except as to life interests in estates—has been almost rigorously excluded from both Federal and State Courts in this country.

He has not been given the opportunity to tell what he knows, unless called upon to testify in corroboration of the results of printed tables of whole life contingencies. His *opinion* as an expert otherwise is, as a rule, severely ignored.

Mr. Ogden's contention is, that the value of that portion of an income which goes to the family of the earner and to savings, when cut off by casualty, should be computed upon the value of a whole life annuity of one dollar.

In the past nine years I have been called into Court five times to give "expert" testimony as to the value of a life annuity of one dollar. In each case the claim was against a corporation for

damages for permanent disability or death occasioned by accident through mismanagement, or neglect by the servants of the defendant. In one case the judge declined the testimony, saying it was improper, giving no reasons. In another the testimony was refused, because it was "irrelevant." In a third the Court would not admit the evidence, because if "other testimony sustained the facts alleged in the petition, the statute fixed the damages." In the fourth the judge said simply that the "plaintiff's earning capacity could not be measured by a whole life annuity." The fifth involved the question of extraordinarily high value in the plaintiff's earning, for which there was "no basis of estimate" because of the special hazard in an employment in which no one ever remained but a few years. The judge, who declined the testimony in this case, told me after he had retired from the bench, that if "there were trustworthy tables showing the average duration of the work-life of artisans in different occupations—the duration of wages-earning ability—the Courts would probably give due weight to the results." He added that if "such tables included reasonable estimates of the portion of wages which went to the family of the earner independent of his personal indulgences, and of the portion saved for future accumulation, they would have further value." He said also, that "upon the question: what is the remainder over after deducting the personal wants of the workman from his wages, of fifty witnesses pursuing the same avocation for the same earning, not more than two or three would testify to similar results."

The judges were all right. The value of a continuing net income, beyond the time of its cessation, cannot be properly measured by the whole life annuity, except in cases where its enjoyment is for life certain, or for life without labor. The measure should not be more than the present value of the temporary annuity, from the age when the event making the determination necessary occurs to the limiting age of the wage or income producing life of the claimant. If the working years of people who have to work for a living always ran to the end of life, the value of the whole life annuity would be the true factor. But the work-years not only do not run through life, and while all fall short of that limit, there are very many occupations in which peculiar conditions destroy the ability to labor within periods far less than the ordinary "expectation."

In 1889, there was put for my consideration, the question: "What is the value to a man's estate of a future income earned by his daily labor, cut off by his total disability or his death?" There were two problems to be solved: one which, under the disability feature, involved the man's future support; the other in which, his death occurring, left his future out of consideration. I worked out the "sum" as best I could, on insufficient data, upon the basis of the temporary annuity value. Afterwards I began, and during the time since have, at intervals, continued to gather information as to certain employments, in respect to the duration of the labor period, and as a result the limiting age when the living workman, through

adverse physical conditions of his environments or through the ill-effects of his self-indulgences, becomes unable to pursue his avocation; or, when having laid up a capital to live upon without labor, he voluntarily retires. I had no systematic course of inquiry.* Some facts, not many, were picked up in books, a few in the trade journals, the rest of them through conversations with representatives of the several trades, of which I had selected thirty. As to twelve of them, I did not obtain information worth recording. Of the eighteen in the table following, for the want of a sufficient number of inquiries, I doubt the accuracy of column two. For the present, however, I assume the table to be true, as it will exploit the chief point in the argument I make adverse to Mr. Ogden's.

Employments are entered in the order of the limiting age. Terminations by death are not included in the columns of "working life."

The beginning of employment as journeyman or expert has no especial value, except as a matter of observation. It might be useful in the case of the value of a child's life, in determining the mean age at which fixed or level wages begin to be paid, if, as Mr. Ogden claims, there is a commutable value to the earnings of a minor.

TABLE 1.

SHOWING MEAN AGE OF CERTAIN WAGE-EARNERS, WHEN ENTERING UPON THEIR WORK AS JOURNEYMEN OR EXPERTS; THE MEAN DURATION OF THEIR WAGE-EARNING LIFE, AND THE AGE WHEN PHYSICAL POWERS BEING EXHAUSTED BY THEIR EMPLOYMENTS, OR BY REASON OF ACQUIRED CAPITAL, THEY RETIRE.

EMPLOYMENT.	Entry as Journeymen.	Working-Life—Years.	Age Limit.
	1	2	3
Ore Reducers (Zinc),	21	18	39
" " (Lead),	20	21	41
" " (Silver),	23	23	46
Cabinet Makers,	22	25	47
Stove Moulders,	22	27	49
Weavers (Cotton),	20	32	52
Glass Blowers,	24	31	55
Stone Cutters,	20	36	56
Weavers (Wool),	24	35	59
Day Laborers,	18	44	62
Engineers (Locomotive),	23	40	63
Carpenters,	20	44	64
Bricklayers,	21	44	65
Pressmen (Newspaper),	22	45	67
Engineers (Stationary),	20	48	68
Bookkeepers,	19	50	69
Compositors (Newspaper),	19	51	70
Government Clerks,	27	46	73

* When I started the investigation, I had directly in mind the preparation of an original paper to be read in the A. S. A. Mr. Ogden has anticipated me in bringing up the subject, hence I use what little I have gathered to illustrate with, and trust he or some other member will pursue the inquiry.

Before, however, considering final results from this table, it is well to ascertain: What is the "remainder over" in wages after taking out the earner's unnecessary personal outgo? There seems to be no uniformity in the amount of this remainder. Men receiving the same wages spend varying amounts upon themselves, and have materially differing notions as to family expenses, as well as to the savings bank account. Mr. Ogden puts it generally at fifty per cent.

As a rule, the poorer and more ignorant workingmen expend the least relative amounts in family expenses—put the least ratio into savings—and spend more in personal indulgences. As workers rise in the scale of intelligence, usually concurrent with the higher nature of their employment, pay attention to economics and take part in matters of public interest, they get more individual satisfaction out of the waste of a lower per cent. of wages, their families live better, and the building association or savings bank takes a larger rate on deposit. The illiterate or bestial, the grumbler and dissatisfied, the listener to the harangues of the walking delegate, wastes his substance. The man of affairs, knowing how the mickle makes the muckle, keeps the substance and gives to his life a higher standard in money value. To the man whose earnings are discontinued by disability through no fault of his own, while he lives on, the portion he has habitually spent in rushing the schooner, or in other sheer wastefulness, has no value whatever in commutation. His necessary personal expense is properly subject to a reduction to present value in case of disablement, while in case of his death by the casualty, that portion of the income must be eliminated. All of these matters of detail are elements in the general problem, and properly taken into account, would necessitate perhaps the determination of every case on its individual merits.

I tabulate again upon the theory that the higher the grade of labor, the higher the intellectual ability to undertake it; and the higher the brain equipment of the artisan, the less will be his mere wastefulness, but a bit higher the necessary personal expenditure for social amenities, dress, etc. I assume temporarily that the proportion of total income will be the same for all wages, for the three R's: rent, rations and raiment. The item of "waste" assumes not only the beer, whiskey and tobacco for the head of the family, but the candy, soda water and little extravagances of the wife and kids.

TABLE 2.

ESTIMATED DISTRIBUTION OF ANNUAL INCOME UPON AN ASSUMED INCREASE OF STANDARD OF INTELLIGENCE IN PROPORTION TO GRADE OF EMPLOYMENT.

1	2		3		4		5	
ANNUAL INCOME.	TOTAL WASTE.		PERSONAL EXPENSE.		FAMILY EXPENSES.		SAVINGS BANK.	
	Rate %	Amount.	Rate %	Amount.	Rate %	Amount.	Rate %	Amount.
$375	20	$75.00	15	$56.25	60	$225.00	5	$18.75
600	16⅔	100.00	10⅔	100.00	60	360.00	6⅔	40.00
900	12½	112.50	20	180.00	60	540.00	7½	67.50
1,200	10	120.00	21	252.00	60	720.00	9	108.00
1,500	8	120.00	22½	337.50	60	900.00	9½	142.50
1,800	5	90.00	25	450.00	60	1,080.00	10	180.00

Columns 3 and 4 make the total family expense, because the items are both necessary to the family living.

Now to apply. Suppose a day laborer, aged 35, is permanently disabled by an engine, and the jury finds that the railway corporation running the engine is liable for damages. What are they? Plainly, the present value of the man's wages, less his waste from the time of the casualty to the limit of his working-life—at age 62—$375 − $75 × 13.988* = $4,196.40.

But suppose the laborer's death had been caused by the accident. His personal share of the family expense is taken out of the problem; and only the amounts in columns four (4) and five (5) are to be taken. The value is then $243.75 × 13.988 = $3,379.58. Suppose the victim of the accident, a silver ore reducer, aged 35, earning $1,800. He wastes $90.00 a year. Living, his remainder for himself, family and saving is $1,710.00. Dead from the accident, $450.00 more is eliminated from consideration. But his wage-producing life terminates at age 46. The measure of the value in either case is the present value of $1.00 to run eleven years into the net income; on $1,710.00 × 7.706 = $13,177.26 in the first, and $1,260.00 × 7.706 = $9,709.56 in the second. Take a bookkeeper, aged 35, meeting a similar casualty. He earns $900, wastes $112.50, expends for himself $180.00 and for his family $540, putting into a building association $67.50. His retiring age is 69. The temporary annuity for the complement of his working-life is worth $15.084 per unit. If he is killed, the present worth of his wages cut off is $6,429.78. If only disabled it is $9,144.90.

It is not necessary to multiply examples. Mr. Ogden's cases take into account death only. I take disability cases as well.

* I use actuaries' table with 4 per cent. interest because it is at hand and is the standard for certain life contingencies in nearly all the States; and the temporary life annuity because discontinuances of income by death are not included in column two of Table 1, and probable mortality must be considered somewhere.

For all the rest he and I agree that the measure should be the present worth of an annuity of $1.00, and disagree as to the possible period of the income. I believe my theory is the true one.

In another particular I totally disagree with him. I cannot conceive of value in a child's life, to its parents or to any one else, beyond the time when the child becomes of age, and in the eye of the law is his own master and entitled to all his own earnings. Prior to attaining legal majority, there are cases where the parents of poor children put them out to some inferior service, or procure apprenticeships for them, and claim and take a portion or perhaps all of their wages. But in such cases, the value in their earnings, to the beneficiaries, should, as in the cases of adults, be dependent upon the temporary annuity to the limit of child-life, age 21, in all cases of minors.

In Holland and some other countries on the Continent, the obligation to care for the indigent parent, or other helpless relatives, is made obligatory by law. Mr. Ogden cites New Jersey law for "similar requirements in this country." There is something of the same sort I am told in Rhode Island and in Connecticut. I think none in any other of the United States.

But such filial, fraternal or parental duty, whether imposed legally or morally, cannot be considered under the same terms, as in the cases of free agency in the disposition of one's earnings to his natural dependents, the family. There is a merger of a second problem into the first. Say, under the law of New Jersey, a workingman obliged to support an indigent mother, in addition to his own family, finds his income cut off by disability. It is not only the present value of the loss to the worker himself and his family that is to be ascertained upon an agreed scale of expense and savings, but the cost of the mother's support by the value of a whole life annuity at her then age, which comes into the computation.

The general problem is one of great perplexity when carried to its ultimate solution. Meantime, if the Labor Unions and National and State Labor Commissioners could embody in their records reliable statistics of the duration of wages-producing life and classify in four or five groups the distribution of wages income, amplifying Table 1 hereinabove to include all the occupations in which men and women engage, and use such a number of observations as would preclude any more doubt of the accuracy of the final columns than we now have of the sufficiency of the standard mortality tables, there would soon be compiled a volume of infinite value in the determination of questions touching the pecuniary present worth of lives, which come before the Courts and to the desks of the actuaries.

Mr. C. C. Hall: The value of actuarial science in determining certain classes of questions that come before the Courts has, as Mr. Ogden points out, been long recognized in England, and this fact is referred to in the preamble to the Royal Charter of the Institute of Actuaries, granted by the Crown in 1884, where the profession of actuary is mentioned as "one requiring a wide and varied

training in respect to the frequent employment of its members in the several capacities of expert witnesses, valuers," etc., and again as "one to which important matters are frequently referred by the Chancery Division of the High Court of Justice and other Legal and Administrative Departments of the State." It is also true that in this country there is a growing recognition of the value of the expert testimony of actuaries in cases where interests involving life contingencies are concerned. In the Federal Courts in the administration of the old bankruptcy law, it was customary in the event of the sale by the assignee of land belonging to the bankrupt, he being a married man, to require the testimony of an actuary as to the value of the wife's potential right of dower, awarding to her, out of the proceeds of sale, such sum as should be shown to be equitable by such expert testimony.

In most of the States there are, probably, established rules of chancery, by which the value of dower rights, whether consummate or inchoate, are determined; but the State Courts also have occasion to avail of the testimony of actuaries for the purpose of ascertaining the value of life estates, reversions, annuities, policies of life insurance, etc. But most frequently is the actuary called upon to testify in the class of cases especially referred to by Mr. Ogden, where action is brought for injury resulting to survivors from the death of a person caused by the "wrongful act, neglect or default" of another.

At the common law all personal actions died with the person. In consequence, however, of the hardship resulting in cases where death was caused by negligence, and thus the parties dependent upon the labor of the person killed were left without support and without remedy for the serious injury so occasioned to them, Parliament, in 1848, passed an act, commonly called Lord Campbell's Act (9 and 10 Victoria, c. 93), entitled "An Act for compensating families of persons killed by accidents." Similar statutes have been enacted in most of the American States. In Maryland (with whose laws I am more familiar than with those of other States) an act with the same title and in substantially the same terms as Lord Campbell's Act, was passed by the Legislature in 1852 (Cap. 299). In this statute the right of action is limited to the surviving wife, husband, parent and child of the person killed, and suit must be brought in the name of the State for the use of the claimant. Under the English act, the suit is to be brought by and in the name of the administrator of the estate of the deceased.

It is provided in the Maryland statute that "the jury may give such damages as they may think proportioned to the injury resulting from such death to the parties respectively for whom and for whose benefit such action shall be brought." But damages are properly allowable only for such injury as is of pecuniary estimation. Statutes which have been enacted in some States restricting or limiting the amount that may be recovered for the death of a person, are generally the result of extravagant verdicts given by juries, where regard was not had to the pecuniary loss sustained,

but exemplary or punitive damages were awarded in a spirit of resentment against the defendant—usually a corporation—for what may have seemed like, and perhaps was, reckless disregard for the safety and protection of employes.

For fairly regulating the amount of damages to be awarded to the claimant, the expert skill of the actuary is most valuable, and if the measure of damages awarded in cases of this kind had been always determined by such expert testimony, the occasion for restrictive statutes of the sort referred to would never have arisen. The problem, however, is not always a simple one.

Mr. Ogden's assumption that a man who has attained his maximum earning capacity at a certain age, say 50, will continue to earn the same income during the remainder of his natural life, does not seem to me tenable. A large proportion of the persons killed by accidents are laborers, and as a class they become superannuated for work earlier in life than persons in other employments. Men can practise medicine or law long past the age at which pickaxe and shovel can be wielded with success and remuneration. This fact is recognized in the establishment of plans for old age pensions for certain classes of employes, by the managements of some of the great railway corporations, and in the compulsory provision for pensions for laborers and operatives under the laws of New Zealand and of several countries in Europe. The laborer's wage-earning period is in general considerably less than the term of his natural life; and the same is true of persons in other occupations in which bodily strength and activity, or physical endurance and ability to withstand exposure, are required. A present wage-earner may himself in turn become a dependent, before his life's end, upon those whom he now supports.

Then, too, the value of the support to a dependent, a wife, for instance, is contingent upon the continuance of her own life, as well as that of the wage-earner. A wife cannot be said to have a pecuniary interest in the earnings of her husband measured solely as an annuity on *his* life, for he may outlive her. The value *to her* is, therefore, contingent upon the joint continuance of both lives, and is more nearly related to the value of an annuity upon the joint lives. The same is true of other claimants, such as parent, or child, should such there be. Suppose, for example, claim were made on behalf of a parent aged 80, for damages resulting from the death of his son killed by accident at the age of 30. The parent's interest in a support derived from the son's earnings should obviously not be measured solely by the value of an annuity on the life of the latter.

In respect to the right of action of a parent for damages sustained by the death of a minor child, who at the time of death was in fact supported by the parent, it seems to be settled law that such right of action exists under Lord Campbell's Act, both English and American decisions agreeing upon this point. But in the case of State, use of *Coughlan* vs. *B. & O. R. R. Co.* (24 Md, 107), the Court, in passing upon the measure of damages, limited the re-

covery to the pecuniary loss to the parent *during the minority of the child.* This rule would seem to follow logically from the fact that the parent has a legal right to the child's earnings only during minority, while at the same time he is charged with responsibility for the child's support. The object of the Court was to restrain the jury from awarding speculative damages based on a conjecture that the child, had he lived, might have been the support of his parent (the mother in this case) after his majority. On the other hand, it is not required that the plaintiff should have a legal claim on the deceased for support, *if the support were actually afforded* at the time of the death (*R. R. Co.* vs. *Barron*, 5 Wallace, 90).

A claim based upon statutes similar to the New Jersey statute quoted by Mr. Ogden, imposing the support of "every poor, old, blind, lame, and impotent person, or other poor person not able to work," upon the immediate ancestors or descendants of such person, they being of sufficient ability, would seem hardly to admit of actuarial valuation, unless the prescribed conditions, or some of them, existed at the time of the death occurring; for the right to such support would depend not only upon the continuance of the claimant's life, but also, by the terms of the statute, upon his incapacity for work. The probability of the occurrence of this latter condition, and the time of its commencement, when the right to support would accrue, would alike be difficult of estimation. But the object and effect of such a statute would apparently be not so much to create a new or extended interest in the life of a child or parent, as to relieve the State or County of what would otherwise be a public charge.

MORTALITY EXPERIENCE AMONG LIVES ACCEPTED AT AGES OVER SIXTY BY THE EQUITABLE LIFE ASSURANCE SOCIETY OF THE UNITED STATES.

R. G. HANN.

PAGE 54, VOL. VI.

MR. BRADSHAW: Mr. Hann's paper is one of those productions which immediately recall to our mind the motto of our Society, for he has substituted "facts for appearances and demonstrations for impressions."

The subject treated is one of practical interest, and the results of his investigation will, no doubt, assist some of us in determining our policy in respect to the admission of those lives which have exceeded the three-score limit.

The experience covers a period of twenty-six years, ending in December, 1895. The number of policies involved is 2 107, of which 563 on 532 lives, terminated by death. The aggregate amount of the assurances is $12,631,896, of exposures, $54,411,276, and of policies terminated by death, $2,621,545. The average length of exposure was 4.31 years, and no less than 66½ per cent., or $36,121,255, of the exposures belong to the first five years of assurance.

It will thus be seen that the experience is a somewhat limited one and does not admit, as Mr. Hann intimates, of trustworthy deductions.

The comparison of the actual claims by amounts and lives is made by a mixed table of mortality, the American Experience Table, and as a comparatively large number of the lives had apparently been recently selected, a favorable mortality might have been expected.

The utilization of an aggregate or mixed table of mortality comprising exposures of all duration, as a standard for the purpose of comparing the actual experience with the expected experience for lives which have been insured from one to five years, appears to be exceedingly inappropriate, and is calculated to lead to wrong conclusions. It is, I think, to be regretted that there was no suitable standard select life table available for the purpose of comparison.

The following table exhibits the percentage of actual claims to expected claims for lives, after breaking up the experience into two sections; one in respect to persons entering between ages sixty-one and sixty-five, and the other in respect to those entering beyond age sixty-five:

Ages at Issue.	Years of Assurance.	Percentage of Actual to Expected on lives.
61—65	1— 5	95.28
	6—10	99 27
	11—26	93.40
66—75	1— 5	104 29
	6—10	96.85
	11—26	93.65

For all ages at issue we have:

Years of Assurance.	Percentage of Actual to Expected on lives.
1— 5	97 25
6—10	98.75
11—26	93.44
Total Experience for lives,	96.87 for amounts 91.00

A noticeable feature of the above results is that in both sections, that is, for ages at issue sixty-one to sixty-five, and for ages at issue sixty-six to seventy-five, also for all ages at issue the percentage of actual deaths to expected is less in the eleventh to the twenty-sixth years of assurance than in either the first to the fifth or the sixth to the tenth years of assurance.

It would not have been surprising had the percentage of actual

deaths to expected deaths been less in the third period than in the second quinquennium, since the results of the investigations into the effects of selections on assured lives made by Dr. Sprague, Mr. King and others, tend to prove that in the early years of assurance a very light mortality is experienced; that as a consequence of a large number of healthy lives withdrawing, deterioration in the remaining number sets in and a gradually increasing rate of mortality results until a maximum mortality is reached, after which a more favorable mortality is experienced.

The percentage of actual deaths to expected deaths for lives is 96.87, that for amounts 91.00; the actual loss ratio, according to lives, is .0528, that according to amounts, .0482; indicating that the losses have mainly been, as Mr. Hann states, on policies carrying small amounts of assurance.

In this connection, it may be interesting to recall the fact that in nearly every American experience the percentage of mortality is greater by amounts than by lives. The following table gives the exact figures:

Company.	Percentage of Mortality.		Difference.
	By amounts.	By lives.	
Mutual Benefit	1.29	1.12	.17
Mutual Life	1.24	1.18	.06
Connecticut Mutual	1.16	1.12	.04
Thirty American Offices	1.10	1.03	.07
John Hancock	1.10	.92	.18
Provident Life and Trust	.81	.78	.03

In Great Britain the investigations made into this question tend to show that the mortality experience among the lives assured for the smaller sums is least favorable, an experience contrary to that of the American companies.

Mr. B. J. Miller (p. 364, Vol. 5) gives the results of an investigation of his company's (Mutual Benefit) experience for the six years (1891-6) on all policies issued at ages above sixty. He included such policies as had been issued before 1891, but which were in force during the whole or any part of the period covered by the examination. He found that on policies less than six years old the actual loss was 51.6 per cent. of the tabular. The rest of the experience being in respect of policies more than five years old and less than thirty-three years old, showed an annual loss of 99.4 per cent. of the tabular. The total figures, including all years of membership from the first to the thirty-second, inclusive, showed that the actual loss was 83.1 per cent. of the tabular. The table employed as the standard for comparison was the American Experience Table.

I have made an investigation of the mortality experience of assured lives, entering at ages sixty-one and over, in the Connecticut Mutual Life and Mutual Benefit Life, from the data given in the

published experiences of these companies. Taking all ages (sixty-one and over) at issue and grouping by years of assurance, we have :

Years of Assurance.	Percentage of Actual to Expected on lives.
1— 5	67.94
6—10	76.93
11—26	94.05
Total Experience,	78.98

It will be observed that Mr. Hann's percentage of actual to expected deaths is greater than that given by Mr. Miller, and likewise greater than that of the combined experiences of the two companies referred to by me, but it must not be overlooked that slightly over ten per cent. of the deaths embraced in Mr. Hann's observations were of persons residing either in the Southern States or in the tropics.

The material contained in this combined experience, like that in Mr. Hann's, is too limited to warrant us in forming definite conclusions, but I think the results of the three investigations would tend to the belief that a not unfavorable experience, according to the American Experience Table, has resulted from insuring lives beyond sixty years of age.

Table IV., "Intensity of Diseases," discloses that 23.30 per cent. of the total claims were due to "Diseases of Brain and Nerves"; 21.75 per cent. to "Constitutional Diseases," such as pneumonia, phthisis, sarcoma, bronchitis, dropsy, rheumatism; 13.10 per cent. to "Diseases of the Urinary Organs." We have, therefore, no less than 58.15 per cent. of the total claims coming under the three broad heads referred to. This table I consider of value as indicating the diseases from which persons are likely to die who were insurable lives at the time of entry at ages beyond sixty. If a medical examiner knows the disease that is most likely to make inroads upon an applicant who has reached a certain period of life, he will naturally make a searching examination and investigation of the applicant and of his history in relation to such disease.

In publications treating with the diseases from which assured lives die, the deaths are usually recorded according to the age at the time of death. While the statistics thus set forth are of value, yet I think that a table exhibiting the causes of death according to groups of ages at entry would be equally as valuable. It would enable an estimate to be formed of the diseases at given ages of assurance at entry, to be most feared as the cause of death. The Washington Life has published such a table, from which I extract the following synopsis :

Ages at time of issue of policy.	Diseases from which there was Highest Mortality.	Percentage of total deaths according to ages at entry.
12—29	1 Consumption.	29.72
	2 Accidents and Injuries..	10.58
	3 Zymotic Diseases.	8 82
30—39	1 Consumption.	20.56
	2 Zymotic Diseases.	10.13
	3 Pneumonia.	9.10
40—49	1 Apoplexy, Paralysis, etc.	13.78
	2 Consumption.	13.22
	3 Diseases of Heart and other Diseases of Circulatory System.	10.24
50—59	1 Apoplexy, Paralysis, etc.	17.79
	2 Diseases of Heart and other Diseases of Circulatory System.	12.08
	3 Diseases of Kidneys and other Diseases of Urinary System.	9.40
*60—69	1 Apoplexy, Paralysis, etc.	25 00
	2 Diseases of Heart and other Diseases of Circulatory System.	14 29
	3 Pneumonia	11.90

As might naturally have been expected, the disease to be most feared in the early period of life is consumption, with zymotic diseases closely following. Accidents and injuries are comparatively numerous in the first age group. After the age of forty, at entry, apoplexy, paralysis, diseases of the heart and other diseases of the circulatory system and diseases of kidneys and other diseases of the urinary system are the most numerous. Mr. Hann's table of "Intensity of Diseases" in respect of those entering beyond age sixty, is well sustained by the experience of the Washington Life.

I do not think that in the insuring of lives beyond ages sixty, the results brought out by this table can be lightly regarded.

*The entrants between ages sixty and sixty-nine were too few to afford reliable averages.

PAGE 95, VOL. VI.

MR. MACAULAY: Mr. Nitchie's note on the experience of the National Life of the United States is a very interesting contribution to our statistics in regard to lives which have been assured for a long term of years. I am probably correct in saying that to most of us the figures have come as a surprise, for we would naturally have supposed that the mortality in a company which has ceased for such a long time to issue new policies would be decidedly in excess of the tabular expectation. To find, therefore, that so far from this being the case, the death rate is actually considerably under that predicted by the Seventeen Offices' experience, is noteworthy. The figures given apply to the mortality of one year only, and we have no means of judging whether that year was an exceptional one or not. Assuming, however, that it was an average year, there are still some points which have to be borne in mind. I think I am right in believing that for many years the National Life of the United States not only issued no new policies, but took very active measures to cancel the assurances already on its books, even going so far as to send representatives to interview its policy-holders for the purpose of persuading them to surrender their policies for cash. It is just possible that this course may have affected the mortality, for the company may have succeeded, perhaps by offering special inducements, in buying up the policies on most of its bad lives. I cannot say that this is so, for I am not familiar with the inner workings of the company, but it is at least possible, if not indeed probable. The very large proportion of paid-up assurances is also a prominent feature of the experience. The mortality on this class of assurance is usually favorable, and while we would expect the mortality under old premium-paying policies to be heavy, we would also expect that on old paid-up policies to be light, and it is not impossible that in a company having sixty per cent. of its assurances paid-up, and only forty per cent. premium-paying, the favorable mortality of the one group might cover up an unfavorable mortality in the other. I am afraid we must come to the conclusion that while the showing is a very good one in the way of indicating the quality of the business of this particular company, the circumstances of that company are sufficiently peculiar to make it unwise for us to attach much importance to the experience as indicating what the mortality in other companies will probably be on old business.

As a matter of curiosity I have had worked out roughly a comparison between this experience and the $H^{M(5)}$ Table of the Twenty English Offices. As the easiest way of obtaining the expected deaths by the $H^{M(5)}$ Table, I found the percentage which the q_x of the latter table bore to the q_x of the Seventeen Offices' experience, taking in each case the central age of the group as given by Mr. Nitchie. The expected amounts of claims in his table were multiplied by the

percentages thus found, and the results were assumed with, I think, sufficient accuracy, to be the expected claims by the $H^{M(5)}$ Table. It is surprising to note that the National Life's experience is only eighty-eight per cent. of the amount thus found. Possibly, however, the feature of the table which will impress us most is the closeness of the claims predicted by the $H^{M(5)}$ and by the Combined Tables, there being a difference of only about 2 per cent. This close agreement at the higher ages we have of course all been aware of theoretically, but it is not often that we have the peculiar character of all aggregate tables brought home to us in such a practical and emphatic way.

EXPERIENCE OF NATIONAL LIFE OF U. S.

Ages.	Exposures.	Expected Claims {Combined Experience.}	Central Age.	Percentage which qx by $H^{M(5)}$ is of that by Combined Table at central age.	Expected Claims. $H^{M(5)}$	Actual Claims.	% Actual of Expected $H^{M(5)}$	% Actual of Expected {Combined Experience.}
38-50	$131,094	$1,888	48	107.4	$2,028	$914	45	48
51-55	344 729	6,744	53	104.3	7,034	6,600	94	98
56-60	389,571	10,362	58	100.7	10,434	9,000	86	87
61-65	468,909	17,950	63	101.8	18,273	17,367	95	97
66-70	223,920	12,348	68	97.9	12,089	15,885	131	128
71-75	116,364	9,458	73	102.0	9,647	4,102	43	43
76-80	58,928	7,371	78	103.5	7,629	1,008	13	14
81-88	21,407	3,700	83	105.6	3,907	7,794	199	211
Total	$1,754,922	$69,821			$71,041	$62,670	88	90

Abstract from the Minutes of the Fall Meeting of the Actuarial Society of America, held in Hartford, Conn., on Thursday and Friday, October 19th and 20th, 1899.

FIRST SESSION.

Board of Trade Room, Phœnix Mutual Life Insurance Company's Building,
Thursday, October 19th, 11 A. M., 1899.

The meeting was called to order by the President, Mr. Thomas B. Macaulay, who, after the roll call, addressed the Society. (See page 177.)

The President announced that the Secretary, Mr. Tatlock, being absent in Europe, Mr. Pierson, at the request of the Council, would take his place temporarily.

Officers and members present at the roll call or during the sessions :

Thomas B. Macaulay, President ; Oscar B. Ireland, 1st Vice-President ; Israel C. Pierson, 2d Vice-President ; John B. Lunger, Treasurer. Other members of Council—David Parks Fackler, Howell W. St. John, Emory McClintock, Elbert P. Marshall, Joel G. Van Cise, James M. Lee, Walter S. Nichols, James M. Craig, Wm. C. Macdonald. Members—Thomas Bradshaw, John Jones Brinkerhoff, James Camp Crawford, Charles Hildebrand, John Marshall Holcombe, Robert Watkinson Huntington, Jr., Charlton Thomas Lewis, William McCabe, William Campbell Macdonald, Hiram John Messenger, Maximilian Heinrich Peiler, Gardner Ladd Plumley, Edward James Sartelle, William Simeon Smith, William Eli Starr, Archibald Ashley Welch, Daniel Halsey Wells, Walter Channing Wright.

The minutes of the meeting of May 18th and 19th were approved as printed.

A summary of the business transacted by the Council was presented and approved.

The matter having been referred to the Council, they recommended the adoption of an amendment to the Constitution, which had been reported to the members in accordance with Art. XIII. of the Constitution. It is an addition to Art. IX., as printed below.

The President announced that Messrs. Fackler, Nichols and Gore had consented to conduct the examinations for admission.

The Council approved names proposed for examination for admission as Associates.

On motion the amendment of the Constitution was adopted as follows :

Addition of the following sentence to Article IX., "Associates" : "When the Council finds that a candidate has passed an equivalent examination required by another recognized society of actuaries, it may waive the examination hereby required."

The invitation of the "Institut des Actuaires Français" to send delegates to the Third International Congress of Actuaries, in Paris, on June 25th, 1900, was accepted. The Council were authorized to appoint representatives.

New papers were then presented. (See pp. 137–176.)

Adjourned at 1 P. M.

During the recess the members were entertained at luncheon at the Colonial Club as guests.

SECOND SESSION.

THURSDAY, October 19th, 1899, 2.30 P. M.

President Macaulay in the chair.

The members proceeded to discuss papers which had been read at previous meetings. (See pages 185–245.)

It was announced that the dinner would take place at the Allyn House.

The Society adjourned at 5.30 P. M.

THURSDAY, October 19th, 1899, 7 P. M.

The usual dinner was given in the large dining-room of the Allyn House. President Macaulay presided.

Postprandial speeches were made by Messrs. J. M. Holcombe, Jacob L. Greene, Jas. G. Batterson, H. W. St. John, Thos. W. Russell, Charlton T. Lewis and J. B. Lunger.

On the same evening the ladies were the guests of Mrs. John M. Holcombe.

THIRD SESSION.

FRIDAY, October 20th, 1899, 9.30 A. M.

President Macaulay in the chair.

The discussion of papers was continued.

The following resolution, offered by Mr. Craig, was unanimously adopted :

"*Resolved*, That the thanks of the Actuarial Society of America

are due and are now tendered to the officers and directors of the Phœnix Mutual Life Insurance Company, whose generous hospitality, expressed by the most thoughtful provision for the needs and convenience of our meetings, and by watchful but quiet attentions to the hourly comfort of our members, has rendered this gathering one of the most enjoyable in our history; and to all the managers of companies and other citizens of Hartford who have done so much to welcome and entertain us."

The Society adjourned at 12 o'clock.

At 12.30 the members and their ladies were conveyed to Farmington and were entertained at luncheon at the Country Club.

ISRAEL C. PIERSON,
2d Vice-President and Secretary, pro tem.

JOHN TATLOCK, JR.,
Secretary,
32 Nassau St., New York.

Constitution and By-Laws of the Actuarial Society of America, as Amended October 19th, 1899.

Constitution.

Art I.—This organization shall be called "The Actuarial Society of America."

Art. II.—*Object.*

The Actuarial Society of America has for its object the promotion of actuarial science by personal intercourse, presentation of appropriate papers, discussion, and such other methods as may be found desirable.

Art. III.—*Membership.*

The membership of the society shall consist of the subscribers and of such other persons, connected with actuarial pursuits, as shall be duly elected or admitted.

Art. IV.—*Officers.*

The officers shall consist of a President, a first and a second Vice-President, a Secretary and a Treasurer.

Art. V.—*Council.*

The officers, with ex-presidents and six other members shall constitute the Council. At the first election of members of Council, after the adoption of this article, two members shall be elected to serve for three years, two for two years and two for one year; and annually thereafter two members shall be elected for three years. No retiring member of the Council shall be eligible for re-election at the same meeting.

Art. VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices. The duties of the Council shall be in general to manage the affairs of the Society in the intervals between its meetings, to call meetings and to make arrangements for the same, to decide upon papers offered for reading and discussion, and to pass upon candidates to be presented to the Society for election.

Art. VII.—*Election of Officers and Council.*

The officers shall be elected by ballot at the annual meeting for the term of one year. The President and Vice-Presidents shall

not be eligible for the same office for more than two consecutive years.

ART. VIII.—*Meetings.*

The annual meetings of the Society shall be held on the first Thursday after the fourteenth day of May in each year. Other meetings may be called by the Council from time to time, and shall be called by the President at any time upon the written request of ten members; twenty days' notice of such meetings shall be given by the Secretary.

ART. IX.—*Associates.*

Any one twenty-one years of age, pursuing actuarial studies and looking forward to future membership, may, on the conditions hereinafter stated, be enrolled as an associate, and in such capacity may be present, without participation, at the meetings of the Society, and receive its publications. The conditions are, that he shall be nominated to the Council by at least two members of the Society; that notice of such nomination shall be given by the Secretary to each member of the Society; that, not less than twenty days after such notice has been given, his nomination shall be approved by the Council, with not more than one negative vote, and that he shall subsequently pay such examination fee and pass such preliminary examination as may be prescribed by the Council. When the Council finds that a candidate has passed an equivalent examination required by another recognized society of actuaries it may waive the examination hereby required.

ART. X.—*Admission to Membership.*

Any associate, who has been such for at least one year, and is twenty-five years of age, may apply to the Council for membership. Every member of the Council shall be notified confidentially of such application at least twenty days before the meeting at which it shall be acted upon. If his application be approved by the Council he shall become a member on paying such examination fee and passing such final examination as shall be prescribed by the Council. Otherwise, no one shall be admitted as a member unless by the unanimous recommendation of the Council, followed by a unanimous ballot of the Society.

ART. XI.—*Quorum.*

Four members of the Council shall constitute a quorum. Ten members of the Society at any duly called meeting shall constitute a quorum.

ART. XII. No resolution expressive of opinion shall be entertained at any meeting.

ART. XIII.—*Amendments.*

This Constitution may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of such proposed amendment shall have been sent to each member by the secretary.

BY-LAWS.

ART. I.—*Order of Business.*

At the meeting of the Society the following order of business shall be observed, unless the members vote otherwise for the time being :

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. The minutes of the last meeting read and passed.
5. Summary of the business transacted by the Council since the last meeting of the Society presented for its information and approval.
6. Candidates for membership considered and voted upon.
7. Reports of Committees and Officers.
8. At annual meeting, election of Officers and Council.
9. Unfinished business ; new business.
10. Reading of papers prepared for presentation to the Society.
11. Discussion of papers presented.
12. Reading of a summary of the business just transacted at the meeting for correction if necessary.
13. The Council meetings shall proceed in this order as far as it may apply to them.

ART. II.—Meetings of the Council shall be called whenever the President or four members of the Council so request, but not without sending notice to each member of the Council ten or more days before the time appointed ; such notice shall state the subjects intended to be brought before the meeting, and should

other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ART. III.—*Duties of Officers.*

The President, or, in his absence, the next Vice-President in order, shall preside at meetings of the Society and of the Council. At the Society meetings he shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The duties of the Secretary shall consist in keeping a full and accurate record of the proceedings at the meetings of the Society and of the Council, sending out calls for the said meetings, and with the approval of the President and Council, carrying on the correspondence of the Society.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Society or the Council; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a Committee of the Council.

ART. IV.—*Dues.*

Each member shall pay ten dollars, and each associate five dollars, upon entrance, and a like amount at each annual meeting thereafter, except in the case of members not residing on this continent, who shall pay five dollars at the times stated.

It shall be the duty of the Treasurer to notify by mail any member or associate whose dues may be six months in arrear, and to accompany such notice by a copy of this article. If such member or associate shall fail to pay his dues within three months from the date of mailing such notice, the Treasurer shall report the delinquent to the Secretary, by whom his name shall be stricken from the rolls, and he shall thereupon cease to be a member or associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ART. V.—*Amendments.*

These By-Laws may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of the proposed amendment shall have been sent to each member by the Secretary.

Information Regarding Admission

TO THE

ACTUARIAL SOCIETY OF AMERICA.

The Constitution of the Actuarial Society states that its object is "the promotion of actuarial science by personal intercourse, presentation of appropriate papers, discussion and such other methods as may be found desirable."

There are two grades in the Society, viz. : Members and Associates, and, practically speaking, no one can become a Member without having passed through the grade of Associate. Anyone, 21 years of age, who has been pursuing actuarial studies with the intention of entering the actuarial profession and is favorably known to two Members of the Society, may make application to the Council, and, if his application is approved by the Council and he can then pass the examination prescribed by the Council, he will be admitted as an Associate.

The examination for admission as an Associate has the following scope :

1. Arithmetic, Algebra and Plane Geometry, including the theory and use of Logarithms.

2. The Principles of Double Entry Bookkeeping ; the Elements of Compound Interest, including Annuities Certain.

3. The Doctrine of Probabilities ; the Elements of Interpolation.

4. The Application of Mathematics to Life Contingencies, including the Theory and Use of Commutation Tables, the Computation of Premiums for Term, Ordinary and Limited Payment Life and Endowment Policies, and also for Policies with return of Premium ; and the different modes of computing Reserves ; all the above for single and for joint lives.

5. Practical Examples in all the foregoing subjects.

6. General Nature of Life Insurance Contracts ; the outlines of the History of Life Insurance, and the source and character of the principal Mortality Tables.

Any Associate, who has been such for at least one year and is 25 years of age, may apply to the Council for Membership, and if

his application is approved and he passes a further examination, he will then become a Member.

The examination for admission as a Member has the following range of subjects :

1. Methods of Constructing and Graduating Mortality Tables and the use of the Formulas of Gompertz and Makeham.

2. Methods of Loading Premiums to provide for Expenses and Contingencies.

3. Valuation of the Liabilities and Assets of Life Insurance Companies.

4. The Assessment of Expenses and the Distribution of Surplus.

5. Practical Treatment of cases of Alteration or Surrender of Life Insurance Contracts.

6. Application of the Calculus of Finite Differences and of the Differential and Integral Calculus to Life Contingencies.

7. Laws of the United States and Canada relating to Life Insurance.

8. Insurance of Under-Average Lives and Extra Premiums for Special Hazards.

The Council have directed that examinations shall take place on the first Thursday in May in each year, and the custom has been to hold them in several places and in such localities as will be convenient to two or more candidates. Examinations thus far have been exclusively in writing.

An application for admission to examination must be submitted to the Council, which cannot act thereon until at least twenty days after the Secretary has given notice of such candidacy to all the Members of the Society ; and, as the Council meets only at long intervals, any person who wishes to apply for admission to the examination should do so some months ahead of the time for the examination, if possible ; otherwise it may not be practicable to give the prescribed notice to the Members twenty days before some Council meeting preceding the examination day, and his nomination may, therefore, have to be held over until after the examination, so that he would have to wait until next year.

The fee for admission to examination is ten dollars. The annual dues of Associates are five dollars, and of Members ten dollars (except for Members not residing on this continent, who pay five dollars only).

If an approved candidate for admission to examination as Associate is found to have passed an equivalent examination required by another recognized society of actuaries, the Council may waive the examination which is prescribed, and in such case no examination fee will be required, although the regular form of application must be made.

Applications for admission should be made on the Society's blank form, which may be obtained by enclosing a stamped and directed envelope to the Secretary of the Society, or to the Chairman of the Examination Committee. At this date (March, 1900) the Secretary is Mr. JOHN TATLOCK, No. 32 Nassau Street, New York City, and the Chairman of the Examination Committee is Mr. D. P. FACKLER, No. 35 Nassau Street, New York City.

Third International Congress of Actuaries.

Copy of Circular Letter Sent to the Members.

141 Broadway, New York, March 21, 1900.

Dear Sir:

Referring to the "PROGRAMME OF THE THIRD INTERNATIONAL CONGRESS OF ACTUARIES (PARIS, 1900)," a copy of which was sent some time ago to the members, I call your attention again to the matter.

The Permanent Committee of International Actuarial Congresses, and the Organizing Committee (appointed by the French Institute of Actuaries), Messrs. Paul Guieyesse and Léon Marie being President and Secretary, respectively, have decided upon the Programme and Regulations of the Third International Congress of Actuaries which will be held in Paris on June 25–30, 1900. The undersigned, Secretary of the Congress for Canada and the United States, has been requested to act as local Treasurer.

As in previous years those not eligible for membership under the rules may, nevertheless, be subscribers for copies of the proceedings.

Papers for the Congress (see Art. 7 of Regulations) should be sent without delay to the Secretary of the Organizing Committee, Mr. Léon Marie, 32 Rue Jouffroy, Paris, or the local Secretary.

Subscriptions (see Art. 3 of the Regulations) and inquiries from Canada or the United States should be sent to the address given below. Should you desire to possess a copy of the proceedings it is advisable to subscribe now unless you have already done so.

Yours faithfully,

ISRAEL C. PIERSON,

Secretary of the Congress for Canada and the United States,

141 Broadway, New York.

Programme of the Third International Congress of Actuaries (Paris, June 25-30, 1900).

SUBJECTS FOR DISCUSSION.

1. Insurance against invalidity, whether arising from sickness, old age, or accident—Definition of the risk—Investigation and estimation of claims—Form of the indemnity—Statistics and necessary tables—Calculation of premiums—&c.
2. Methods of valuation and distribution of surplus of life offices—The results obtained by different methods.
3. Comparison of the rates of mortality in various countries—Extra premiums for travel and residence in countries where the rate of mortality is abnormal, more especially in the Tropics.
4. Comparison of the rates of mortality in various occupations—Extra premiums for the more dangerous occupations.
5. Methods employed for the valuation of the negotiable securities included in the assets of a company—Estimated values based on the purchase price—Estimated values based on market values—Mixed systems—&c.

Regulations of the Third International Congress of Actuaries (Paris, 1900).

Art. 1. The Congress shall confine itself to the consideration of questions relating to Actuarial Science.

Art. 2. The following persons shall alone be qualified to become members of the Congress :

1. Delegates nominated by their respective Governments to act as their official representatives at the Congress.

2. Members of established Actuarial Societies.

3. Members of the Permanent Committee of Actuarial Congresses.

4. Members of the Congresses held in Brussels in 1895, and in London in 1898.

5. Actuaries in active practice in public or private institutions known to the Organizing Committee.

6. Persons exceptionally admitted by the Organizing Committee on the nomination of a member of the Congress previously inscribed on the list of members and belonging to one of the five foregoing classes.

ART. 3. In order to become a member of the Congress, any person belonging to one of the classes set out in Article 2 must forward before June 1, 1900, to the Secretary of the Organizing Committee a written intimation that he desires to become a member, together with a minimum subscription of 20 francs.*

Correspondents have been appointed in various countries to receive and forward applications and subscriptions.

ART. 4. The Organizing Committee may appoint Presidents, Vice-Presidents, and honorary members of the Congress.

ART. 5. Every member of the Congress (as defined in Art. 3) shall receive a card entitling him to take part in the meetings and to receive free of charge the volume of the proceedings which will be published under the authority of the Organizing Committee. Any surplus copies of the volume will be sold at a minimum price of 20 francs each.

ART. 6. The programme of the business of the Congress shall be drawn up by the Organizing Committee in concert with the Executive Council of the Permanent Committee (regulations of Permanent Committee, Art. 2), and will be forwarded to all members in good time before the Congress.

ART. 7. The Organizing Committee in concert with the Executive Council of the Permanent Committee, will nominate a list of Referees for any questions appearing in the programme. Such Referees will represent as far as possible the various countries which take part in the Congress.

The Organizing Committee may accept for discussion by the Congress papers on subjects having relation to Actuarial Science although not mentioned in the programme.

All papers and reports should be sent to the Secretary of the Organizing Committee not later than April 1, 1900, *at the very latest*, so that they may be translated and printed before the opening of the Congress.

ART. 8. The official language of the Congress shall be French, but papers and speeches may be in English or German. The Organizing Committee will endeavor to have as many as possible of the papers and reports translated.

ART. 9. The meetings will be held in the "Palais des Congres" of the Exhibition† on the days and at the times undermentioned :

* Equivalent to 4 dollars, 16 shillings, 16 marks, 19 crowns, 7½ roubles, 9½ florins, &c.

† Near the "Place de l'Alma."

Monday,	June 25,	1900,	at 2.00 P. M.
Tuesday,	June 26,	1900,	at 9.30 A. M.
Wednesday,	June 27,	1900,	at 9.30 A. M. and 2.30 P. M.
Thursday,	June 28,	1900,	at 9.30 A. M.
Friday,	June 29,	1900,	at 9.30 A. M. and 2.30 P. M.
Saturday,	June 30,	1900,	at 9.30 A. M.

ART. 10. The Bureau of the Permanent Committee provisionally installed in Office by the Organizing Committee will declare the Congress open and proceed to the election of officers of the Congress as follows:

1. All the members present will elect a President and a General Secretary, an absolute majority of those present being required. If there be no such absolute majority, then a ballot shall be taken and the election shall be determined by a bare majority of votes.

2. Each group of members of the same nationality shall elect out of their own number a Vice-President and a Secretary, provided that there be fifteen such persons of the same nationality members of the Congress and five present at the time of voting.

The members of other nationalities shall form another group and shall also elect a Vice-President and a Secretary in the same manner.

ART. 11. It shall be the duty of the President to direct the work of the Congress and act as chairman at the meetings. He may be replaced by one of the Vice-Presidents to be nominated by him, or failing such nomination, by the Bureau.

ART. 12. The Congress shall have full power to decide any point which is not provided for by the Regulations of the Permanent Committee or by the present Regulations.

ART. 13. The accounts shall be kept in the name of the Organizing Committee, which alone shall have full power to regulate the receipts and expenses and to dispose of any surplus (Rules of the Permanent Committee, Art. 7).

No member of the Congress shall be liable for any claim or demand beyond the subscription fixed by Art. 3.

ART. 14. Membership of the Congress is subject to the present Regulations and to those of the Permanent Committee.

Proceedings of the Second International Congress of Actuaries, London, 1898.

THE Institute of Actuaries, at considerable expense, undertook the publication of the Transactions of the International Actuarial Congress, held in London in May, 1898. At the commencement of the volume there are various technical papers, while from page 185 onwards the majority of the subjects dealt with are exciting wide interest. Authors of many nationalities have contributed authoritative papers on Life Assurance Legislation, pages 185–347; Friendly Societies, pages 348–581; Workmen's Compensation for Accidents, pages 648–756; Old Age Pensions, pages 757–875. Each writer deals with the matter from the point of view of his own country, and in popular language, with the result that the Transactions contain a mass of authentic information which cannot be found elsewhere. For the benefit of English readers translations are given of all the papers written in foreign languages.

The price is thirty shillings per copy, which may be obtained from the publishers, Messrs. C. & E. Layton, or their agents.

VOL. VI. NO. 23.

PAPERS AND TRANSACTIONS.

MAY 17TH AND 18TH, 1900.

MORTALITY EXPERIENCE—THE NORTHWESTERN MUTUAL LIFE INSURANCE COMPANY.

C. A. LOVELAND.

THE mortality experience of the Company with which I am connected, now partially tabulated, presents features of possible interest to the Society. The period covered by the investigation is from the commencement of the Company's business in 1858 to the end of 1895, or 38 years. The comparison will be between the actual payment on account of deaths and the expected payment according to the actuaries' table of mortality. The table submitted herewith is arranged by years of issue, and I will call attention briefly to the points of greatest interest.

Whole number of policies under observation, .	281,797
Whole number of policies terminated by death,	15,394
Expected payment on account of deaths, .	$51,306,792
Actual payment on account of deaths, . .	35,256,473
Percentage of actual to expected, . . .	68.71

For the first half of this period—19 years—although I must assume that the general conditions under which the insurance was placed were similar, we find great variety in the percentages of actual payment on account of mortality, with 65.5 per cent. on the issues of 1863 as a minimum and 98.10 per cent. on the issues of 1870 as a maximum. The high rate on 1870 issues, however, is exceptional and due to special causes. From 1877 to the end of the period the improvement is marked, the rate, with a single exception, 1878, being under 71 per cent. of the expected loss.

In 1881 the Company adopted the tontine dividend plan. It had been confidently expected that this plan of insurance, involving a

selection in favor of the Company on the part of members accepting deferred dividend contracts, would result in a decided improvement in the mortality rate. During the period of 15 years, 1881–1895, included in the table, a very large portion of all business written was on the deferred dividend plan. The items for the period are as follows :

Number of Policies.	Expected Payment.	Actual Payment.	Number of Policies Terminated by Death.	Per Cent. of Actual to Expected.
196,660	$24,971,163	$14,407,634	5,078	57.69

Of this fifteen-year period the last five years, 1891–1895, were within the influence of medical examination at date assumed for this investigation, 1895, and the items relating to the five-year period are as follows :

Number of Policies.	Expected Payment.	Actual Payment.	Number of Policies Terminated by Death.	Per Cent. of Actual to Expected.
97,742	$5,315,518	$2,626,120	988	49.40

Omitting the issues of the last five years from the fifteen-year period we have :

Number of Policies.	Expected Payment.	Actual Payment.	Number of Policies Terminated by Death.	Per Cent. of Actual to Expected.
98,918	$19,655,645	$11,781,514	4,090	59.94

In point of mortality the anticipations with respect to this class of insurance seem to have been fully met.

Under the theory that lapses and surrenders are largely from the best class of risks, impaired lives being most persistent, it might be expected that the mortality rate in any distinct class of insurance would respond to this influence, and increase as time passes. The present investigation, however, suggests a counteracting force in the early weeding out of weak and inferior lives by death, and with a considerable portion of the insurance here mentioned the mortality rate is found to be improving with the age of the classes under observation. In the table submitted we have

assumed thirty-eight distinct classes or bodies of insurance, each year's issue constituting a class. In an additional table I will show the ratio of actual to expected payment at three dates, to wit: in 1875, 1885 and 1895, and by comparison the distinct classes showing increase or decrease in mortality rate will appear. Excluding all classes involving tontine dividend business and all in which the effect of medical examination may be observed, we find an average percentage of actual to expected for each examination, viz.:

For 1875, ...	78.59	per cent.
" 1885, ...	78.56	"
" 1895, ...	78.64	"

This result indicates the absence of any permanent effect of adverse selection.

Percentage of Actual to Expected Mortality.

(Three Examinations.)

Year of Issue.	To 1875.	To 1885.	To 1895.
1858	75.40	80.10	71.07
1859	71.12	67.50	69.90
1860	73.79	78.70	77.55
1861	67.48	70.30	74.28
1862	72.43	74.20	80.14
1863	58.08	65.40	65.50
1864	62.29	64.50	69.88
1865	68.01	67.20	70.13
1866	76.69	78.40	78.09
1867	78.97	78.70	83.64
1868	77.20	76.70	76.16
1869	76.71	86.00	85.71
1870	106.89	106.80	98.10
1871	96.93	89.20	87.43
1872	76.06	80.90	85.75
1873	85.33	86.40	80.46
1874	112.64	84.50	83.17
1875	56.55	81.70	79.32
1876	.....	87.50	83.83
1877	.....	73.80	70.12
1878	.....	81.90	75.10
1879	.....	64.00	65.02
1880	.....	54.20	65.21
1881	.....	56.90	62.58
1882	.....	57.00	68.06
1883	.....	49.40	69.13
1884	.....	49.40	70.21
1885	.....	9.60	57.36
1886	.....	.....	53.21
1887	.....	.....	56.45
1888	.....	.....	63.93
1889	.....	.....	46.88
1890	.....	.....	59.46
1891	.....	.....	45.69
1892	.....	.....	55.65
1893	.....	.....	48.40
1894	.....	.....	49.95
1895	.....	.....	40.35
	79.13	77.20	68.71

Mortality Experience by Year of Issue.

Year of Issue.	Number of Policies.	Expected. Amount.	Actual.		Per Cent. Actual to Expected.
			Number of Exits.	Amount.	
1858	51	$33,768	8	$24,000	71.07
1859	215	179,827	51	125,700	69.90
1860	385	230,798	92	179,000	77.55
1861	584	171,090	107	127,100	74.28
1862	837	282,235	185	226,200	80.14
1863	1,737	588,036	322	385,180	65.50
1864	3,648	1,273,038	658	889,637	69.88
1865	4,607	1,567,664	828	1,099,421	70.13
1866	4,900	1,686,933	815	1,317,366	78.09
1867	7,731	2,551,068	1,061	2,133,805	83.64
1868	9,153	2,713,110	1,050	2,066,557	76.16
1869	7,679	2,357,710	913	2,020,996	85.71
1870	6,064	1,581,281	691	1,551,280	98.10
1871	4,341	1,289,949	486	1,127,844	87.43
1872	4,633	1,276,916	457	1,094,962	85.75
1873	3,773	1,126,225	334	906,262	80.46
1874	4,063	1,116,661	352	928,762	83.17
1875	4,861	1,306,437	443	1,036,296	79.32
1876	4,006	1,205,599	365	1,010,623	83.83
1877	2,945	822,897	261	577,059	70.12
1878	2,613	841,155	252	631,725	75.10
1879	2,891	1,069,651	288	695,504	65.02
1880	3,420	1,063,581	297	693,560	65.21
1881	5,245	1,698,854	370	1,063,158	62.58
1882	5,781	1,628,354	408	1,108,389	68.06
1883	5,695	1,585,507	373	1,096,200	69.13
1884	5,747	1,606,176	366	1,127,652	70.21
1885	7,341	1,972,831	391	1,131,775	57.36
1886	9,696	2,094,290	416	1,114,365	53.21
1887	11,019	2,039,330	434	1,151,284	56.45
1888	12,939	2,328,345	459	1,488,555	63.93
1889	15,884	2,350,171	422	1,101,531	46.88
1890	19,571	2,351,787	451	1,398,605	59.46
1891	21,493	2,031,964	364	928,409	45.69
1892	22,292	1,631,725	322	908,145	55.65
1893	17,137	867,958	172	420,092	48.40
1894	17,307	553,581	92	276,551	49.95
1895	19,513	230,290	38	92,923	40.35
Total,	281,797	$51,306,792	15,394	$35,256,473	68.71

Note upon Annuity Contracts.

CLAYTON C. HALL.

IN January last a decision was rendered by the Court of Appeals of Maryland in which, for the first time in this country, the nature of a contract of annuity granted by an insurance company for the life of the annuitant was authoritatively defined. The case is of interest chiefly for the reason that the conclusion of the Court is at first glance at variance with the doctrine upon this subject that had prevailed undisputed for three hundred years, confirmed in repeated decisions by the Courts in which the Common Law of England is administered. This decision, however, while it appears to be an overruling of the older decisions, is, in fact, based rather upon a clear discrimination between the modern form of contract of life annuity and the older form which had been the subject of judicial interpretation hitherto.

The decision was rendered in a suit which had been brought against the Maryland Life Insurance Company of Baltimore, by one Cahill, administrator of a deceased annuitant, for the recovery of the consideration money paid for the purchase of the annuity. The contention was that a valid contract of annuity, executed under power given in the charter of the Company to "grant annuities," could only be made by deed under seal, and that the annuity contract executed by the Insurance Company, being without seal, did not constitute a valid *grant*, and, therefore, that the consideration failed.

It was my fortune to be associated with the general counsel of the Insurance Company, the Hon. William A. Fisher, in the trial of this case, and as it is the general, if not the universal, custom of American life insurance companies to issue policies of annuity as simple contracts, that is to say, without seal, it occurred to me that a brief report of the points involved in this case might not be without interest to the members of this Society.

The defense of the Insurance Company was upon two principal points :

First. That the policy of annuity executed by the Company under authority given by its charter, and in the manner prescribed by its by-laws, though without seal, did constitute a valid contract of annuity.

Second. That after a contract had been (as in this case) fully executed by one of the parties thereto, there was no right of action in the other party for the recovery of the consideration on account of defect in the form of the contract, if defect there were.

There was a third defense, which was sustained by the trial Judge, that the party plaintiff was not the proper plaintiff in an action for money had and received, on account of lack of privity of contract; but this was apart from the merits of the case.

The case was fully argued on all the points involved, for while a decision upon the grounds expressed in either the second or third defense would have fully disposed of this particular case, it was thought desirable, in view of the large interests involved in existing contracts of annuity made by American insurance companies, amounting probably to between $35,000,000 and $40,000,000, to present strongly to the Court the broad question of the validity of the form of annuity contract generally in use, and secure, if possible, a decision sustaining the validity of these contracts.

The argument upon this point was somewhat technical. Anciently, especially before the rise and development of corporations formed for the insurance of lives and granting of annuities, annuities were frequently granted by individuals, generally in consideration of borrowed money. These annuities were to be paid for the life of the *borrower*, that is the grantor, and as he might survive the lender, or grantee, they were made payable to the latter and his *heirs*, following in this respect the order of descent of title to land and not that of interest in personal property, which would pass to the administrator for distribution and not to the heir.

With this form of annuity in view, the ancient law upon this subject was developed.

Going back to the time of Lord Coke, we read (Coke on Littleton, Book 2, Section 144 b,) that "*a writ of annuity* is a writ for the recovery of an annuity. An annuity is a yearly payment of a certain sum of money, granted to another in fee, for life, or years, charging the person of the grantor only. But not only the grantee, but his heir, and his and their grantee also, shall have a writ of annuity. But if a rent-charge be granted to a man and his heirs, he shall not have a writ of annuity against the heir of the grantor, albeit he hath assets, unless the grant be for him and his heirs."

Sir William Blackstone (Commentaries, Book 2, p. 20,) includes annuities in the class of incorporeal hereditaments, stating that these hereditaments "are principally of ten sorts, advowsons, titles,

commons, ways, offices, dignities, franchises, corodies or pensions, annuities, and rents." And again (Ibid., Book 2, p. 40), "An annuity is a thing very distinct from a rent-charge with which it is frequently confounded, a rent-charge being a burthen imposed upon and issuing out of *lands*, whereas an annuity is a yearly sum chargeable only on the person of the grantor. . . . . And yet a man may have a real estate in it, though his security is merely personal."

It may be remarked in passing that the distinction drawn by this commentator upon the Common Law, between a rent-charge and an annuity, is apparently very much the same as that recognized by the Civil Law between a *rente foncière* and a *rente constituée*. (Pothier, *Traité de Constitution de Rente.*)

A number of cases from the English decisions were cited in support of the appellant's contention, but perhaps the strongest was that rendered *in re* Locke, bankrupt, decided in the Court of King's Bench in 1823 (2 Dowling and Ryland, 603), in which the Court used this language: "A grant of annuity *must be by deed*, and no action at law can be maintained upon any other instrument for the arrears of an annuity. . . . In order to constitute an annuity deed there must be positive words of grant immediately and presently; the word grant must be used. . . ."

Equally emphatic is the language in 2 Broom's Commentaries, 54: "It (an annuity) is chargeable only upon the person of the grantor, unlike a rent-charge, which always issues out of specific land. It is, of course, incorporeal in its nature, and may in a limited sense be described as an hereditament, because the law permits it to be limited to a man and his heirs, or to a man and the heirs of his body, so that it in that case descends, upon death and intestacy, to the person who would succeed to land limited in the same manner, and it is forfeitable for treason, and moreover, like incorporeal hereditaments strictly so called, *it can only be granted or transferred by deed.*"

Later commentators obviously recognized the incongruity between the older decisions rendered in respect to annuities made for the life of the *grantor* to the grantee and his heirs, and the rules which should govern modern annuities granted by corporations for the life of the annuitants. But in the absence of judicial determination, there was an apparent hesitancy to assert the distinction. Evidences of its recognition can, however, be seen in Kent's Commentaries (Vol. III., p. 460), where the Chancellor, after quoting

the language of Lord Coke, above cited, says: "*If* the annuity be "agreed to be paid to the annuitant *and his heirs*, it is a personal "fee, and transmissible by descent like an estate in fee, and for- "feitable for treason as an hereditament, and *for that reason* be- "longs to the class of incorporeal hereditaments."

And in Chitty on Pleadings, 124, we find this qualified language: "*It should seem* that no action can be supported at law upon "an annuity unless it be granted by deed, and there must be an "express grant in such deed."

In the case under consideration, a Court of final resort was called upon to distinguish between the ancient form of annuity made for the life of the grantor to the grantee and his heirs, which was the subject of the older decisions and of the definitions of the older commentators, and the modern form of annuity granted by a corporation to an annuitant for his life. The question was squarely met and the Court decided that the latter form of contract is a mere *chose in action* for the payment of money.

In support of this view ancient authority for the doctrine that an annuity is a mere *chose in action* was cited in argument on behalf of the appellee—the defendant Company; for it was so held in the case of *Gerard vs. Boden*, decided in the Court of Common Bench in the third year of Charles I. (Hetley's Reports, p. 80.)

The particular suit against the Maryland Life Insurance Company might have been decided, as has been already remarked, on much narrower grounds than those taken by the Court; for it will be observed that it was a suit brought by the administrator of a deceased annuitant, to whom the annuity had been fully paid, to recover the consideration money on the ground of defect in the form of the contract. The principle of law relied upon is well established, and its applicability is illustrated by the decision in the nearly parallel case of *Davis vs. Bryan*, decided in the Court of King's Bench in 1827. (6 Barnewall & Cresswell, 651.) The statute 53, George III., Cap. 141 (repealed by Act 17 and 18, Vict., Cap. 90) was then in force in England, by the terms of which it was required that every grant of annuity made by a natural person should be enrolled in Chancery, and unless such enrollment were made within the time specified in the Act, the annuity should be "null and void to all intents and purposes." The annuity which was the subject of litigation had not been enrolled. But the Court held that where one had purchased an annuity *for his life*, which was regularly paid up to the *time of his death*, but no memorial of

the annuity was enrolled, the annuitant's executors could not, on that ground, insist that the contract was void, and recover back the consideration money paid for the annuity, for the contract was executed; the testator had enjoyed the *full consideration* for his advance, and the claim of the executors was against equity and *good conscience*. (Chitty on Contracts, 625.)

In the decision in this case, rendered by Judge Bayley, the Court characterized the claim in somewhat scathing terms. The Judge's language (quoting a portion only) was as follows: "This appears to be a clear case on principles both of law and honesty. This is an action for money had and received, and I learned many years ago that such an action could not be maintained if it were against equity and good conscience that the money should be recovered.

"Here a bargain was made, and the testator paid a consideration of 300*l.* and the defendant agreed for that to pay a certain annuity. The testator received the whole of that he bargained for, and now his representative says that the contract was void from the beginning. Is there anything like good conscience in the claim? Then is the contract void? The Act of Parliament says that unless a memorial be duly enrolled, the deed of which no memorial is enrolled shall be void; but in many cases such words have been held to make the instrument voidable only at the will of the party, and I think we are at liberty to put that construction upon them in the present case . . . . . On these grounds I think the action cannot be maintained."

This decision was made simply on the ground that after a contract had been fully executed, the party who had received the benefit of its performance,—who had "got that which he bargained for,"—could not be heard to deny the consideration on account of defect in the form of the contract. And the case of Cahill, Admr., against the Maryland Life Insurance Company might have been decided on similar grounds, for in this case, as in the other, the contract had been fully executed. Upon that point there was no dispute.

But the Court of Appeals of Maryland, in its wisdom, without ignoring the other grounds of defense, decided the case upon the broad principle that there was no such defect, as was claimed, in the form of contract. I quote briefly from the language of the Court's decision, rendered by Judge Fowler.

"We might almost take judicial notice of the fact that it is now

usual, and has been for many years, the general custom of insurance companies, to issue just such documents as the one issued to Mrs. Cahill, not under seal, but in the form of a written contract signed by officers designated by the By-laws. But 'it is settled law in this State that a policy of life insurance is a mere *chose in action* for the payment of money' (*Rittler vs. Smith*, 70 Md. 265), and so is a contract for a life annuity unless it be a life annuity issuing out of or charged upon lands. . . . . What is an annuity like the one in this case but a contract on the part of the Insurance Company, in consideration of a sum certain, to pay the annuitant specified sums annually during life? There is certainly no feature of inheritance about such a contract as this, and we can conceive of no reason why such a *chose in action* may not, as any other *chose in action*, be the subject of a parol or written contract, unless there be some statute requiring such a contract to be by deed. But so far from this being the case . . . . . we are of opinion that the charter of the defendant Company authorizes just such a contract as we are considering.

"Our conclusion is that, the contract of annuity being valid, there is no right of action in the plaintiff or any other person to recover the purchase money paid by Mr. Cahill for the annuity."

In this opinion the law as to what constitutes a valid contract of annuity granted by a life insurance company for the life of the annuitant was for the first time judicially determined in this country; and it bears, therefore, not only upon the particular annuity which was the subject of litigation in this case, but upon all other subsisting annuities similarly executed under similar charter powers.

Duties of an Actuary from a Practical Standpoint.

JOEL G. VAN CISE.

IT goes without saying that an Actuary should be a good mathematician, but unless he is something more than a mathematician he cannot hope to occupy a position of much importance, or exercise any perceptible influence in the administration of the affairs of the company by which he may be employed. To properly honor his profession he should be a man of good judgment and inflexible honesty, and by familiarizing himself as far as possible with all the details of the business of life assurance, should make his influence felt in favor of all measures which tend to make his company prosperous and secure.

The burdens of the American Actuary have been largely increased in the last few years, owing to the great demand for new plans of assurance and new kinds of policies by managers and agents, and the complicated privileges which have consequently been given in many of the new contracts now being issued. The changes in the rate of interest used in the computation of premiums and reserves, and the great extension of the investment feature in policies of assurance which has been granted in recent years, have also tended to add both to the labor and the responsibility of Actuaries.

These changes have largely resulted from the competition for new business, which in the past has never been keener than it is at present. This very competition makes an Actuary's position more important than ever before, for he must bear the brunt of the responsibility, if insufficient premiums are charged on any form of policy issued by his company. Now is the time, therefore, to strongly oppose any dangerous innovations, no matter what rival institutions may do, and no Actuary should consent to the issue of contracts on which less than net premiums are charged, merely for the sake of enabling his company to do a larger new business than some other company. In the getting up of any new plan, the question of safety should take precedence of the question of popularity, and the accumulated surplus of the past should not be relied upon to meet the losses of the future, or the profits of a high premium policy be used to make up the shortage on policies issued at less than the cost of the contract privileges given.

In other words, Actuaries should never yield to the pressure which may be brought against them by Presidents or Vice-Presidents, or Superintendents of Agencies, to get up some form of policy that will "beat the other fellows," when those "other fellows" are already issuing as liberal contracts as any company can afford to issue. It is very desirable to have a large volume of new business, for the blessings of life assurance are thus more generally diffused, but there are some things better even than a large new business, or a large amount of assurance in force, among which are economy of management, liberal dividends for policy-holders, and a large surplus for their future benefit and security.

The question of how much a company ought or can afford to pay for new business is one difficult of solution, and probably there will always be honest differences of opinion on this subject; but Actuaries should take a vital interest in it, and see to it that their influence is always in favor of a wise economy, and against the demands which may easily become inordinate on the part of solicitors, or officers whose energies are entirely devoted to the obtaining of a large number of new applications. We all recognize that there must be a limit, beyond which the percentages of commission should not go, and we should, therefore, earnestly endeavor to ascertain this limit, and contend for its maintenance.

Another practical duty of an Actuary, in my judgment, is to oppose all improper forms of competition among the various companies. There is room enough for all, and business enough for all, and considering the beneficent office of life assurance, competition should be conducted on a high plane, and not on a low one. Bitter and personal controversies should never be engaged in, unless in self-defense, and our influence as Actuaries should always be exercised in favor of the reëstablishment of harmonious relations. While feeling a natural pride in our own companies, and while thoroughly loyal to them, we should ever be willing to recognize the merits of our rivals, and should steadily oppose the publication of any unfair comparisons, or any unjust criticisms. We should not only hope, but work for the coming of the happy day when the Presidents and Vice-Presidents of all American companies shall live in as harmonious relations, and have as friendly feelings for one another, as the members of the Actuarial Society of America.

Notwithstanding the tendencies of the times in other branches of business, there is no probability, for many obvious reasons, that any attempt will ever be made to form a combination, or

trust, of American life assurance companies. Nevertheless, there is no good reason why the regular companies, which we represent, should not live together in unity and work together for the common good in the endeavor to eradicate any evils which may arise in connection with the business. There will always be a difference in the magnitude of assets and amount of assurance in force, but we should always remember that we have a common object in view, and like the human body, of which, when even one of the smallest members suffers, all the other members suffer with it, so when one company is unjustly assailed, all other companies should feel the attack and present a united front to the common enemy. The enormous accumulation of trust funds in the shape of reserve and surplus, which will naturally increase for many years to come, will offer great temptations to the greedy and unprincipled politicians who are too often sent to our State Legislatures, and the instinct of self-protection and self-preservation should lead us to act together in resisting the imposition of the increased burdens of taxation which are constantly threatened.

There is another great and growing evil which menaces the prosperity of all companies to some extent, and which in my judgment all Actuaries should unite in an effort to remedy, and that is the practice of "twisting" business from one company to another. This method of obtaining so-called new business not only largely increases the percentages of expenses and death losses, but also brings discredit to some extent upon the whole business of life assurance. The only defense that can be made by any officer for encouraging or sanctioning this endeavor on the part of soliciting agents to make men give up policies in one company merely for the purpose of taking out policies in another company, is that his company will probably pay larger dividends in the future. Saying nothing about the loss of reserve and interest in surplus which is generally involved in making such changes, and the double initial expense for commissions, medical examinations, etc., which must be incurred, does this hope of larger future dividends justify the change? The experience of the past may not be duplicated in the future, for the circumstances which have made large profits possible in some companies are constantly changing, and the elements contributing to success in this direction are not necessarily permanent. Furthermore, the question of management must always be considered, for those who have been preëminent must die, and in the future great administrative talent may be de-

veloped where it has been wanting in the past. For these and other reasons which might be given, it seems to me that we, as Actuaries, should use our influence in favor of having our soliciting agents instructed not only to stop "twisting," but also to advise every holder of a policy in another regular company to keep his contract in force, unless they shall have been instructed from their home office, that this other company is not able to carry out its contracts. The adoption of a rule of this kind would not only remedy a great evil, but would also tend to the establishment on a permanent basis of harmonious relations among the different companies.

It is a matter of regret that the changing of policies from one form to another is sometimes encouraged by companies' officers, and possibly by actuaries, provided that the new policies issued in lieu of the old ones are taken out in the same company. This is particularly true when the old contracts terminated are paid-up policies, and the proceeds are applied to pay premiums on new contracts. I speak from experience, when I say that changes of this kind are not justifiable as a rule, even if the full reserve should be allowed in purchasing the old policy. We all know the deserved popularity of limited payment contracts, by means of which a time is reached when the burden of premium payments is lifted, and if a paid-up policy for a large amount is a good thing to have, so is a paid-up policy for a small amount a good thing to keep, and its holder should not be tempted to part with it simply that an agent may add a large first year's commission to the expenses of his company. The only proper course to adopt, in my judgment, is to advise the holder of a policy in any solvent institution to keep it in force if possible until its maturity. Let him take new assurance if he is able to carry it, but let him cling to his old policy as he would to an old friend, even if it be a small paid-up policy, or a contract taken in another company which does not contain all the modern guarantees and privileges, which may not all be beneficial, even if they are new.

This talk of the evils connected with the changing of policies leads me to say that the men who spend their time laboring to accomplish such changes, do not generally belong to the highest class of soliciting agents, and this also leads to the suggestion that actuaries should use their influence and lend their assistance in elevating the standard adopted in the selection of solicitors. There is no more honorable calling than that of a life assurance agent,

and while it is not necessary that they should all be church members or Sunday-school teachers, they ought to be men of integrity, whose word should be as good as their bond. They must be men of ability in order to succeed, but quality as well as quantity of business should be considered in judging an agent's success. The value of the risks taken must depend to a great extent upon the character of the man who takes them, for an agent naturally seeks applications among men who are his congenial associates. Let it then be part of an actuary's duty to help in the good work now going on, until no solicitor shall be allowed to canvass who is ever known to resort to misrepresentation in order to obtain applications for policies.

And, coming nearer home, let us be careful never to approve the declaration of dividends, which in our judgment have not been fairly earned on the kind or class of policies on which such dividends may be declared. Honest differences of opinion as to methods will always exist, and agreement cannot be reached as to how and when profits should be divided, but so long as the amount of surplus returned to policy-holders is such an important factor in the competition for new business, the temptation to temporarily pay too much will always exist to some extent, and we must be careful, therefore, to resist it. Let us remember that permanent benefit is always better than temporary advantage, and we must all hope that long as our lives may be, the existence of our companies may be much longer.

Let us be careful, also, not to sanction the publication of extravagant or misleading estimates, or statements of past results without a correct description of such results, and a frank explanation to new assurants that, owing to the various changed conditions of the business, the dividends of the future are likely to be less than those of the past. Life assurance is valuable enough to stand on its own merits, and in the future these merits, and the liberal privileges and guarantees contained in the new contracts issued now by nearly all companies, must be substituted for the expectation of very large dividends which companies cannot reasonably hope to earn.

In closing this lengthy paper, I wish to say that I have had no thought of assuming the rôle of a preacher, or of claiming that my standard of duty is any higher than that of the youngest or most unpretentious member of our Society. My practical experience of thirty-three years' continuous service in one company is, however,

longer than that of most of my associates, and I feel so deeply all that I have said, that I trust that any seeming egotism may be excused. My earnest desire is that every member of this Society may be a power for good in the profession which he has chosen, and that to the laudable desire for success as mathematicians may be added the determination to faithfully perform our duties to the policy-holders and their families, whose interests and whose future prosperity have been in some measure committed to our care.

Note on Reversionary Annuities.

J. H. NITCHIE.

MY attention has recently been called to a problem involving a peculiar form of deferred, increasing, reversionary annuity.

Instead of the usual form, beginning, after the death of (x), with a payment of k and increasing h per annum during the life of (y), the case in point is an annuity of k, payable after the death of (x) and during the life of (y), *provided* (x) dies at the age of $x+n$.

If (x) dies at age $x+n+1$, (y) receives an annuity of $k+h$; if (x) dies at age $x+n+2$, (y) receives an annuity of $k+2h$, etc.; if (x) dies at or after age $x+n+t$, (y) receives an annuity of $k+th$, the present value of these annuities may be expressed thus:

$$(A)\ k\left(\frac{l_{x+n}}{l_x}\cdot\frac{\mathsf{N}_{y+n+1}}{\mathsf{D}_y}-\frac{{}^{n+1}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)+h\left(\frac{l_{x+n+1}}{l_x}\cdot\frac{\mathsf{N}_{y+n+2}}{\mathsf{D}_y}-\frac{{}^{n+2}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)$$
$$+h\left(\frac{l_{x+n+2}}{l_x}\cdot\frac{\mathsf{N}_{y+n+3}}{\mathsf{D}_y}-\frac{{}^{n+3}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)+*\ *\ *\ *\ *+h\left(\frac{l_{x+n+t}}{l_x}\cdot\frac{\mathsf{N}_{y+n+t+1}}{\mathsf{D}_y}-\frac{{}^{n+t+1}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right);$$

or, making $l_x\mathsf{N}_{y+1}=\mathsf{N}'_{x:y+1}$ (which may be tabulated for all needed terms):

$$(B)\ k\left(\frac{{}^{n}\mathsf{N}'_{x:y+1}}{l_x\mathsf{D}_y}-\frac{{}^{n+1}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)+h\left(\frac{{}^{n+1}\mathsf{N}'_{x:y+1}}{l_x\mathsf{D}_y}-\frac{{}^{n+2}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)+\ *\ *\ *$$
$$+h\left(\frac{{}^{n+t}\mathsf{N}'_{x:y+1}}{l_x\mathsf{D}_y}-\frac{{}^{n+t+1}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right).$$

The positive and negative numerators of the h terms may be summed over the respective common denominators within required limits; if the number of cases with like age differences is sufficient to justify forming the S' and S columns, (B) may be reduced to

$$k\left(\frac{{}^{n}\mathsf{N}'_{x:y+1}}{l_x\mathsf{D}_y}-\frac{{}^{n+1}\mathsf{N}_{xy}}{\mathsf{D}_{xy}}\right)+h\left(\frac{{}^{n+1}\mathsf{S}'_{x:y+1}-{}^{n+t+1}\mathsf{S}'_{x:y+1}}{l_x\mathsf{D}_y}-\frac{{}^{n+2}\mathsf{S}_{xy}-{}^{n+t+2}\mathsf{S}_{xy}}{\mathsf{D}_{xy}}\right).$$

In some cases h was variable. But results sufficiently accurate were obtained by assuming h to be the average annual difference.

In a few cases the benefit was not to be deferred, making $n=0$.

If the above seems deserving of attention, credit is largely due to Mr. Peiler, at whose kind suggestion it has been reshaped, upon my attention being called by him to an error in the above expressions as first prepared.

Present Values of One Dollar for any Number of Years from One to One Hundred, at 3, 3½ and 4 Per Cent. Annual Compound Interest.

EMMA WARREN CUSHMAN.

THE tables which follow were called out by the need that has from time to time arisen in the Massachusetts Insurance Department for commutation columns true to a greater number of decimals than those commonly in use. The eight places given in the usual text books will afford generally only three reliable decimals when the mortality table is applied in constructing the D column, and usually only two in the summation for N, which are not sufficient for annuities to the degree of correctness often desirable. While the work was in hand, it was thought best to carry the results to the extreme that any conditions could possibly call for. These tables were made by actual division, employing the full number of places needed to secure accuracy to the twenty-fifth decimal in the one-hundredth value. They were tested by logarithms, proving correct throughout. A test was also made by the usual formula after the commutation columns were constructed.

Perhaps a little more solicitude that these figures should be exact has been felt on account of dismal experience in past years with the results of careless computations. For the tables of premiums and reserves required in the valuation of the great variety of policies issued by the companies, it has often been necessary to construct values from apocryphal premises or to accept tables from uncertain sources, and so errors and inconsistencies often have been disclosed. In case of commutation columns, joint or single, giving four to six decimals, in very rare cases are more than two correct. Considering the present and prospective magnitude of the business and the very considerable effect of an apparently trivial error when magnified by the great volumes of insurance, as under group methods, it would seem that at least a workmanlike degree of accuracy should be attempted. Since it is just as easy to make tables correct as incorrect, the publication of false values seems no less than a crime.

Present Value of One Dollar due x Years Hence. Three Per Cent.

Years.		Years.	
1	.97087 37864 07766 99029 12621	51	.22146 31842 61896 87726 03722
2	.94259 59091 33754 35950 60797	52	.21501 28002 54268 81287 41478
3	.91514 16593 53159 57233 59998	53	.20875 02915 08998 84745 06289
4	.88848 70479 15688 90518 05823	54	.20267 01859 31066 84218 50766
5	.86260 87843 84163 98561 22158	55	.19676 71708 06861 01183 01715
6	.83748 42566 83654 35496 33163	56	.19103 60881 62000 98235 93898
7	.81309 15113 43353 74268 28314	57	.18547 19302 54369 88578 58153
8	.78940 92343 13935 67250 76033	58	.18006 98351 98417 36484 05974
9	.76641 67323 43626 86651 22362	59	.17482 50827 16910 06295 20363
10	.74409 39148 96725 11311 86759	60	.16973 30900 16417 53684 66372
11	.72242 12765 98762 24574 62873	61	.16478 94077 82929 64742 39196
12	.70137 98801 92973 05412 26090	62	.15998 97162 94106 45380 96307
13	.68095 13399 93177 72244 91350	63	.15532 98216 44763 54738 79910
14	.66111 78058 18619 14800 88689	64	.15080 56520 82294 70620 19330
15	.64186 19473 96717 61942 60863	65	.14641 32544 48829 81184 65369
16	.62316 69392 20114 19361 75595	66	.14214 87907 27019 23480 24630
17	.60501 64458 44771 06176 46209	67	.13800 85346 86426 44155 57893
18	.58739 46076 16282 58423 74960	68	.13398 88686 27598 48694 73683
19	.57028 60268 11924 83906 55301	69	.13008 62802 20969 40480 32702
20	.55367 57541 86334 79520 92525	70	.12629 73594 37834 37359 54079
21	.53754 92759 09062 90797 01481	71	.12261 87955 70712 98407 32116
22	.52189 25008 82585 34754 38331	72	.11904 73743 40498 04278 95258
23	.50669 17484 29694 51217 84787	73	.11557 99750 87862 17746 55590
24	.49193 37363 39509 23512 47366	74	.11221 35680 46468 13346 17078
25	.47760 55692 61659 45157 74142	75	.10894 52116 95600 12957 44736

Present Value of One Dollar due x Years Hence. Three Per Cent.

Years.		Years.	
26	.46369 47274 38504 32191 98196	76	.10577 20501 89903 03842 18190
27	.45018 90557 65538 17662 11841	77	.10269 13108 63983 53244 83680
28	.43707 67531 70425 41419 53244	78	.09970 03018 09692 74994 98718
29	.42434 63623 01383 89727 70140	79	.09679 64095 23973 54364 06523
30	.41198 67595 15906 69638 54504	80	.09397 70966 25217 03266 08274
31	.39998 71451 61074 46251 01461	81	.09123 98996 36133 04141 82791
32	.38833 70341 36965 49758 26661	82	.08858 24268 31197 12759 05622
33	.37702 62467 34917 95881 81224	83	.08600 23561 46793 32775 78274
34	.36604 48997 42638 79496 90509	84	.08349 74331 52226 53180 37159
35	.35538 33978 08387 17958 16028	85	.08106 54690 79831 58427 54523
36	.34503 24250 56686 58211 80610	86	.07870 43389 12457 84881 11188
37	.33498 29369 48239 40011 46223	87	.07641 19795 26658 10564 18629
38	.32522 61523 76931 45642 19634	88	.07418 63878 89959 32586 58863
39	.31575 35459 97020 83147 76344	89	.07202 56193 10640 12219 98896
40	.30655 68407 73806 63250 25576	90	.06992 77857 38485 55553 38734
41	.29762 80007 51268 57524 52016	91	.06789 10541 15034 51993 57994
42	.28895 92240 30357 84004 38850	92	.06591 36447 71878 17469 49509
43	.28054 29359 51803 72819 79466	93	.06399 38298 75609 87834 46125
44	.27237 17824 77479 34776 49967	94	.06212 99319 18067 84305 30219
45	.26443 86237 64543 05608 25211	95	.06032 03222 50551 30393 49727
46	.25673 65279 26740 83114 80787	96	.05856 34196 60729 42129 60900
47	.24925 87649 77418 28266 80376	97	.05685 76889 90999 43815 15437
48	.24199 88009 48949 78899 80948	98	.05520 16397 97086 83315 68385
49	.23495 02921 83446 39708 55289	99	.05359 38250 45715 37199 69306
50	.22810 70797 89753 78357 81834	100	.05203 28398 50209 09902 61462

PRESENT VALUE OF ONE DOLLAR DUE x YEARS HENCE. THREE AND ONE-HALF PER CENT.

Years.		Years.	
1	.96618 35748 79227 05314 00966	51	.17299 84294 95898 82811 44541
2	.93351 07003 66402 94989 38132	52	.16714 82410 58839 44745 35789
3	.90194 27056 68022 17381 04475	53	.16149 58850 81004 29705 65979
4	.87144 22276 98572 14860 91280	54	.15603 46715 75849 56237 35246
5	.84197 31668 58523 81508 12831	55	.15075 81367 88260 44673 77049
6	.81350 06443 07752 47833 94040	56	.14566 00355 44212 99201 71062
7	.78599 09606 83818 81965 15980	57	.14073 43338 59142 98745 61413
8	.75941 15562 16250 06729 62300	58	.13597 52017 96273 41783 20205
9	.73373 09721 89613 59159 05604	59	.13137 70065 66447 74669 76044
10	.70891 88137 09771 58607 78361	60	.12693 43058 61302 17072 23231
11	.68494 57137 29247 90925 39479	61	.12264 18414 11886 15528 72687
12	.66178 32982 89128 41473 81139	62	.11849 45327 65107 39641 28200
13	.63940 41529 36355 95626 87091	63	.11448 74712 70635 16561 62512
14	.61778 17902 76672 42151 56610	64	.11061 59142 71145 08755 19335
15	.59689 06186 24804 27199 58077	65	.10687 52794 89029 07009 84865
16	.57670 59117 14786 73622 78335	66	.10326 11396 02926 63777 63154
17	.55720 37794 34576 55674 18682	67	.09976 92170 07658 58722 34932
18	.53836 11395 50315 51376 02591	68	.09639 53787 51360 95383 91238
19	.52015 56903 86778 27416 45015	69	.09313 56316 43827 00854 02163
20	.50256 58844 31669 83011 06295	70	.08998 61175 30267 64110 16582
21	.48557 09028 32531 23682 18643	71	.08694 31087 24896 27159 58051
22	.46915 06307 56068 82784 72118	72	.08400 30035 98933 59574 47392
23	.45328 56335 80742 82883 78858	73	.08116 23223 17810 23743 45306
24	.43795 71338 94437 51578 53969	74	.07841 77027 22521 96853 57784
25	.42314 69892 69987 93795 69053	75	.07576 58963 50262 77153 21530

PRESENT VALUE OF ONE DOLLAR DUE x YEARS HENCE. THREE AND ONE-HALF PER CENT.

Years.		Years.	
26	.40883 76707 92258 87725 30486	76	.07320 37645 89625 86621 46406
27	.39501 22423 11361 23406 09165	77	.07072 82749 65822 09296 10054
28	.38165 43403 97450 46769 17067	78	.06833 64975 51518 93039 71067
29	.36874 81549 73382 09438 81225	79	.06602 56014 99052 10666 38712
30	.35627 84106 02301 54047 16159	80	.06379 28516 89905 41706 65422
31	.34423 03484 08020 81204 98704	81	.06163 56054 97493 15658 60311
32	.33258 97086 06783 39328 48990	82	.05955 13096 59413 67786 08996
33	.32134 27136 29742 40897 09169	83	.05753 74972 55472 15252 26083
34	.31047 60518 16176 24055 16105	84	.05559 17847 87895 79953 87520
35	.29997 68616 58141 29521 89474	85	.05371 18693 60285 79665 57991
36	.28983 27165 77914 29489 75337	86	.05189 55259 51966 95329 06271
37	.28003 16102 20207 04821 01775	87	.05014 06047 84509 13361 41325
38	.27056 19422 41745 94029 96884	88	.04844 50287 77303 51073 82922
39	.26141 25045 81397 04376 78149	89	.04680 67910 89182 13597 90263
40	.25257 24681 94586 51571 76955	90	.04522 39527 43171 14587 34554
41	.24403 13702 36315 47412 33773	91	.04369 46403 31566 32451 54158
42	.23577 91016 77599 49190 66447	92	.04221 70437 98614 80629 50878
43	.22780 58953 40675 83759 09611	93	.04078 94142 98178 55680 68481
44	.22010 23143 38817 23438 74020	94	.03941 00621 23844 01623 85006
45	.21265 92409 07069 79167 86493	95	.03807 73547 09028 03501 30440
46	.20546 78656 10695 45089 72457	96	.03678 97146 94713 07730 72889
47	.19851 96769 18546 32936 93195	97	.03554 56180 62524 71237 41922
48	.19180 64511 29030 26992 20478	98	.03434 35923 30941 75108 61760
49	.18532 02426 36744 22214 69061	99	.03318 22148 12504 10732 96387
50	.17905 33745 28255 28709 84600	100	.03206 01109 29955 65925 56895

Present Value of One Dollar due x Years Hence. Four Per Cent.

Years.		Years.	
1	.96153 84615 38461 53846 15385	51	.13530 05916 66576 64831 49838
2	.92455 62130 17751 47928 99408	52	.13009 67227 56323 70030 28691
3	.88899 63586 70914 88393 26354	53	.12509 30026 50311 25029 12203
4	.85480 41910 29725 84993 52264	54	.12028 17333 17606 97143 38656
5	.82192 71067 59351 77878 38715	55	.11565 55128 05391 31868 64093
6	.79031 45257 30145 94113 83380	56	.11120 72238 51337 80642 92397
7	.75991 78132 02063 40494 07096	57	.10693 00229 33978 66002 81151
8	.73069 02050 01984 04321 22208	58	.10281 73297 44210 25002 70337
9	.70258 67355 78830 81078 09815	59	.09886 28170 61740 62502 59940
10	.67556 41688 25798 85652 01745	60	.09506 04010 20904 44714 03788
11	.64958 09315 63268 13126 93986	61	.09140 42317 50869 66071 19027
12	.62459 70495 80065 51083 59602	62	.08788 86843 75836 21222 29834
13	.60057 40861 34678 37580 38079	63	.08450 83503 61380 97329 13302
14	.57747 50828 21806 13058 05845	64	.08125 80291 93635 55124 16636
15	.55526 45027 13275 12555 82543	65	.07813 27203 78495 72234 77535
16	.53390 81756 85841 46688 29368	66	.07512 76157 48553 57918 05322
17	.51337 32458 51770 64123 35931	67	.07223 80920 65916 90305 82040
18	.49362 81210 11317 92426 30703	68	.06945 97039 09535 48370 98116
19	.47464 24240 49344 15794 52599	69	.06678 81768 36091 81125 94342
20	.45638 69462 01292 45956 27499	70	.06421 94008 03934 43390 33021
21	.43883 36021 16627 36496 41826	71	.06174 94238 49936 95567 62520
22	.42195 53866 50603 23554 24833	72	.05937 44460 09554 76507 33193
23	.40572 63333 17887 72648 31570	73	.05709 08134 70725 73564 74224
24	.39012 14743 44122 81392 61125	74	.05489 50129 52620 89966 09830
25	.37511 68022 53964 24415 97235	75	.05278 36663 00597 01890 47914

Present Value of One Dollar due x Years Hence. Four Per Cent.

Years.		Years.	
26	.36068 92329 36504 08092 28111	76	.05075 35252 89035 59510 07610
27	.34681 65701 31253 92396 42414	77	.04880 14666 24072 68759 68855
28	.33347 74712 80051 84996 56168	78	.04692 44871 38531 43038 16207
29	.32065 14146 92357 54804 38623	79	.04511 96991 71664 83690 54045
30	.30831 86679 73420 71927 29445	80	.04338 43261 26600 80471 67351
31	.29646 02576 66750 69160 86005	81	.04171 56981 98654 61991 99376
32	.28505 79400 64183 35731 59620	82	.04011 12482 67937 13453 84016
33	.27409 41731 38637 84357 30404	83	.03856 85079 49939 55244 07707
34	.26355 20895 56382 54189 71542	84	.03708 51037 98018 80042 38180
35	.25341 54707 27290 90567 03406	85	.03565 87536 51941 15425 36712
36	.24366 87218 53164 33237 53275	86	.03428 72631 26866 49447 46838
37	.23429 68479 35734 93497 62764	87	.03296 85222 37371 62930 25806
38	.22528 54307 07437 43747 71889	88	.03170 05021 51318 87432 94044
39	.21662 06064 49459 07449 72970	89	.03048 12520 68575 84070 13504
40	.20828 90446 62941 41778 58625	90	.02930 88962 19784 46221 28369
41	.20027 79275 60520 59402 48678	91	.02818 16309 80561 98289 69586
42	.19257 49303 46654 41733 16036	92	.02709 77220 96694 21432 39986
43	.18516 82022 56398 47820 34650	93	.02605 55020 16052 12915 76910
44	.17804 63483 23460 07519 56395	94	.02505 33673 23127 04726 70106
45	.17119 84118 49480 84153 42687	95	.02408 97762 72237 54544 90486
46	.16461 38575 47577 73224 44892	96	.02316 32464 15613 02447 02391
47	.15828 25553 34209 35792 73934	97	.02227 23523 22704 83122 13837
48	.15219 47647 44432 07493 01860	98	.02141 57233 87216 18386 67151
49	.14634 11199 46569 30281 74865	99	.02059 20417 18477 09987 18414
50	.14071 26153 33239 71424 75832	100	.01980 00401 13920 28833 83091

Massachusetts Annuities.

EDWARD J. SARTELLE.

A MORTALITY table, recently published, based upon the population of the State of Massachusetts, and giving the deaths among males and females separately, suggests a possible basis for annuities of some interest to insurance companies. The tables are here given entire as they appear in a pamphlet entitled, "Vital Statistics of Massachusetts for 1897, with a Life Table based upon the Experience of the Five-Year Period, 1893–97, by Samuel W. Abbott, M. D., Secretary of the Board."

"From the Thirtieth Annual Report of the State Board of Health of Massachusetts for 1898."

The Commonwealth of Massachusetts takes a census of its population five years after the National Census, and that taken in 1895, together with the deaths registered in the years 1893, 1894, 1895, 1896 and 1897, some 240,215 in number, forms the material for the basis of the table for all ages above five years.

The figures for the first five years of life were compiled from the births, and from the deaths which occurred among children under five years of age during the same period.

The basis number of the table is taken as 100,000, that being the round number nearest the exact number of births annually in the State. Of this basis number 51,350 are males and 48,650 females.

The limitations which affect the accuracy of such a table are discussed at some length by Dr. Abbott, who says great pains were taken to make the table as accurate as possible from the data at hand. For further explanation of the table, or its method of construction, reference is made to the pamphlet referred to, which is one of the "Public Documents" of Massachusetts, and can be had of Dr. Abbott on application.

In forming the D_x columns the interest tables used were those of Mr. Levi W. Meech, as published in his "Systems and Tables of Life Insurance," edition of 1898.

In performing the work the arithmometer was used, which limited the number of figures used in the divisor, in computing the annuities, to EIGHT. In each case the *first eight* figures were taken, the corresponding number in the N_x column being used entire.

The D_x and N_x columns, and the Annuities, are given with 3½ per cent. and 3 per cent. interest assumptions.

MASSACHUSETTS LIFE TABLE.

Based on the Mortality of the Five Years, 1893–97.

Table No. 1.—Males.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age. P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
0	8,849	51,350	46,343	2,264,048	44.09
1	1,794	42,501	41,604	2,217,705	52.18
2	818	40,707	40,298	2,176,101	53.46
3	559	39,889	39,609	2,135,803	53.54
4	424	39,330	39,118	2,096,194	53.30
5	316	38,906	38,748	2,057,076	52.88
6	252	38,590	38,464	2,018,328	52.30
7	205	38,338	38,235	1,979,864	51.64
8	170	38,133	38,048	1,941,629	50.92
9	146	37,963	37,890	1,903,581	50.14
10	123	37,817	37,755	1,865,691	49.33
11	110	37,694	37,639	1,827,936	48.49
12	104	37,584	37,532	1,790,297	47.63
13	111	37,480	37,424	1,752,765	46.76
14	135	37,369	37,301	1,715,341	45.90
15	159	37,234	37,154	1,678,040	45.07
16	181	37,075	36,984	1,640,886	44.26
17	195	36,894	36,796	1,603,902	43.47
18	211	36,699	36,593	1,567,106	42.70
19	226	36,488	36,375	1,530,513	41.94
20	241	36,262	36,141	1,494,138	41.20
21	255	36,021	35,893	1,457,997	40.48
22	268	35,766	35,632	1,422,104	39.76
23	280	35,498	35,358	1,386,472	39.06
24	289	35,218	35,073	1,351,114	38.36
25	296	34,929	34,781	1,316,041	37.68
26	301	34,633	34,482	1,281,260	37.00
27	305	34,332	34,179	1,246,778	36.32
28	309	34,027	33,872	1,212,599	35.64
29	313	33,718	33,561	1,178,727	34.96
30	316	33,405	33,247	1,145,166	34.28
31	318	33,089	32,930	1,111,919	33.60
32	319	32,771	32,611	1,078,989	32.93
33	319	32,452	32,292	1,046,378	32.24
34	320	32,133	31,973	1,014,086	31.56
35	322	31,813	31,652	982,113	30.87
36	325	31,491	31,328	950,461	30.18
37	328	31,166	31,002	919,133	29.49

MASSACHUSETTS LIFE TABLE—*Continued.*

TABLE No. 1.—Males.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age. P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
38	331	30,838	30,672	888,131	28.80
39	334	30,507	30,340	857,459	28.11
40	337	30,173	30,004	827,119	27.41
41	341	29,836	29,665	797,115	26.72
42	346	29,495	29,322	767,450	26.02
43	352	29,149	28,973	738,128	25.32
44	359	28,797	28,617	709,155	24.63
45	368	28,438	28,254	680,538	23.93
46	379	28,070	27,880	652,284	23.24
47	393	27,691	27,494	624,404	22.55
48	410	27,298	27,093	596,910	21.87
49	429	26,888	26,673	569,817	21.19
50	448	26,459	26,235	543,144	20.53
51	466	26,011	25,778	516,909	19.87
52	483	25,545	25,303	491,131	19.23
53	502	25,062	24,811	465,828	18.59
54	520	24,560	24,300	441,017	17.96
55	539	24,040	23,770	416,717	17.33
56	561	23,501	23,220	392,947	16.72
57	585	22,940	22,647	369,727	16.12
58	608	22,355	22,051	347,080	15.53
59	636	21,747	21,429	325,029	14.95
60	659	21,111	20,781	303,600	14.38
61	677	20,452	20,113	282,819	13.83
62	691	19,775	19,429	262,706	13.28
63	709	19,084	18,729	243,277	12.75
64	729	18,375	18,010	224,548	12.22
65	748	17,646	17,272	206,538	11.70
66	769	16,898	16,513	189,266	11.20
67	789	16,129	15,734	172,753	10.71
68	810	15,340	14,935	157,019	10.24
69	827	14,530	14,116	142,084	9.78
70	840	13,703	13,283	127,968	9.34
71	845	12,863	12,440	114,685	8.92
72	847	12,018	11,594	102,245	8.51
73	842	11,171	10 750	90,651	8.11
74	831	10,329	9,913	79,901	7.74
75	816	9,498	9,090	69,988	7.37
76	794	8,682	8,285	60,898	7.01
77	769	7,888	7,503	52,613	6.67

MASSACHUSETTS LIFE TABLE—*Continued.*

TABLE NO. 1.—Males.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
78	741	7,119	6,748	45,110	6.34
79	707	6,378	6,024	38,362	6.01
80	672	5,671	5,335	32,338	5.70
81	632	4,999	4,683	27,003	5.40
82	590	4,367	4,072	22,320	5.11
83	546	3,777	3,504	18,248	4.83
84	499	3,231	2,981	14,744	4.56
85	452	2,732	2,506	11,763	4.31
86	402	2,280	2,079	9,257	4.06
87	353	1,878	1,701	7,178	3.82
88	307	1,525	1,371	5,477	3.59
89	263	1,218	1,086	4,106	3.37
90	220	955	845	3,020	3.16
91	181	735	644	2,175	2.96
92	146	554	481	1,531	2.76
93	116	408	350	1,050	2.57
94	88	292	248	700	2.40
95	66	204	171	452	2.22
96	48	138	114	281	2.04
97	33	90	73	167	1.86
98	23	57	45	94	1.65
99	15	34	26	49	1.44
100	9	19	14	23	1.21

MASSACHUSETTS LIFE TABLE, 1893–97.

MALES, 3½ %.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
1	41,063.76790857	907,713.84518748	22.104982
2	38,000.42006490	866,650.07727891	22.806328
3	35,977.59236030	828,649.65721401	23.032382
4	34,273.82290590	792,672.06485371	23.127623
5	32,757.80815202	758,398.24194781	23.151678
6	31,392.98969760	725,640.43379579	23.114729
7	30,133.32142448	694,247.44409819	23.039194
8	28,958.64063615	664,114.12267371	22.933194
9	27,854.62881411	635,155.48203756	22.802511
10	26,809.18263777	607,300.85322345	22.652718
11	25,818.34359274	580,491.67058568	22.483691
12	24,872.46354720	554,673.32699294	22.300699
13	23,964.86754200	529,800.86344574	22.107398
14	23,085.88771051	505,835.99590374	21.911048
15	22,224.62534508	482,750.10819323	21.721406
16	21,381.37161325	460,525.48284815	21.538632
17	20,557.47625932	439,144.11123490	21.361771
18	19,757.31547686	418,586.63497558	21.186412
19	18,979.44081672	398,829.31949872	21.013755
20	18,224.04394056	379,849.87868200	20.843337
21	17,490.74938890	361,625.83474144	20.675262
22	16,779.64143258	344,135.08535254	20.509085
23	16,090.73329374	327,355.44391996	20.344346
24	15,423.97420434	311,264.71062622	20.180578
25	14,780.10121371	295,840.73642188	20.016151
26	14,159.27502511	281,060.63520817	19.849931
27	13,561.56022368	266,901.36018306	19.680727
28	12,986.55222718	253,339.79995938	19.507857
29	12,433.45045888	240,353.24773220	19.331179
30	11,901.48028605	227,919.79727332	19.150542
31	11,390.23805115	216,018.31698727	18.965215
32	10,899.29738641	204,628.07893612	18.774429
33	10,428.21362492	193,728.78154971	18.577370
34	9,976.52691465	183,300.56792479	18.373184
35	9,543.16384718	173,324.04101014	18.162115
36	9,127.12218552	163,780.87716296	17.944416
37	8,727.46515726	154,653.75497744	17.720352
38	8,343.58910572	145,926.28982018	17.489630
39	7,974.91144257	137,582.70071446	17.251940

MASSACHUSETTS LIFE TABLE, 1893-97—*Continued.*

MALES, 3½ %.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
40	7,620.86913731	129,607.78927189	17.006956
41	7,280.91995532	121,986.92013458	16.754327
42	6,954.30455450	114,706.00017926	16.494244
43	6,640.31417910	107,751.69562476	16.226897
44	6,338.28650904	101,111.38144566	15.952478
45	6,047.60346712	94,773.09493662	15.671182
46	5,767.48311090	88,725.49146950	15.383745
47	5,497.20845888	82,958.00835860	15.090933
48	5,235.93247210	77,460.79989972	14.794079
49	4,982.89061312	72,224.86742762	14.494572
50	4,737.57338142	67,241.97681450	14.193337
51	4,499.86216273	62,504.40343308	13.890293
52	4,269.80179080	58,004.54127035	13.584832
53	4,047.40999518	53,734.73947955	13.276327
54	3,832.21149520	49,687.32948437	12.965706
55	3,624.22568560	45,855.11798917	12.652390
56	3,423.15660004	42,230.89230357	12.336827
57	3,228.44553020	38,807.73570353	12.020563
58	3,039.72559600	35,579.29017333	11.704770
59	2,857.05583647	32,539.56457733	11.389194
60	2,679.71021841	29,682.50874086	11.076760
61	2,508.27091168	27,002.79852245	10.765503
62	2,343.22933075	24,494.52761077	10.453320
63	2,184.87887748	22,151.29828002	10.138456
64	2,032.56734625	19,966.41940254	9.823251
65	1,885.92119088	17,933.85205629	9.509333
66	1,744.90674372	16,047.93086541	9.197013
67	1,609.17774938	14,303.02412169	8.888405
68	1,478.70512920	12,693.84637231	8.584434
69	1,353.26070390	11,215.14124311	8.287494
70	1,233.07980236	9,861.88053921	7.997763
71	1,118.34922393	8,628.80073685	7.715658
72	1,009.54805400	7,510.45151292	7.439419
73	906.66427672	6,500.90345892	7.170133
74	809.97642330	5,594.23918220	6.906669
75	719.62451820	4,784.26275890	6.648276
76	635.55504432	4,064.63824070	6.395415
77	557.90459376	3,429.08319638	6.146361
78	486.48754350	2,871.17860262	5.901854
79	421.11127680	2,384.69105912	5.662852
80	361.76925235	1,963.57978232	5.427713

MASSACHUSETTS LIFE TABLE, 1893–97—*Continued.*

MALES, 3½ %.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
81	308.11641439	1,601.81052997	5.198718
82	260.06057077	1,293.69411558	4.974587
83	217.31913750	1,033.63354481	4.756293
84	179.61707349	816.31440731	4.544748
85	146.74082884	636.69733382	4.338924
86	118.32180840	489.95650498	4.140881
87	94.16406558	371.63469658	3.946672
88	73.87867075	277.47063100	3.755761
89	57.01067022	203.59196025	3.571120
90	43.18887225	146.58129003	3.393959
91	32.11556040	103.39241778	3.219387
92	23.38824570	71.27685738	3.047550
93	16.64208336	47.88861168	2.877561
94	11.50773752	31.24652832	2.715262
95	7.76778144	19.73879080	2.541110
96	5.07698136	11.97100936	
97	3.19910580	6.89402800	
98	1.95758463	3.69492220	
99	1.12819548	1.73733757	
100	.60914209	.60914209	

MASSACHUSETTS LIFE TABLE, 1893–97.

Males, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
1	41,263.10694879	1,005,028.84820778	24.356597
2	38,370.25170837	963,765.74125899	25.117525
3	36,504.08567574	925,395.48955062	25.350463
4	34,944.19567650	888,891.40387488	25.437455
5	33,560.65719468	853,947.20819838	25.444889
6	32,318.51759340	820,386.55100370	25.384412
7	31,172.30231038	788,068.03341030	25.281034
8	30,102.54216759	756,895.73109992	25.143914
9	29,095.47832099	726,793.18893233	24.979592
10	28,139.39939447	697,697.71061134	24.794335
11	27,230.94735138	669,558.31121687	24.588139
12	26,360.66140992	642,327.36386549	24.366891
13	25,522.05622320	615,966.70245557	24.134681
14	24,705.31144189	590,444.64623237	23.899502
15	23,899.08784630	565,739.33479048	23.672006
16	23,103.91430050	541,840.24694418	23.452314
17	22,321.47690630	518,736 33264368	23.239338
18	21,556.79479239	496,414.85573738	23.028232
19	20,808 59666264	474,858.06094499	22.820283
20	20,077.39040912	454,049.46428235	22.614964
21	19,363.06261488	433,972.07387323	22.412368
22	18,666.00715500	414,609.01125835	22.211981
23	17,986.54374150	395,943.00410335	22.013290
24	17,324.92245532	377,956.46036185	21.815766
25	16,682.28495453	360,631.53790653	21.617635
26	16,059 13958409	343,949.25295200	21.417664
27	15,455.89080792	327,890.11336791	21.214573
28	14,872 41057225	312,434.22255999	21.007639
29	14,308.11056648	297,561.81198774	20.796723
30	13,762.41771780	283,253.70142126	20.581682
31	13,235.17480635	269,491.28370346	20.361748
32	12,726.19281013	256,256.10889711	20.136118
33	12,235.25586500	243,529.91608698	19.903951
34	11,762.12077170	231,294.66022198	19.664368
35	11,305.81210420	219,532.53945028	19.417671
36	10,865.41625313	208,226.72734608	19.164174
37	10,440.07830804	197,361.31109295	18.904198
38	10,029.32401370	186,921.23278491	18.637470
39	9,632.69354985	176,891.90877121	18.363701

MASSACHUSETTS LIFE TABLE, 1893-97—*Continued.*

MALES, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N}{D_x}$
40	9,249.73953332	167,259.21522136	18 082586
41	8,880.02900800	158,009.47568804	17.793801
42	8,522.85219390	149,129.44668004	17.497598
43	8,177.54615806	140,606.59448614	17.194228
44	7,843.49014866	132,429.04832808	16.883944
45	7,520.10547556	124,585.55817942	16.566996
46	7,206.59439710	117,065.45270386	16.244212
47	6,902.22432316	109,858.85830676	15.916442
48	6,606.08324240	102,956.63398360	15.585125
49	6,317.34339752	96,350.55074120	15.251751
50	6,035.48522972	90,033.20734368	14.917310
51	5,760.47877498	83,997.72211396	14 581726
52	5,492.50197600	78,237.24333898	14.244372
53	5,231.69976798	72,744.74136298	13.904609
54	4,977.57962080	67,513.04159500	13.563427
55	4,730.28276680	62,535.46197420	13.220237
56	4,489.53915109	57,805.17920740	12.875526
57	4,254.72607420	53,315.64005631	12.530922
58	4,025.46104965	49,060.91398211	12.187651
59	3,801.92101476	45,035.45293246	11.845446
60	3,583.23526299	41,233.53191770	11.507347
61	3,370.27301332	37,650.29665471	11.171289
62	3,163.79671300	34,280.02364139	10.835090
63	2,964.31428488	31,116.22692839	10.496939
64	2,771.05381875	28,151.91264351	10.159280
65	2,583.60820950	25,380.85882476	9.823803
66	2,402.03025342	22,797.25061526	9.490825
67	2,225.93958037	20,395.22036184	9.162522
68	2,055.38926580	18,169.28078147	8.839824
69	1,890.15364840	16,113.89151567	8.525175
70	1,730.65272408	14,223.73786727	8.218713
71	1,577.24549577	12,493.08514319	7.920825
72	1,430.71129266	10,915.83964742	7.629659
73	1,291.14384487	9,485.12835476	7.346298
74	1,159.05396453	8,193.98450989	7.069545
75	1,034.76160458	7,034.93054536	6.798600
76	918.31293810	6,000.16894078	6.533904
77	810.02905328	5,081.85600268	6.273671
78	709 76643570	4,271.82694940	6.018637
79	617.36750298	3,562.06051370	5.769757
80	532.94413410	2,944.69301072	5.525331

MASSACHUSETTS LIFE TABLE, 1893-97—*Continued.*

MALES, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
81	456.10826010	2,411.74887662	5.287667
82	386.83947181	1,955.64061652	5.055431
83	324.83091372	1,568.80114471	4.829593
84	269.78019633	1,243.97023099	4.611051
85	221.47086404	974.19003466	4.398727
86	179.44589520	752.71917062	4.194686
87	143.50169844	573.27327542	3.994888
88	113.13424475	429.77157698	3.798775
89	87.72720516	316.63733223	3.609340
90	66.78102990	228.91012707	3.427771
91	49.89992175	162.12909717	3.249085
92	36.51615656	112.22917542	3.073411
93	26.10948264	75.71301886	2.899828
94	18.14193956	49.60353622	2.734191
95	12.30534528	31.46159666	2.556742
96	8.08175196	19.15625138	
97	5.11719210	11.07449942	
98	3.14649348	5.95730732	
99	1.82218988	2.81081384	
100	.98862396	.98862396	

MASSACHUSETTS LIFE TABLE.

Based on the Mortality of the Five Years, 1893–97.

Table No. 2.—Females.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age. P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
0	7,151	48,650	43,907	2,267,469	46.61
1	1,662	41,499	40,668	2,223,562	53.58
2	763	39,837	39,455	2,182,894	54.79
3	544	39,074	38,802	2,143,439	54.83
4	385	38,530	38,337	2,104,637	54.62
5	318	38,145	37,986	2,066,300	54.17
6	250	37,827	37,702	2,028,314	53.62
7	206	37,577	37,474	1,990,612	52.97
8	170	37,371	37,286	1,953,138	52.26
9	147	37,201	37,127	1,915,852	51.50
10	129	37,054	36,990	1,878,725	50.70
11	113	36,925	36,868	1,841,735	49.88
12	113	36,812	36,755	1,804,867	49.03
13	123	36,699	36,637	1,768,112	48.18
14	146	36,576	36,503	1,731,475	47.34
15	172	36,430	36,344	1,694,972	46.53
16	195	36,258	36,160	1,658,628	45.74
17	206	36,063	35,960	1,622,468	44.99
18	218	35,857	35,748	1,586,508	44.24
19	230	35,639	35,524	1,550,760	43.51
20	241	35,409	35,288	1,515,236	42.79
21	251	35,168	35,042	1,479,948	42.08
22	255	34,917	34,790	1,444,906	41.38
23	261	34,662	34,531	1,410,116	40.68
24	265	34,401	34,268	1,375,585	39.99
25	269	34,136	34,001	1,341,317	39.29
26	274	33,867	33,730	1,307,316	38.60
27	278	33,593	33,454	1,273,586	37.91
28	282	33,315	33,174	1,240,132	37.22
29	286	33,033	32,890	1,206,958	36.54
30	290	32,747	32,602	1,174,068	35.85
31	294	32,457	32,310	1,141,466	35.17
32	297	32,163	32,014	1,109,156	34.48
33	301	31,866	31,715	1,077,142	33.80
34	302	31,565	31,414	1,045,427	33.12
35	306	31,263	31,110	1,014,013	32.43
36	308	30,957	30,803	982,903	31.75
37	312	30,649	30,493	952,100	31.06

MASSACHUSETTS LIFE TABLE—*Continued.*

TABLE No. 2.—Females.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age. P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
38	315	30,337	30,180	921,607	30.38
39	318	30,022	29,863	891,427	29.69
40	320	29,704	29,544	861,564	29.00
41	324	29,384	29,222	832,020	28.31
42	326	29,060	28,897	802,798	27.62
43	329	28,734	28,570	773,901	26.93
44	332	28,405	28,239	745,331	26.24
45	335	28,073	27,905	717,092	25.54
46	346	27,738	27,565	689,187	24.85
47	354	27,392	27,215	661,622	24.15
48	367	27,038	26,854	634,407	23.46
49	379	26,671	26,481	607,553	22.78
50	395	26,292	26,094	581,072	22.10
51	410	25,897	25,692	554,978	21.43
52	423	25,487	25,275	529,286	20.77
53	437	25,064	24,845	504,011	20.11
54	457	24,627	24,398	479,166	19.46
55	476	24,170	23,932	454,768	18.81
56	494	23,694	23,447	430,836	18.18
57	512	23,200	22,944	407,389	17.56
58	530	22,688	22,423	384,445	16.95
59	550	22,158	21,883	362,022	16.34
60	572	21,608	21,322	340,139	15.74
61	588	21,036	20,742	318,817	15.16
62	606	20,448	20,145	298,075	14.58
63	625	19,842	19,530	277,930	14.01
64	644	19,217	18,895	258,400	13.45
65	665	18,573	18,240	239,505	12.90
66	689	17,908	17,563	221,265	12.36
67	715	17,219	16,861	203,702	11.83
68	743	16,504	16,132	186,841	11.32
69	771	15,761	15,375	170,709	10.83
70	793	14,990	14,593	155,334	10.36
71	809	14,197	13,792	140,741	9.91
72	821	13,388	12,977	126,949	9.48
73	825	12,567	12,154	113,972	9.07
74	824	11,742	11,330	101,818	8.67
75	818	10,918	10,509	90,488	8.29
76	806	10,100	9,697	79,979	7.92
77	790	9,294	8,899	70,282	7.56

MASSACHUSETTS LIFE TABLE—*Continued.*

TABLE No. 2.—Females.

AGE. x	Dying in each Year of Age. d_x	Born and Surviving at each Age. l_x	Population or Years of Life Lived in each Year of Age. P_x	Years of Life Lived in and above each Year of Age. Q_x	Expectation of Life at each Year of Age. E_x
78	768	8,504	8,120	61,383	7.22
79	742	7,736	7,365	53,263	6.89
80	711	6,994	6,638	45,898	6.56
81	678	6,283	5,944	39,260	6.25
82	640	5,605	5,285	33,316	5.94
83	600	4,965	4,665	28,031	5.65
84	559	4,365	4,085	23,366	5.35
85	518	3,806	3,547	19,281	5.07
86	476	3,288	3,050	15,734	4.79
87	434	2,812	2,595	12,684	4.51
88	393	2,378	2,181	10,089	4.33
89	351	1,985	1,810	7,908	3.98
90	310	1,634	1,479	6,098	3.73
91	270	1,324	1,189	4,619	3.49
92	232	1,054	938	3,430	3.25
93	194	822	725	2,492	3.03
94	160	628	548	1,767	2.81
95	129	468	403	1,219	2.60
96	100	339	289	816	2.41
97	77	239	200	527	2.20
98	55	162	134	327	2.02
99	40	107	87	193	1.80
100	27	67	53	106	1.58

MASSACHUSETTS LIFE TABLE, 1893-97.

Females, 3½ %.

AGE.	l_x	D_x	N_x	$1+a_x = \frac{N_x}{D_x}$
0	48,650			
1	41,499	40,095.65197143	893,506.31265015	22.284369
2	39,837	37,188.26575590	853,410.66067872	22.948386
3	39,074	35,242.50905980	816,222.39492282	23.160167
4	38,530	33,576.66912190	780,979.88586302	23.259599
5	38,145	32,117.06656965	747,403.21674112	23.271217
6	37,827	30,772.28870928	715,286.15017147	23.244490
7	37,577	29,535.18230392	684,513.86146219	23.176219
8	37,371	28,379.96903505	654,978.67915827	23.078907
9	37,201	27,295.52581497	626,598.71012322	22.956096
10	37,054	26,268.27758574	599,303.18430825	22.814712
11	36,925	25,291.62034175	573,034.90672251	22.657105
12	36,812	24,361.56683960	547,743.28638076	22.483911
13	36,699	23,465.49290085	523,381.71954116	22.304314
14	36,576	22,595.98675104	499,916.22664031	22.124116
15	36,430	21,744.72528660	477,320.23988927	21.951081
16	36,258	20,910.20288478	455,575.51460267	21.787236
17	36,063	20,094.43991814	434,665.31171789	21.631124
18	35,857	19,304.01539698	414,570.87179975	21.475888
19	35,639	18,537.82863591	395,266.85640277	21.322177
20	35,409	17,795.35524492	376,729.02776686	21.170076
21	35,168	17,076.55741120	358,933.67252194	21.019089
22	34,917	16,381.33254771	341,857.11511074	20.868700
23	34,662	15,711.78650706	325,475.78256303	20.715390
24	34,401	15,066.16322913	309,763.99605597	20.560244
25	34,136	14,444.54565064	294,697.83282684	20.402015
26	33,867	13,846.10536989	280,253.28717620	20.240586
27	33,593	13,269.64617832	266,407.18180631	20.076434
28	33,315	12,714.81433710	253,137.53562799	19.908866
29	33,033	12,180.85796928	240,422.72129089	19.737750
30	32,747	11,667.04909227	228,241.86332161	19.562947
31	32,457	11,172.68446995	216,574.81422934	19.384313
32	32,163	10,697.08284273	205,402.12975939	19.201697
33	31,866	10,239.90679686	194,705.04691666	19.014339
34	31,565	9,800.17651825	184,465.14011980	18.822634
35	31,263	9,378.17657418	174,664.96360155	18.624618
36	30,957	8,972.35151304	165,286.78702737	18.421791
37	30,649	8,582.68881489	156,314.43551433	18.212758
38	30,337	8,208.03757378	147,731.74669944	17.998424
39	30,022	7,848.12637522	139,523.70912566	17.777964

MASSACHUSETTS LIFE TABLE, 1893–97—*Continued.*

FEMALES, 3½ %.

AGE.	l_x	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
40	29,704	7,502.41264888	131,675.58275044	17.551098
41	29,384	7,170.61777608	124,173.17010156	17.316941
42	29,060	6,851.74064600	117,002.55232548	17.076325
43	28,734	6,545.77473060	110,150.81167948	16.827773
44	28,405	6,252.00639960	103,605.03694888	16.571486
45	28,073	5,969.98284452	97,353.03054928	16.307087
46	27,738	5,699.26777806	91,383.04770476	16.034173
47	27,392	5 437.85107456	85,683.77992670	15.756919
48	27,038	5,186.06279510	80,245 92885214	15.473381
49	26,671	4,942.67612104	75,059.86605704	15.186078
50	26,292	4,707.67146696	70,117.18993600	14.894240
51	25,897	4,480.14034171	65,409.51846904	14.599881
52	25,487	4,260.10719288	60.929.37812733	14.302311
53	25,064	4,047.73298696	56,669.27093445	14.000249
54	24,627	3,842.66581809	52,621.53794749	13.694018
55	24,170	3,643.82424380	48,778.87212940	13.386724
56	23,694	3.451.26898776	45,135.04788560	13.077812
57	23,200	3,265.03645600	41,683 77889784	12.766711
58	22,688	3,085.00533760	38,418.74244184	12.453379
59	22,158	2,911.05178758	35,333.73710424	12.137790
60	21,608	2,742.79657048	32,422.68531666	11.821032
61	21,036	2,579.89374624	29,679.88874618	11.504306
62	20,448	2,422.97614944	27,099.99499994	11.184590
63	19,842	2,271.66037974	24,677.01885050	10.862988
64	19,217	2,125.70594247	22,405.35847076	10.540196
65	18,573	1.984.99457544	20,279.65252829	10.216477
66	17,908	1,849.20049512	18,294.65795285	9.893280
67	17,219	1,717.92619918	16,445.45745773	9.572855
68	16,504	1,590.90935152	14,727.53125855	9.257304
69	15,761	1,467.91066443	13,136.62190703	8.949197
70	14,990	1,348.89193880	11,668.71124260	8.650590
71	14,197	1,234.33133267	10,319.81930380	8.360655
72	13,388	1,124.63216400	9,085.48797113	8.078631
73	12,567	1,019.96687544	7,960.85580713	7.805014
74	11,742	920.78063340	6,940.88893169	7.538048
75	10,918	827.21209620	6,020.10829829	7.277587
76	10,100	739.35797600	5,192.89620209	7.023520
77	9,294	657.34854138	4,453 53822609	6.775002
78	8,504	581.13359600	3,796.18968471	6.532387
79	7,736	510.77404160	3,215.05608871	6.294478
80	6,994	446.16719290	2,704.28204711	6.061140

MASSACHUSETTS LIFE TABLE, 1893-97—*Continued.*

FEMALES, 3½ %.

AGE.	l_x	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
81	6,283	387.25653763	2,258.11485421	5.831056
82	5,605	333.78509255	1,870.85831658	5.604978
83	4,965	285.67368750	1,537.07322403	5.380520
84	4,365	242.65816335	1,251.39953653	5.157047
85	3,806	204.42737722	1,008.74137318	4.934473
86	3,288	170.63250264	804.31399596	4.713720
87	2,812	140.99539532	633.68149332	4.494341
88	2,378	115.20228134	492.68609800	4.276704
89	1,985	92.91147815	377.48381666	4.062832
90	1,634	73.89593430	284.57233851	3.850987
91	1,324	57.85170336	210.67640421	3.641662
92	1,054	44.49677070	152.82470085	3.434512
93	822	33.52890324	108.32793015	3.230882
94	628	24.74951768	74.79902691	3.022241
95	468	17.82020448	50.04950923	2.808582
96	339	12.47171508	32.22930475	
97	239	8.49540318	19.75758967	
98	162	5.56366158	11.26218649	
99	107	3.55049754	5.69852491	
100	67	2.14802737	2.14802737	

MASSACHUSETTS LIFE TABLE, 1893-97.

Females, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
0			
1	40,290.29141121	990,636.61981328	24.587477
2	37,550.19326667	950,346.32840207	25.308693
3	35,758.24522284	912,796.13513540	25.526871
4	34,233.40603650	877,037.88991256	25.619358
5	32,904.21191310	842,804.48387606	25.613878
6	31,679.51710302	809,900.27196296	25.565423
7	30,553.53967127	778,220.75485994	25.470723
8	29,501.01233433	747,667.21518867	25.343781
9	28,511.46877273	718,166.20285434	25.188677
10	27,571.65574114	689,654.73408161	25.013177
11	26,675.40539475	662,083.07834047	24.819982
12	25,819.19614256	635,407.67294572	24.609893
13	24,990.23322666	609,588.47680316	24.393068
14	24,181.04501856	584,598.24357650	24.175888
15	23,383.03083850	560,417.19855794	23.966834
16	22,594.78691052	537,034.16771944	23.768057
17	21,818.70823635	514,439.38080892	23 577902
18	21,062.20853077	492,620.67257257	23.388842
19	20,324.42382317	471,558.46404180	23.201567
20	19,605.10498584	451,234.04021863	23.016151
21	18,904.53307904	431,628.93523279	22.832033
22	18,222.92042250	412,724.40215375	22.648642
23	17,562.94943850	394,501.48173125	22.462143
24	16,923.01258974	376,938.53229275	22.273725
25	16,303.54373752	360,015.51970301	22.082041
26	15,703.94942091	343,711.97596549	21.886977
27	15,123.20109258	328,008.02654458	21.689060
28	14,561.21192625	312,884.82545200	21.487555
29	14,017.43330988	298,323.61352575	21.282328
30	13,491.33042972	284,306.18021587	21.073250
31	12,982.38292755	270,814.84978615	20.860181
32	12,490.08389589	257,832.46685860	20.642974
33	12,014.31848250	245,342.38296271	20.420833
34	11,554.20726850	233,328.06448021	20.194208
35	11,110.35123420	221,773.85721171	19.961012
36	10,681.16893551	210,663.50597751	19.722888
37	10,266.89212806	199,982.33704200	19.478371
38	9,866.38571255	189,715.44491394	19.228464
39	9,479.55307810	179,849.05920139	18.972314

MASSACHUSETTS LIFE TABLE, 1893-97—*Continued.*

FEMALES, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
40	9,105.96437536	170,369.50612329	18.709661
41	8,745.50115200	161,263.54174793	18.439599
42	8,397.15493320	152,518.04059593	18.163061
43	8,061.12083796	144,120.88566273	17.878517
44	7,736.72041090	136,059.76482477	17.586232
45	7,423.58537926	128,323.04441387	17.285858
46	7,121.35786914	120,899.45903461	16.977023
47	6,827.69595392	113,778.10116547	16.664201
48	6,543.16355440	106,950.40521155	16.345366
49	6,266.35918459	100,407.24165715	16.023218
50	5,997.39134736	94,140.88247256	15.696971
51	5,735.23197246	88,143.49112520	15.368775
52	5,480.03123360	82,408.25915274	15.037917
53	5,232.11726856	76,928.22791914	14.703078
54	4,991.15852286	71,696.11065058	14.364623
55	4,755.86249890	66,704.95212772	14.025837
56	4,526.40911646	61,949.08962882	13.686144
57	4,302.94877600	57,422.68051236	13.344960
58	4,085.42430304	53,119.73173636	13.002255
59	3,873.77412264	49,034.30743332	12.658019
60	3,667.59260872	45,160.53331068	12.313399
61	3,466.51002876	41,492.94070196	11.969658
62	3,271.46979456	38,026.43067320	11.623653
63	3,082.05428844	34,754.96087864	11.276557
64	2,898.03217605	31,672.90659020	10.929108
65	2,719.33329225	28,774.87441415	10.581591
66	2,545.60053132	26,055.54112190	10.235518
67	2,376.36887807	23,509.94059058	9.893220
68	2,211.35231048	21,133.57171251	9.556854
69	2,050.28985908	18,922.21940203	9.229046
70	1,893.19742640	16,871.92954295	8.911870
71	1,740.81896163	14,978.73211655	8.604417
72	1,593.80618956	13,237.91315492	8.305849
73	1,452.49348299	11,644.10696536	8.016633
74	1,317.61173894	10,191.61348237	7.734914
75	1,189.46380278	8,874.00174343	7.460505
76	1,068.29770500	7,684.53794065	7.193255
77	954.41303514	6,616.24023565	6.932261
78	847.85135120	5,661.82720051	6.677853
79	748.81702776	4,813.97584931	6.428774
80	657.27583740	4,065.15882155	6.184859

MASSACHUSETTS LIFE TABLE, 1893–97—*Continued.*

FEMALES, 3%.

AGE.	D_x	N_x	$1+a_x=\frac{N_x}{D_x}$
81	573.26029170	3,407.88298415	5.944739
82	496.50452015	2,834.62269245	5.709157
83	427.00171740	2,338.11817230	5.475664
84	364.46628195	1,911.11645490	5.243602
85	308.53517882	1,546.65017295	5.012881
86	258.77986992	1,238.11499413	4.784433
87	214.87048776	979.33512421	4.557792
88	176.41523542	764.46463645	4.333325
89	142.97085570	588.04940103	4.113072
90	114.26199252	445.07854533	3.895245
91	89.88775020	330.81655281	3.680329
92	69.47297656	240.92880261	3.467949
93	52.60292826	171.45582605	3.259435
94	39.01759604	118.85289779	3.046135
95	28.22990976	79.83530175	2.828039
96	19.85299938	51.60539199	
97	13.58898791	31.75239261	
98	8.94266568	18.16340470	
99	5.73453874	9.22073902	
100	3.48620028	3.48620028	

NOTE.

MR. BRADSHAW, having discovered some errors in the net premiums based upon Dr. Sprague's Select Life Table with interest at 3½%, as printed on pages 235-238 of volume V. of the Transactions, has requested the insertion of the following tables, which have been carefully computed and checked, and which may be relied upon as being accurate.—ED.

Net Annual Premiums, Life.

Select Life, H^M 3½ %.

Age.	Whole Life.	10-Payment Life.	15-Payment Life.	20-Payment Life.	25-Payment Life.	30-Payment Life.
20	$14.71	$36.57	$26.95	$22.26	$19.55	$17.83
1	14.92	36.88	27.17	22.44	19.71	17.99
2	15.14	37.23	27.42	22.65	19.90	18.17
3	15.40	37.63	27.72	22.90	20.12	18.38
4	15.69	38.08	28.05	23.18	20.37	18.62
25	16.00	38.58	28.42	23.49	20.66	18.89
6	16.36	39.14	28.84	23.84	20.98	19.20
7	16.75	39.76	29.31	24 24	21.34	19.55
8	17.18	40.44	29.82	24.68	21.74	19.93
9	17.64	41.17	30.37	25.14	22.17	20.34
30	18.14	41.95	30.96	25.65	22.64	20.79
1	18.68	42.78	31.59	26.19	23.13	21.26
2	19.21	43.58	32.19	26.71	23.61	21.73
3	19.78	44.42	32.83	27.26	24.13	22.24
4	20.37	45.27	33.48	27.83	24.66	22.76
35	21.00	46.16	34.16	28.43	25.22	23.31
6	21.66	47.09	34.87	29.05	25.81	23.89
7	22.37	48.05	35.62	29.71	26.44	24.52
8	23.12	49.05	36.41	30.41	27.10	25.18
9	23.92	50.11	37.24	31.15	27.81	25.89
40	24.77	51.21	38.12	31.94	28.57	26.65
1	25.67	52.35	39.03	32.76	29.36	27.45
2	26.62	53.53	39.97	33.61	30.19	28.29
3	27.61	54.74	40.95	34.50	31.07	29.18
4	28.65	55.98	41.94	35.42	31.98	30.12
45	29.74	57.23	42.97	36.37	32.92	31.09
6	30.88	58.52	44.02	37.36	33.92	.....
7	32.09	59.84	45.12	38.40	34.97	.....
8	33.37	61.22	46.28	39.51	36.09	.....
9	34.73	62.65	47.49	40.67	37.27	.....
50	36.17	64.13	48.76	41.91	38.55	.....
1	37.71	65.67	50.10	43.22	.....	.....
2	39.34	67.27	51.52	44.62	.....	.....
3	41.08	68.94	53.01	46.10	.....	.....
4	42.94	70.70	54.60	47.70	.....	.....
55	44.93	72.55	56.29	49.41	.....	.....
6	47.04	74.48	58.07	.....	.....	.....
7	49.28	76.50	59.96	.....	.....	.....
8	51.66	78.59	61.93	.....	.....	.....
9	54.16	80.75	64.01		.....	.....
60	56.81	82.98	66.19	.....	.....	.....
1	59.59	85.26	.....	.....	.....	.....
2	62.55	87.65	.....	.....	.....	.....
3	65.79	90.25	.....	.....	.....	.....
4	69.29	93.05	.....	.....	.....	.....
65	72.83	95.78	.....	.....	.....	.....

Net Annual Premiums, Endowment.

Select Life, H^M 3½ %.

Age.	10 Years' Endowment.	15 Years' Endowment.	20 Years' Endowment.	25 Years' Endowment.	30 Years' Endowment.
20	$86.80	$55.08	$39.60	$30.66	$24.99
1	86.68	54.96	39.51	30.59	24.95
2	86.57	54.85	39.43	30.53	24.92
3	86.46	54.76	39.37	30.49	24.92
4	86.37	54.70	39.33	30.48	24.94
25	86.29	54.65	39.31	30.49	24.99
6	86.23	54.63	39.32	30.53	25.07
7	86.21	54.65	39.36	30.61	25.19
8	86.23	54.70	39.43	30.72	25.34
9	86.26	54.76	39.52	30.85	25.51
30	86.32	54.85	39.64	31.01	25.72
1	86.41	54.96	39.79	31.20	25.95
2	86.48	55.05	39.91	31.37	26.18
3	86.55	55.15	40.06	31.57	26.44
4	86.63	55.25	40.21	31.79	26.72
35	86.70	55.37	40.39	32.03	27.03
6	86.78	55.51	40.60	32.29	27.38
7	86.88	55.67	40.83	32.60	27.77
8	86.99	55.86	41.09	32.94	28.20
9	87.13	56.08	41.39	33.32	28.68
40	87.30	56.34	41.72	33.75	29.21
1	87.50	56.62	42.09	34.22	29.79
2	87.72	56.93	42.49	34.73	30.42
3	87.96	57.27	42.93	35.30	31.11
4	88.22	57.63	43.40	35.90	31.84
45	88.49	58.01	43.91	36.54	32.63
6	88.78	58.41	44.46	37.24	.....
7	89.09	58.86	45.06	38.00	.....
8	89.44	59.36	45.73	38.84	.....
9	89.83	59.91	46.46	39.75	.....
50	90.26	60.53	47.27	40.76	.....
1	90.74	61.21	48.17	.....	.....
2	91.28	61.98	49.15	.....	.....
3	91.88	62.83	50.24	.....	.....
4	92.57	63.79	51.44	.....	.....
55	93.34	64.85	52.78	.....	.....
6	94.21	66.01	.....	.....	.....
7	95.17	67.28	.....	.....	.....
8	96.22	68.66	.....	.....	.....
9	97.35	70.15	.....	.....	.....
60	98.56	71.77	.....	.....	.....
1	99.83	.....	.....	.....	.....
2	101.22	.....	.....	.....	.....
3	102.82	.....	.....	.....	.....
4	104.65	.....	.....	.....	.....
65	106 44	.....	.....	.....	.....

Net Annual Premiums, Limit Pay't Endowment.

Select Life, H^M 3½ %.

Age.	10-Payment 15 End't.	10-Payment 20 End't.	10-Payment 25-End't.	10-Payment 30 End't.	20-Payment 25 End't.	20-Payment 30 End't.
20	$74.74	$65.06	$57.35	$51.26	$34.91	$31.20
1	74.60	64.93	57.23	51.16	34.82	31.13
2	74.47	64.80	57.12	51.08	34.75	31.07
3	74.36	64.70	57.03	51.03	34.70	31.05
4	74.27	64.62	56.98	51.02	34.67	31.05
25	74.19	64.56	56.95	51.04	34.67	31.07
6	74.15	64.54	56.96	51.11	34.70	31.13
7	74.15	64.56	57.03	51.24	34.77	31.24
8	74.18	64.62	57.14	51.41	34.87	31.37
9	74.23	64.71	57.28	51.63	34.99	31.53
30	74.32	64.83	57.47	51.90	35.14	31.73
1	74.43	64.99	57.70	52.21	35.32	31.96
2	74.52	65.12	57.89	52.49	35.48	32.17
3	74.62	65.27	58.12	52.82	35.67	32.42
4	74.71	65.43	58.36	53.16	35.87	32.68
35	74.82	65.60	58.62	53.54	36.10	32.97
6	74.94	65.79	58.91	53.96	36.35	33.29
7	75.08	66.01	59.24	54.42	36.63	33.65
8	75.25	66.27	59.61	54.93	36.96	34.06
9	75.45	66.56	60.03	55.50	37.32	34.51
40	75.69	66.90	60.50	56.14	37.73	35.01
1	75.95	67.27	61.02	56.82	38.18	35.55
2	76.25	67.68	61.58	57.56	38.67	36.14
3	76.57	68.12	62.19	58.35	39.20	36.77
4	76.91	68.60	62.84	59.18	39.76	37.45
45	77.26	69.09	63.52	60.06	40.37	38.17
6	77.64	69.63	64.25	.....	41.02	.....
7	78.06	70.21	65.03	.....	41.73	.....
8	78.52	70.86	65.89	.....	42.52	.....
9	79.03	71.56	66.81	.....	43.38	.....
50	79.60	72.34	67.82	.....	44.32	.....
1	80.24	73.18	.....	.....	.....	.....
2	80.94	74.11	.....	.....	.....	.....
3	81.72	75.13	.....	.....	.....	.....
4	82.59	76.26	.....	.....	.....	.....
55	83.58	77.51	.....	.....	.....	.....
6	84.66	.....	.....	.....	.....	.....
7	85.84	.....	.....	.....	.....	.....
8	87.13	.....	.....	.....	.....	.....
9	88.50	.....	.....	.....	.....	.....
60	89.98	.....	.....	.....	.....	.....

Net Single Premiums, Life and Endowment.

Select Life, H^M 3½ %.

Age.	Life.	10 Years' Endowment.	15 Years' Endowment.	20 Years' Endowment.	25 Years' Endowment.	30 Years' Endowment.
20	$303.18	$719.64	$619.61	$539.39	$475.51	$425.00
1	306.08	719.37	619.10	538.80	474.92	424.56
2	309.29	719.10	618.63	538.29	474 44	424.28
3	312.89	718.85	618.24	537.91	474.16	424.25
4	316.87	718.64	617.95	537.67	474.06	424.48
25	321.22	718.45	617.74	537.55	474.14	424.94
6	326.00	718.31	617.67	537.61	474.49	425.72
7	331.26	718.27	617.76	537.88	475.12	426.86
8	336 92	718.30	617.97	538.34	476.01	428.31
9	342.85	718.36	618.24	538.90	477.06	429.98
30	349.17	718.52	618.63	539.66	478.37	431.97
1	355.78	718.74	619.10	540.56	479.87	434.21
2	362.26	718.89	619.46	541.33	481.26	436.37
3	369.01	719.07	619.89	542.25	482 84	438.80
4	375.90	719.23	620.32	543.22	484 52	441.38
35	383.05	719.41	620.83	544.32	486.40	444.25
6	390.44	719.60	621.41	545.55	488.48	447.39
7	398.10	719.82	622.10	546.95	490.81	450.87
8	406.05	720.09	622.91	548.54	493.41	454.70
9	414.28	720.40	623.84	550.32	496.29	458.89
40	422.81	720.80	624.92	552.34	499.50	463.48
1	431.52	721.26	626.09	554.51	502.97	468.37
2	440.42	721.75	627.37	556.86	506.70	473.59
3	449.50	722.32	628.76	559.40	510.71	479.14
4	458.68	722.90	630.21	562.08	514.93	484.95
45	467.93	723.51	631.73	564.91	519.36	491.04
6	477.33	724.16	633.35	567.96	524.08	.....
7	486.89	724.86	635.11	571.25	529.11	.....
8	496.70	725.65	637.07	574.87	534.54	.....
9	506.66	726.51	639.19	578.75	540.34	.....
50	516.85	727.46	641.56	582.98	546.57	....
1	527.22	728.52	644.15	587.54	.....	.....
2	537.77	729.68	647.00	592.43	.....	.....
3	548.47	730.97	650.10	597.68	.....	.....
4	559.42	732.43	653.53	603.38	.....	.....
55	570.54	734.06	657.26	609.51	.....	.....
6	581.75	735.87	661.25	.....	.....	.....
7	593.05	737.84	665.51	.....	.....	.....
8	604.36	739.95	670.01	.....	.....	.....
9	615.63	742.19	674.75	.....	.....	.....
60	626.85	744.54	679 75	.....	.....	.....
1	637.96	746.98	.....	.....	.....	.....
2	649.09	749.58	.....	.....	.....	.....
3	660.49	752.51	.....	.....	. ....	.....
4	672.01	755.77	.....	.....	.....	.....
65	682.90	758.90	.....	.....	.....	.....

Address of the President, Mr. Thomas B. Macaulay, in opening the Annual Meeting, May 17th, 1900.

Gentlemen of the Actuarial Society:

I am sure that my predecessors in the Presidential chair will sympathize with me in the difficulty which I have experienced in finding a suitable theme to which to draw your attention on this occasion. As the Inaugural Address is not, as a rule, thrown open for subsequent discussion, it is tacitly understood that it shall, as far as possible, avoid not merely all subjects which are of a controversial nature but even those which are debatable, and this fact reduces to narrow limits the field from which something appropriate and profitable may be selected. I propose to refer very briefly to a few of the fallacies with which we are sometimes brought in contact in connection with actuarial matters, and which reveal to us something of the cloud of ignorance which even yet remains for us to dispel. This line of thought will necessarily take us back in many instances to the very alphabet of the science, but I trust that some of the points may be found interesting, if nothing more.

The subject was suggested by a recent publication of the Chicago Board of Health, in which a number of elaborate tables and charts were presented, showing, among other things, that a decided change has taken place in regard to the ages at death of those who die in that city. As compared with the years gone by, a smaller proportion of the deaths occur in infancy, and a larger proportion at the more advanced ages. The average age of those who died in 1872 was about 15 years; this has gradually increased, until in 1898 the average was over 29 years. The Health Commissioner, Dr. Reynolds, evidently considered that this indicated a wonderful improvement in the sanitary condition of the city; and I would be sorry indeed to say that this may not have had some influence in bringing about the result. The process of reasoning, however, appealed to me as a rather striking instance of the liability of our good friends, the doctors, to fall into error when they undertake to deal with statistics. One of the first things that an actuarial student is taught is that there is little to be learned with regard to the healthfulness of a city from a mere consideration of the ages of those that die in it. In the case now before us, for example, the facts are capable of a very different explanation. In 1872 Chicago

was a city of young men. It was notorious that comparatively few elderly people were to be found in it. On the other hand the proportion of children was, we believe, great. At the present time the survivors of those young men are nearly thirty years older than they then were, and the proportion of elderly people must have greatly increased, and will probably continue to increase for years to come. In other words, the average age of the entire population is greater now than formerly. Is it anything more than natural, then, that the average age of such of the inhabitants as die should also be greater now than was the case 25 or 30 years ago?

And even a lowering of the death rate per thousand of the total population does not necessarily imply an improvement in healthfulness. A city which is growing as rapidly as Chicago naturally draws to itself immigrants from the entire surrounding country and from other cities. These are chiefly young people, for neither children nor elderly persons migrate freely. The city therefore is continually receiving additions to its population, a large proportion of these being between the ages of 15 and 25. This is the period of life when the rate of mortality is at about its lowest. This fact, coupled with the gradually reducing proportion of children, with their high infantile mortality, might be sufficient, without any other explanation, to account for the striking results which have been set before us. I do not say that the healthfulness of Chicago has not been much improved, but it is evident that the figures on which the claim has been based do not prove it.

Of a somewhat similar nature are some statistics which have been scattered far and wide throughout the Dominion, and, I presume, in other parts of the world, proving to the entire satisfaction of the compilers, that not only moderate drinkers, but even those who are decidedly intemperate, live to a greater age than the deluded abstainers. These statements are so interesting that I quote a few extracts. They are from an attractively prepared card, issued by a Canadian firm of wholesale liquor merchants, and entitled "Alcohol and Old Age—Truth vs. Fiction."

"The British Medical Association appointed a committee to "make inquiries in order to ascertain the average age of the "different categories of drinkers—that is to say, those who refrain "completely from alcoholic drink, those who indulge more or less "in moderation, and those who drink to excess. This committee "has handed in its report. Its conclusions are drawn from 4,234 "deaths, which are divided into five categories of individuals, "with the average of age attained by each :

"1. Total abstainers..... .....51 years 22 days.
"2. Habitually temperate drinkers...63 years 13 days.
"3. Careless drinkers.....59 years 67 days.
"4. Free drinkers ....57 years 59 days.
"5. Decidedly intemperate drinkers..53 years 3 days.

"These figures show, singularly enough, that those who reach "the shortest age are those who drink no alcohol whatever. After "them come the drunkards, who only exceed them by a trifle."

We are further informed that the following information was brought to light by the efforts of the same Medical Association respecting persons who had attained the age of eighty and upwards :

Age Attained.	Number of Deaths Reported at these Ages.	Of whom Total Abstainers.	Per cent.	Moderate Drinkers.	Per cent.	Heavy Drinkers.	Per cent.
80 to 90	530	82	15.47	396	74.72	52	9.81
90 to 100	159	23	14.46	126	79.25	10	6.29
80 to 100	689	105	15.25	522	75.76	62	8.99

We are told that these statistics prove that of both males and females those who drink alcohol show the greater longevity. Taking the total 689 persons who are reported on as having died beyond eighty years of age, 15.25 per cent. were total abstainers and 84.75 per cent. non-abstainers, so that the chances of living to beyond eighty are said to be :—men, one abstainer to 11.586 non-abstainers; women, one abstainer to 3.263 non-abstainers; men and women, one abstainer to 5.561 non-abstainers. These results are claimed to be "a complete refutation of the absurd statements so repeatedly made by the total abstinence party," and to show that even heavy drinkers are very much more desirable risks for assurance companies than those who are so foolish as to abstain from the use of this veritable "Water of Life."

It is evident, now that this oracle has spoken, that we will have to revise our opinions about the injurious effects of alcohol, and that we will have to return to the ancient practice of charging an extra premium to all who commit slow suicide by abstaining. It has now been proved that teetotalism shortens the lives of its followers by from two to twelve years, and that it must be ranked among the most destructive death-dealing forces in existence.

The danger incurred by exposure to it is apparently greater than the military risk of the Transvaal war! To avoid the bottle's mouth is more hazardous than to face the cannon's mouth!

But before we finally adopt these new conclusions those of us who have still the courage to continue to be teetotalers, may be allowed to ask a few questions. We would like to know, for example, whether the physicians who, we presume, supplied the information, were men with an extensive practice in teetotal circles or not. People naturally can give details of this kind only about their own acquaintances, and the proportion of total abstainers included in the statistics may be better evidence as to how many teetotal friends these doctors had, than as to the longevity of the drinking and non-drinking classes of the community. We can imagine that a coterie of abstaining doctors might supply similar data, in which the number of aged drinking people known to these men would be so few as to apparently prove on the other hand that even moderate drinking destroyed any chance of long life.

But we would like to ask also what proportion of the population of Great Britain are abstainers? If it were found, for example, that their proportion in the total country was only ten per cent. (I do not know the percentage, and use this figure merely for the purpose of illustration), while their proportion among those dying at over eighty years of age was fifteen per cent., our courage might begin to revive. If further we were to find that the proportion of total abstainers was greater among children than among older people, we might let our heads rise a little higher still. In fact we might draw a conclusion from the statistics entirely different from that deduced by the compilers, and I can imagine an enthusiastic teetotaler retorting on an enthusiastic liquor advocate as follows: "The average age at death of our people is lower than the average age at death of yours, because on our side we have the children, who hold the future in their hands, while you have principally the old men, who are rapidly dropping out to join the past." The one conclusion would be as much justified by the statistics as the other.

I tremble to think, however, what will happen if these medical gentlemen begin to investigate the mortality prevailing among the members of the different professions, including our own. If they combine all into one table they will assuredly find that of those professional men who attain eighty years of age and over, the vast majority are doctors, lawyers and clergymen, and of Actuaries but

very few. This of course will be to them conclusive proof that Actuaries are very short-lived, and that our occupation is extremely unhealthful. The doctors will, of course, reason that as out of every hundred professional men who attain to eighty years of age, less than one per cent. are Actuaries, it follows that the chance which each Actuary has of attaining that age is less than one one-hundredth part as great as would be the case if he were a physician or a lawyer. This result they will probably diagnose to be due to the melancholy produced by continuous brooding over mortality tables and death claims. When the highly dangerous nature of our occupation is once clearly understood by the directors and officers of the companies, we can no longer hope that they will extend to us the favor of assuring our lives on the same terms as those of other people. I am afraid that they will consider that we are worse risks than even the teetotalers. We will all have to use our influence to the utmost to induce some at least of the companies to refrain from imposing extra premiums, on the score of the necessity to the business that some persons should follow this very unhealthy profession.

Turning now from jest to earnest, I would remind you that fallacious reasoning is sometimes indulged in as to the healthfulness of many other occupations. It can readily be shown, for example, that clerks die at a younger average age than merchants. It is hardly necessary to say that this does not in any way prove that they are worse lives than the latter. Most merchants go through the intermediate stage of being clerks or salesmen before beginning business on their own account. The average age of living clerks is therefore less than that of merchants, and the average age of those who die should similarly be less. Sometimes the argument merely takes the form of stating that as the average age at death of persons in certain walks of life is greater than that prevailing in the total population, such classes must be specially long-lived. We all know that the average age at death of those engaged in almost any occupation, must be greater than that of the community as a whole, seeing that the special group contains no children but consists of adults alone.

In connection with Chicago we nave already seen how erroneous are the conclusions which may be drawn from the death rates per thousand prevailing in various cities. Such comparisons can only be fairly made when the circumstances of each are exactly similar. But even when the cities are of equal age and are growing with equal

rapidity it sometimes happens that the very reputation of one of them for healthfulness will have the effect of increasing its death rate, by attracting invalids to it, as in certain cities of Colorado, California and the Southern States. Going one step further, account must also be sometimes taken even of racial differences in the inhabitants. As an illustration, I would refer to the French Canadians, who are, I think, the most prolific race upon the face of the earth. The contrast between Montreal or Quebec and a city of old France, or even a typical American city, is marked. In the one instance the streets swarm with children, while, except in a few localities,these are conspicuously absent in the other. Naturally, therefore, in a French Canadian city the death rate per thousand will be high on account of this large proportion of children, even though the rate among adults be favorable. When we further remember that a high birth rate and a high infantile death rate usually go hand in hand, it becomes evident that the existence of a heavy aggregate mortality rate in such a city gives no information as to whether the adults are healthy and long-lived or not.

This failure to recognize the necessity of taking into account the ages of the persons under observation is responsible for other erroneous conclusions. I remember reading many years ago in an English insurance journal a statement that the reported claim of a respected American office to be more careful in the selection of its risks than British companies were, was easily shown to be unfounded. It was urged that if the supposed claim were correct, it could be proved by the existence of a lower percentage of deaths from consumption in the American office than in British companies. Heart disease and kidney disease it was difficult to guard against if not present at the time of examination, but it was claimed that the deaths from consumption might be lessened if a more severe standard were adopted for the admission of applicants. It was accordingly urged that the proportion which those deaths formed to the total deaths occurring in a company, was a fair index of the care exercised by each office in the selection of its lives. Now, applying this test, it was shown that the percentage of deaths from this disease was actually greater in the American company than in its English rivals, and that the claim to the exercise of any special care must therefore fall to the ground. The writer apparently did not know that the deaths from consumption are a much larger proportion of those which happen at the younger ages than they are of those which happen later in life, and that therefore it would

be strange indeed if a comparatively young company of any nationality, which has a large number of policies on youthful lives, should not have a greater proportion of deaths from tuberculosis than one which is perhaps old and hoary and which has many aged lives on its books.

Turning now for a moment to the question of female life, what a number of fallacies, now mostly exploded, are brought to view. In the early part of the century, and in fact up till about 1843, it was more or less generally supposed that assured women lived longer than men, and many of the companies actually issued policies to them on terms more favorable than those which they exacted from the male sex. It was the publication of the experience of the Seventeen Offices that overthrew this fallacy. There are now no companies, so far as I am aware, which claim that women are better risks for life assurance than men; but on the other hand there are some, including practically all the French companies, which have gone to the other extreme, and issue annuity bonds at the same rates to both sexes. It is quite true that it does not seem reasonable to charge clients of the fair sex an extra premium if they apply for life assurance on the ground that they are not expected to live as long as men, and also to charge those who apply for annuities an extra premium because it is expected that they will live longer. This is one of the facts which no amount of *a priori* reasoning could have foretold, and yet there are few things better established now than the necessity of charging our fair customers higher rates for annuities than are necessary for lives of the sterner sex.

The mention of annuities naturally suggests other thoughts. How difficult it is for the unprofessional mind to grasp the real nature of a life annuity. Those who have occasionally had to give evidence as to the value of the earnings of a man whose life may have been cut short by an accident, will remember that the first question has usually been—"What is the expectation of life for such an age?" This has been followed by—"Was not the value of this man's income equal to the present value of an annuity for that number of years?" There are very few lawyers who can grasp the difference between an annuity for life and an annuity certain for the term of the expectation. This fallacy I am afraid we will find it difficult to ever eradicate from the popular mind. Closely associated with it is the impression that companies accumulate their net premiums on their assurance policies at interest until the

expectation of life is reached, so as to be able to then pay the sum assured to the beneficiaries. Some possibly think all the lives should die off at the exact end of the expectation, and that if they fail to do so, either the mortality table is wrong or the lives are not acting as they should. Reasoning along these lines, some policy-holders claim that as the companies have in hand the whole amount of the premiums on their policies, improved with interest, they should be willing to give as a surrender value, the whole, or nearly the whole, of the amount which has been paid in. This again reminds us of the actuarial ideas of the founder of an important American company, who, as explained by one of our members, believed in those early days that if a person assured for $10,000, and found himself unable to pay the premium for the second year, the policy need not lapse. All that was necessary was that the assurance be reduced to $5,000, when the premium already paid could be applied to cover the two years. Similarly, if again unable to pay the third premium the policy could be reduced to $2,500, and be considered then paid for four years.

The Northampton table, and the method in which it was prepared would naturally occupy a prominent place in any list of actuarial fallacies, but I will not weary you by explaining it, as this is set forth fully in the pages of the Journal of the Institute. English actuaries have, however, referred to some interesting facts which the use of this table has drawn attention to. It is apt to be taken for granted that if a table predicts an abnormally high death rate, it must also, as a matter of course, provide correspondingly high policy values. As is well known, for ages under 50 or 55, the Northampton table is excessive, although for the latter half of life it is not very far from the truth. Mr. Bailey has pointed out that some offices which used it lived in something like a "fool's paradise," thinking they were justified in employing this table, because the errors in it were on what was called the safe side. Mr. Manly and others have, however, shown that the value of an ordinary life policy by the Northampton table instead of being high is very low.

It might then well be supposed that as its successor, the Carlisle table, goes to the other extreme, and gives, especially for the higher ages, too favorable a mortality, it must avoid the error of the Northampton of providing too small reserves. This, however, is another fallacy, for the same result of low policy values is brought about if the mortality table errs either in predicting too heavy a mortality in the younger half of life, or too light a mortality in the

later half of life. The reason for this becomes evident when we consider one of the formulas for the value of a whole life policy :

$$\frac{a_x - a_{x+n}}{1 + a_x}$$

A decrease in the value of the initial annuity, or an increase in the value of the annuity for the age at time of valuation, has the same effect of diminishing the amount of the reserve.

It is difficult to imagine that any thoughtful person could fail to detect the fallacy involved in the supposition that because a table has been found to be abundantly safe for use in life assurance transactions, it is therefore also the correct one to employ for annuity contracts. Yet that is what actually happened in 1808. The British government at that time appears to have reasoned that because the Equitable Society, which based its operations upon the Northampton table, had been highly successful, it was entirely proper to take this table as a basis on which to issue annuities. This error was discovered by the slow teaching of experience, and the fee paid for learning the lesson was a very heavy one.

Among other claims which have been put forward, based upon improper, or at least insufficient, evidence, may be mentioned the exaggerated views sometimes entertained as to the effect of marriage in lengthening life. This conclusion is drawn from the known fact that the mortality among married persons is more favorable than among unmarried, but it overlooks the further fact that those who marry are, as a rule, self-selected risks, the sickly and delicate being usually left to swell the death rate prevailing among the unmarried.

I may refer also to the supposition that the rate of mortality must increase continuously with every advancing year of adult life. Most of our graduation formulas are based upon this theory, and yet its accuracy has still to be proved, at least for the period of young manhood, between ages 20 and 25. There are indications that the rate of mortality at ages 20 to 23 is, at least in many communities or groups, greater than for ages 24 to 27. We are in the habit of brushing aside such peculiarities of an experience under the supposition that they are merely accidental fluctuations, but the persistence with which the same results are found under different circumstances may well make us doubt the soundness of the theory on which we set them aside.

Under this heading come also almost all mortality experiences in which the exposures are obtained by some process of "looking

backward." If, as an illustration, an attempt be made to ascertain the effects of intemperance by trusting to the memories of people as to the ages and lengths of time they have known persons, either living or dead, who have been subject to the habit, it is almost certain that most of the cases thus reported will be people who are either still living or who have died recently ; and if they be considered as exposed for the lengths of time the informants can remember them, erroneous results are sure to be brought out. Memory is too poor a thing to trust, and many deaths will be omitted which should be included, because of their having been under the observation of the informant in the earlier years to which his memory goes back, although he now only recollects those cases which have survived to recent years. In fact, even apart from the question of memory, any system by which the number supposed to have been under observation is in any way dependent upon particular attention having been drawn to such cases, either by their recent death or by their long life, is necessarily unscientific and unreliable.

Yet another fallacy, now happily exploded, is the theory that the profits of a mutual life assurance company should be divided in the same way as the profits of a business enterprise would be among the partners ; in other words, in proportion to the amount invested by each in the company. It was at one time held in some quarters that this was the only businesslike way of arranging the matter, and the surplus was accordingly divided in proportion to the reserve on each policy, that being considered to be the capital standing to the credit of each member or partner. The fallacy lay in forgetting that the profits of the institution are not obtained from outsiders, as in an ordinary business, but are merely the surplus contributions of the partners themselves, and should therefore be divided, not in proportion to the capital invested by each, but in proportion to the contribution made to the surplus made by each. Some very interesting points in connection with this and other matters are brought out in a report made by Mr. Robert Tucker, Mr. A. H. Bailey, Dr. T. B. Sprague and Mr. R. P. Hardy, in reply to questions submitted to them many years ago by the Australian Mutual Provident Society. This is a volume worthy of a place on the shelves of every actuary.

Another danger which might entrap even the most wary is that of supposing, on insufficient evidence, that the mortality experience of any life company exhibits a maximum a few years after the issue of the policies. I will not here express any opinion as to

whether such maximum mortality percentages really exist or not. That is a debatable question. I may, however, point out how easily a wrong conclusion in regard to this matter may be arrived at. If we employ as a standard, or measuring stick if I may so call it, a table which is less favorable to the older ages of life than to the younger, the actual mortality experienced is likely to form a smaller proportion of that expected at the higher ages than at the earlier, for policies of the same duration. This fact has a very important bearing; firstly, because all aggregate tables based on assurance experience have exactly the peculiarity mentioned, and secondly, because the lives under observation in the earlier policy years are always comparatively young, while those assured under policies which have been long in existence are necessarily much older. Policies twenty years in force will, therefore, usually show a lower percentage of actual to expected deaths than those only ten years in force, rather because the lives have reached the ages at which lower percentages are to be expected, than because of any real improvement in the mortality. The gradual wearing out of the benefit of the initial selection, coupled with the later gradual improvement in the percentage due to the cause just mentioned, is likely to produce an apparent maximum, whether such really exists or not.

As a final illustration I can hardly refrain from introducing the latest proof that human life is gradually lengthening. It is an item that caught my eye some days ago in one of our daily papers, and is from the pen of an unknown humorist. It is more amusing and quite as logical as some of the examples of supposed reasoning I have already quoted. Saith Uncle Silas: "Statistics show that the world keeps gittin' healthier and healthier. People live longer than they did fifty or a hundred years ago; anyone can observe the fact." To which Aunt Salina sagely replies: "I've noticed it for a long time. It's got so that one can't pick up a paper any more without readin' about someone bein' cured and seein' their picture."

I wish I could now say that I have exhausted my subject. I have no doubt, however, that other fallacies, some of them probably more glaring than any which I have mentioned, have occurred to you even as I have been speaking. Unhappily we meet them on every hand. Some of them are insidious enough to remind us that we must be on our guard against them continually, but fortunately

most are quite transparent. If some of those I have mentioned are only worthy of a smile, then let me plead that

"A little nonsense now and then
Is relished by the wisest men,"

and on that score I ask your pardon for taking up so much of your time.

Abstract of the Discussion of Papers Read at Previous Meetings.

Concerning some recent mortality tables on annuitants.

RUFUS W. WEEKS.

Page 137, Vol. VI.

Mr. Barker: An examination of the paper on Life Annuities, submitted by Mr. Weeks to the Society last October, seems to show that there is a considerable difference in the mortality to be expected on annuitants in England, in America and in France, the English annuitants living longest and the French the shortest; also that in the case of Mr. McClintock's annuity tables while that for males closely corresponds to, and is slightly lower in mortality than, the actual experience of fifteen of the leading American companies collated and tabulated by the efforts of Mr. Weeks some six years ago, the table for females calls for a higher mortality than the American Actual Experience exhibits, the total variation in the tables for males being as 1408.7 is to 1415, and those for females as 1123.5 is to 1104, the mortality for females being by Mr. McClintock's assumptions less until the age of 58 is reached.

As the experience of no two companies is probably alike on annuities, owing to the great variations in amounts purchased by different individuals, the advice given by the eminent actuary Dr. Price in 1779 to the English Equitable Society will, I think, hold good here, namely, "It is of particular consequence that the Society should distinguish between the cases which require it to compute from observations that give the chances of high life rather than from those which give them low."

Another reason why great care should be taken that annuity rates shall be based on safe tables is that the average duration of human life in recent times has been appreciably lengthening, with the limits of improvement scarcely yet reached, caused doubtless by the greatly improved and more favorable conditions of life now existing; among which may be included the more comfortable habitations, better sanitary conditions, the purer, fresher, more abundant, and by reason of the enlarging transportation and trade facilities, the more equally distributed food supply; the invention of the means of rapid transit, thus diffusing the population, rendering the cities less crowded, and giving to many people healthier and happier surroundings; the growth of wealth, the minimizing through mechanical inventions of the necessity for the more exhaustive labors of man; the cheapening by the same means of the price of suitable and comfortable clothing, bringing it within the reach of nearly every one; the increasing means and facilities for the care of the poor, the friendless and the aged through the philanthropic efforts of society; and last, but not least, the higher

degree of skill in physical diagnosis, therapeutics and surgery attained by physicians, aided by the discoveries and advances made in medical science. All these are powerful forces in the direction of increased longevity.

When an annuitant is taken it can be for nothing short of the balance of his natural life, and no contingency toward the relief of the company from its liability can arise, as by forfeiture or surrender frequently happens in the case of life insurance policies. It is a hard and fast bargain, and the mortality as well as the rate of interest realized cannot be less than required by the basis assumed, or a loss is bound to be sustained.

In addition to this there are expenses of investing and caring for the funds and paying the annuities, all of which must be paid out of the excess of interest and mortality, unless sufficient has been saved out of the loading for the purpose, which in the case of single premium annuities is seldom the case. Therefore in this class of policies there is a much greater necessity for a safe margin in both mortality and interest assumptions than in the case of life policies, where forfeiture and surrender, referred to above, and the higher loading permissible on the premiums give a much greater chance to make good any losses.

It occurs to me that there is another feature of annuities worthy of investigation, namely, to what extent will the annuity experience of companies be affected by the recently introduced practice of granting deferred survivorship annuities in connection with instalment insurance contracts, especially where the annuitant voluntarily discontinues the payment of premiums before paying all required by the contract, especially where the annuities are granted on the same tables with the premiums.

The large accumulation of money and increasing wealth in this country have combined to make a greater demand for annuities in recent years, and the low rates of interest now obtainable on safe investments have made annuities the most popular and safe investments that can be obtained, particularly for elderly people, where a high rate of income is received on the sum invested.

On a question so important we are certainly highly indebted both to Mr. Weeks and Mr. McClintock for their able and painstaking investigations, and they deserve our warmest thanks.

MR. SARTELLE: In this connection I may refer to the paper just presented on "Massachusetts Annuities." In seeking a table of annuities worthy of adoption for use it was thought that the mortality experienced in Massachusetts might furnish a conservative basis.

MR NICHOLS: My attention was drawn to this table by Dr. Abbott, and from a cursory examination I was much impressed with the reliability thereof, with the method on which it was constructed, and with the pains taken in its preparation.

MR. W. C. WRIGHT: Any one who examines that table will see that it is generally parallel with the Farr No. 3 Table.

MR. MACAULAY: I understand this table just referred to is based upon the mortality of the general population of Massachusetts. Now, if that is so, and if the results shown by it agree with the results shown in Mr. Weeks' table of the experience on annuitants in American offices, which I suppose it does, the deduction I would be inclined to draw is that it condemned Mr. Weeks' table for use in granting annuities on lives on this continent, because we expect annuitants to live longer than the general population, and therefore it does not seem wise to grant annuities based on a general population table. If American mortality is as favorable as this table shows, it seems that Mr. Weeks' table is hardly severe enough.

MR. VAN CISE: On this question of annuities, I may say that an actuary of one of the American companies in a discussion in regard to the rate of commission which could be paid, took the ground that his company was justified in paying a higher rate of commission because, firstly, there would be a high rate of mortality among the annuitants; secondly, that high commissions on annuities would attract better agents, and through them a better class of lives for insurance would be secured. It occurred to me that if any company represented here had any experience in that line it would be interesting. If the increased commission works a corresponding increase in mortality, it makes the company just so much more profit. I think that some companies which sell annuities do not expect to make a profit on that business, but merely look upon it as an aid to the insurance side of the business.

MR. W. C. WRIGHT: As far as I know or have observed, Mr. McClintock's table of annuities gives the most conservative values throughout of any table; it gives more conservative values than the Heathy Districts Table.

MR. DE BOER: It is to be noted that the companies do not necessarily develop demands as well as markets, but the desire for annuities in the United States is bound to grow out of the changes in the economic conditions, in the increase of wealth seeking investment and the consequent lowering in the rate of income. This business must be undertaken, and the question of the proper basis has got to be solved, and for that purpose the actuaries have got to take up the matter of mortality. Now this may give rise to an argument. It is wise for us to interest ourselves in the rate of interest, or in what might have been the rate of mortality a hundred years ago in some little place in England, but we are people who are in this business practically, and we must reach some general basis on which we can stand. It is impossible for life offices to be changing their rates every few months; that is why to-day we are working all through New England, as well as in New York, with an antiquated table of mortality. It is impossible to change the basis of our calculations every little while.

I think the Society must agree with opinions expressed some years ago, that we should begin to take a vital, thoughtful, deter-

mined hold on this matter of the tabulation of American mortality experience as a basis for the conduct of life insurance in the next third of a century. We ought to-day to have such a table as the product of this Society. I am interested, whether rightly or wrongly, in a new mortality table. I want a table constructed by a committee of this Society, composed of the strongest men we have the wisest in council, with the widest experience, securing also the coöperation of the officers of all companies who have anything in the way of valuable experience to contribute for the construction of such a table. Furthermore, I would have that experience examined by classes, for there are at least a dozen important factors which determine the question of selection. Family History is one, Habits is another, Personal History in connection with Family History is a third, and Geographical Location is a fourth. Let us so divide the whole experience, and create, if you will, a dozen mortality tables, not necessarily that they should all be used in one office, but that we may have a basis for the better determination of this question of the measurement of the individual risk. This same point, I think, is relevant with reference to annuities. I do not think the annuity experiences as presented to this Society cover the case.

The proposition was asked of me recently to issue a contract with a consideration of $20,000, to insure a man on a non-participating basis for $20,000 and use the balance of the consideration to purchase an annuity. At the rates in use by certain companies, the return on the investment would be between four and four and one-half per cent. The answer I made was this: Those rates cannot be used with reference to their combination in that way. They were not intended for that purpose.

MR. W. C. WRIGHT: I agree entirely with Mr. De Boer in that we could probably have for annuities a better table, but let us be clear in our ideas about any table which we propose to abandon as antiquated. The H^M Table, which was brought out only twenty years later than the Actuaries', has just been adopted in Canada by the companies, and closely conforms to the Actuaries' Table.

Now, in the table prepared by Dr. Abbott, of which Mr. Sartelle has just spoken, if you will compare the ratios of q_x for successive ages, you will find it is decidedly more in agreement with the Actuaries' than with the American Table, and again both agree better with the Farr No. 3 Table. My understanding of a good table is one which is parallel with a really reliable population table, because that has a real rate of mortality. What other differences there are are differences due to selection, and in examining the different tables I find this fact, that all select life tables, like the American Table, are based upon sufficient material only at the middle ages. Therefore, for a safe table, we want one that approximates the population table. Of course, the American Table in that respect is a conservative table, but it has the peculiar characteristic, as a true table of mortality, that it assumes no survival

beyond the age of ninety-six. Now, the Abbott Table, to which Mr. Sartelle has referred, especially discredits the American Table on that point. If we want a table which shall be safe all along the line, and yet which shall be a true table all along this line, it must not be widely divergent from the population table at the high ages.

Mr. Wells: I understood Mr. De Boer to say that a person can, from some company, obtain an insurance for a given sum and use the balance in hand of a sum of the same amount to purchase an annuity, receiving during his life between four and four and one-half per cent. return. In other words, he could make with that company an investment at that rate. If that is so, the company is doing the business on a wrong basis. No company can afford to-day to make a contract to pay a man four and one-half per cent. during his lifetime, and the capital when he dies. It is certain that companies engaged in this business are charging too little for their annuities or too little for their insurances.

There is one point which may possibly be of interest, and that is the fact that the computation of annuities upon a low rate of interest goes a long way to offset the low mortality which we experience on annuities. For annuities we get the money in hand, and have it invested while the interest rate is four, perhaps five, per cent. Now, if the rate falls in the future a fund has been created while the rate is high; and I find that taking a decreasing rate of interest through a long series of years the value of the annuity is equivalent to an annuity computed upon a higher rate of mortality with a much higher interest assumption than would have been supposed.

A SECOND NOTE ON PREMIUM LOADING. WALTER C. WRIGHT. PAGE 153, VOL. VI.

Mr. Craig: In Mr. Wright's Second Note on Premium Loading appears the following:

"I learn from the table of extended insurance of one of our leading companies that the holder of an annual premium whole-life policy, taken at the age of 32, will be entitled to five years' extension of insurance upon lapse after the payment of five annual premiums. The net reserve of this policy, Am. Ex. 4 per cent., at the end of five years is $50.32, and the single premium of five years' ensuing term insurance is $41.92, showing an allowance for expenses (to say nothing of surrender charge, which would seem to be due in this case in some measure) of $8.40 for the entire five years' insurance, or of $1.68 per $1,000 per annum. The only drawback to the liberality of this agreement is that the term insurance is made non-participating, the value of which feature to the company is, of course, uncertain and undeterminate in advance. The initial terms of the policy, according to the same table of mortality and rate of interest, allowed $6.52 on $1,000 annually for expenses

in excess of the net premium. These are small figures, but represent exactly the conditions of a large and growing volume of insurance, and remembering that few companies now have any considerable portion of 'loading' left after meeting their annual expenses, they show a sufficiently decided contrast to confirm my claim of their gross and injurious, if not dangerous, violation of equity."

Mr. Wright has not shown in this note, and, so far as I have been able to discover, no attempt has been made to show, that the margin for expenses in the case cited of a paid-up five-year term policy is insufficient to meet the requirements. It is true he says "that few companies now have any considerable portion of 'loading' left after meeting their annual expenses," but the question in the present case is not whether but few companies have any loading left after meeting their annual expenses, but whether a 20 per cent. loading is sufficient on a non-participating paid-up five-year term policy, free from all the initial expenses incident to procuring new business, except those pertaining to office work, and which after it is once on the books is practically outside the range of the activities and expenses of the business.

It is contended by Mr. Wright that a reserve should be held to cover the annual expenses on a paid-up policy. He does not attempt to show what the expenses actually are, or are likely to be in the future, or how they will probably fall in point of time.

Mr. Wing gave expression to the thought, Vol. V., page 260, that whatever expense attaches to keeping paid-up policies on the books is in hand in the form of surplus, and applicable for the purposes intended. He then goes on to say "if it should be thought best by some process to set it aside as a liability and divert it from surplus, where it now appears, there would be no objection to the plan as a simple statement of fact." Is it necessary to go as far as this? The first question to be determined is, what expenses attach to paid-up policies generally? When the answer is made that the expenses are such as are incident to investments and their contingencies, the whole ground is pretty nearly covered. If this much be conceded, then Mr. Wright himself comes to our aid, for in a note at the end of his 1890 paper he says "the interest earned on the initial reserve for each year, in excess of three per cent., provides for investment expenses and contingencies, the excess being returnable surplus."

As we are discussing non-participating paid-up policies, it is sufficient to know that the excess of interest provides for "investment expenses and contingencies."

It will be observed that in the quotation from Mr. Wright's note he says that "the only drawback to the liberality of this agreement is that the term insurance is made non-participating," which statement, taken in connection with his proposition, means, if it means anything, that it would be more advantageous to the insured, or at least more scientific on the part of the company, if an extension of

four years and ninety-four days were granted with participation in surplus than an extension of five years without participation.

Let us see if we can discover anything to justify this.

The reserve on the policy at the end of five years is, .	$50.32
The net single premium for five years' extended insurance is,	41.92
Leaving a margin for expenses, etc., ...	8.40

As against the above statement of facts we find in his note: "Following the method recommended by me, and waiving all lapse charge, the reserve at lapse of the above policy would be $57.14, and the term of temporary insurance purchasable by it four years and ninety-four days." The question would therefore seem to resolve itself into this: Is it a "gross and injurious, if not dangerous, violation of equity" to grant *non-participating* insurance over a period of five years when Mr. Wright's method would produce *participating* insurance over a period of four years and ninety-four days?

THE NORMAL RATE OF INTEREST. CHARLTON T. LEWIS. PAGE 158, VOL. VI.

MR. ST. JOHN: I shall not try to enlarge the liberal fund of information presented in the scholarly paper on the "Normal Rate of Interest," but instead offer a few comments to gain the ear of those present who, like myself, are fond of dabbling now and then in matters related to political economy.

The easily besetting sin of economic discussions is the failure to distinguish clearly between essentially scientific terms and their usual significance. For example, rent may be employed in a purely theoretical sense as a differential gain or surplus originating in the varied degrees of fertility in cultivated soils, or to difference in location, and yet be interpreted as a sum of money paid by the tenant to the landowner; or interest may be used in an economic sense as a normal surplus above the cost of maintaining an established fund of capital, and yet be interpreted as the price paid for loans of money. It is not an infrequent fault of even trained economists to glide in the course of discussion from the technical to the usual meaning of their terms without a word of warning, to the confusion of thought in their readers.

We have in the earlier portion of Dr. Lewis' admirable essay an examination of the origin and nature of interest—the capitalists' share of the social product in an economic scheme of distribution—and in the later, an acute and extended discussion of the various causes which influence the current *rates* paid for loans of capital in the money markets of the world, two diverse subjects. For the first is presented the theory of the distinguished Austrian economist and financier Böhm-Bawerk, viz., that interest is the difference in value of present and future "goods," *i. e.*, "goods" fitted for the

immediate gratification of wants through consumption, and those in course of preparation to this end through different methods of production—termed in economic phrase "consumption goods" and "production goods." The "exchange" theory of interest, like all others which have preceded it, has been subjected to elaborate criticisms, and although not generally accepted as the final settlement of the vexed question, the origin and nature of interest, as a proximate solution of the problem it has the support of able economists, and it is stated that in the later edition of his great work on the "Positive Theory of Capital" the learned author will somewhat modify his earlier theory in consequence of these criticisms.

It appears to me that the proposition upon which Dr. Lewis has based his hypothesis of a normal rate of interest is tersely enunciated by him as follows—"the normal level of time exchange is par, plus the average rate of productiveness of capital during the interval." I interpret this proposition in an abstract sense, relating to what I shall term *economic* interest, *i. e.*, the capitalist's share of the proceeds of a period of industrial operations conducted under normal conditions, *i. e.*, the conditions which determine normal prices. To determine the *rate* of interest in its usually accepted meaning it is necessary first to fix the pecuniary value of capital—the discounted value of its future annual product—and this itself depends upon the rate of interest adopted in the calculation; and I would urge further that the income derived from a fund of capital productively invested generally includes two distinct elements, profits—the gain of the successful undertaker or speculator—which industrial competition tends to destroy, and pure interest which this economic force tends to preserve. I am unable, therefore, to accept the proposition that the normal rate of loan interest is determined with any considerable degree of exactness by what is commonly, and often vaguely, termed the productiveness of capital, though undoubtedly high rates of both interest and profits depend upon a high degree of efficiency in production, *i. e.*, the increase of utilities or goods.

I will quote here briefly from an essay by President Hadley, of Yale University, entitled "Interest and Profits": "Outside capital is attracted by profits rather than by interest. It seems to go where there are large profits rather than where there is large real interest. . . . . With any given amount of capital the lower the rate of interest the more valuable the control of industry and vice versa. . . . . The rate of interest does not depend so directly, as has been supposed, on a general market for capital, but is the result of commutation of profits in particular lines, the terms of this commutation depending upon the relative numbers of those who are willing to part with such control for the sake of avoiding the risks which it entails."

The hypothesis just presented respecting the commutation of profits suggests the inquiry, whether the State, in pursuit of revenue, and acting within its legitimate sphere, may not justly discriminate

in levying taxes between those citizens who initiate and conduct productive operations and those who attain a life of ease (and security of income) through what is essentially the status of annuitants?

From my point of view no portion of the paper under review is of greater practical value than that which prefigures the recurrence of former thriving industrial and commercial experiences, with its resultant demand for capital. Closely connected with this phase of the question is the relation between a general rise of prices of commodities and the attendant increase of rates for short-time loans of capital, such as the markets of the world are now experiencing. The corelation of these two phenomena has been noted by economists; by Marshall in his classic treatise (unfortunately still unfinished), and it forms the subject of an original and able essay (enriched with mathematical analysis), by Prof. Irving Fisher, entitled "Appreciation and Interest," published by the American Economic Association in 1896. I render a service to students of the interest question by urging upon them a careful study of this thesis.

Corroborative evidence of the favorable views just mentioned in respect to the future is to be found in the increasing variety of gainful occupations among the people of this country; the remarkably efficient extension of their industries within the field of the mechanic and liberal arts, indicated by the changing character of foreign trade, *i. e.*, by the continually increasing proportion of the exports of manufactures to the products of the soil. These facts manifestly reveal conditions of industrial progress favorable to the demand for capital, in marked contrast, for example, to the former status in the Southern States, when by far the greater portion of the social income of that section was appropriated by the landowners in the form of rent, using this term in its technical sense.

One feature of the economic changes now in progress, viz., the limitation of free competition through vast industrial combinations under single systems of administration, has greatly aroused public attention and stimulated political agitation, though I am not aware that any definite relation of this condition to the general rates of loan interest has been determined by competent investigation. It is evident, however, that so far as the element of monopoly is effective in these consolidations the tendency will be to an increase of *profits* rather than to a gain in commercial interest. Take, for example, that typical monopoly, the Standard Oil Company, the very successful exploitation of its field of operations has resulted in extraordinary dividends to its stockholders, gains that certainly could not have been attained by capitalists competing for interest in the open loan markets.

I will offer but one more suggestion. Is not the general and systematic organization of labor in this country, its success in gaining a higher wage and reduced hours of service, the repeated

and aggressive assertion of its interests, directly related to the reduced surplus earnings of capital during the past twenty years, which in popular language we call reduction in the rates of interest?

Finally, I wish to declare my appreciation of the vigor and independence of thought expressed in the essay which has given occasion for these comments, and my gratification at the author's refutation of the subtle fallacy so often lurking under the argument —"*post hoc ergo propter hoc.*"

MR. OGDEN: The Actuarial Society is indebted to Mr. Charlton T. Lewis for his interesting paper on "The Normal Rate of Interest."

His treatment of the subject covers such a vast field of speculation, theory and fact, that it leaves but little for the critic to criticise further than to agree, or differ from Mr. Lewis in his final conclusions.

Mr. Lewis very properly asserts in the beginning of his paper that "the rate of interest assumed is at least as important as the rate of mortality." I think he might have gone even farther, and asserted with equal truth, that, could we but look into the future and foresee the normal rate of interest that will be obtainable during the next fifty years with the same degree of accuracy that we can estimate the rate of mortality that will prevail during the same period, then life insurance would almost cease to be governed by the doctrine of probabilities, and the elimination of the doubtful quantity, interest, would place the science on a basis of almost absolute certainty.

In life we know that there will follow death, but with an investment worth 5 per cent. to-day, where can we find a man who can tell us what its value will be ten, twenty or thirty years hence? We can theorize all we please, but, after all, theory is but a poor source from which to obtain facts.

Mr. Lewis' definition of capital and the nature of interest is clear and precise, and the student of economics would be well repaid by a careful perusal of this particular feature of his paper.

While it is true, as Mr. Lewis says, that the normal level of local exchange is par, the normal level of time exchange is par, plus the average rate of productiveness of capital during the interval. If capital commands interest it is necessary for us to know what the average rate of productiveness is, in order to determine what rate of interest will be obtained from a given capital which is exchangeable for a given length of time. To determine the average rate of productiveness in many of the trades to-day would not be a problem of great magnitude, but to assume that the same conditions and prices will prevail one year from now is entirely another matter.

We know that whenever an actual deficiency of capital is revealed the demand for the same becomes urgent and the rate of interest will rapidly rise. We also know that when a surplus of capital appears on the market, and the demand for it remains at the

same level, the rate of interest will fall. While the fall in the rate may in a few instances lessen the inducement to lend, I very much doubt the assertion that this lessening will be great enough to bring about the withdrawal of a sufficient quantity of the then marketable capital to balance the supply and demand. Unemployed capital has no value, and, as a rule, it is only some extreme emergency which withdraws capital from the markets of the world. I may be pardoned for referring to a case in court as an apt illustration of one way to determine the value of capital. The case on trial involved the value of a horse. An expert was put upon the stand and asked to state what, in his opinion the horse was worth. After stating the value, he was asked how he determined the value of a horse. Answer, "I first find out how anxious the proposed purchaser is to buy, and then, fix the value accordingly."

The disturbing influences which upset the equilibrium between the supply and demand of capital are clearly set forth in Mr. Lewis' paper under the two grouped orders which he figuratively designates "The Wave Forces" and "The Tidal Forces."

We had a striking illustration of the working of these forces four years ago, when uncertainty as to the future financial policy of this Government became so grave that rates were forced up to a very high figure, owing to the hoarding of capital to meet what was generally looked for if certain policies were carried out by the Government, viz., a financial crash. The flurry, however, was soon over, confidence was restored, and with it came the release of a flood of idle capital, followed by a gradually decreasing rate of interest as the inevitable result. While it is true that many of the disturbing influences referred to by Mr. Lewis as "Wave Forces" have the quick, but temporary, effect of increasing the rate of interest, it seems to me that it is equally true that as soon as these disturbing influences are removed the reaction invariably sets in, and the interest rate falls proportionately.

There will always be a large demand for Government securities, no matter how low the rate may go, for, let the credit of a Government fall to pieces, and the probabilities are that the credit of the private individual living under that Government would fall with it. No safer or easier investment can be made by the trustee than a Government bond, which, as Mr. Lewis says, will acquit him of all responsibility.

In Mr. Lewis' discussion of the "Tidal" influences which seem to bear directly upon the rate of interest, he refers to a succession of tidal movements in interest, which have covered the commercial world during the nineteenth century. These periodical fluctuations of interest might form the basis for a strong argument in favor of the belief that the rate of interest would gradually increase during the next twenty-five years, because of the fact that during the last twenty-five years it had slowly but steadily decreased, and the cycle of time which regulated the rise and fall had again rolled around.

In considering these periodical fluctuations we should also consider the periodical fluctuations of prosperity and adversity. I think no one will doubt that we have lately passed through very trying and uncertain periods of the world's history. Besides war, we have had unusual financial problems to unravel. We have new countries to look after, and we will need millions of capital to do it with. We have new interests at stake, and the world recognizes our ability to take care of them. We have passed from the period of adversity to prosperity, and at this cross-road, I fear, Mr. Lewis will meet with a wide divergence of opinion. There will be many who will not agree with him in his general conclusions that in prosperous times the rate of interest advances. He says that "all experience proves that the demand for capital finds its supreme stimulus in the expectation of productiveness": that "this expectation is excited chiefly by discovery and invention, and by a desire to explore and bring under cultivation new tracts of land which will find new and profitable employment for capital"; and "that interest has always been notably higher in newly-opened countries."

The development of a new country does, as a rule, have a tendency to increase the rate of interest on the capital with which the new territory is developed, not necessarily because it is a new country and the prospects of an increased rate of productiveness are great, but largely because the capital is removed from the development of home industries and invested in an uncertain quantity, the value of which we must determine by the productiveness of the time exchange as unraveled in the future.

Should the venture prove successful the day of doubt and adversity will vanish, prosperity will loom up in the distance with such a rush of capital to advance the interests of the successful enterprise that the rate of interest will steadily decline. No better illustration can be given than the experience of those who have had occasion to loan money during the last few years on what are popularly known as "Western Farm Loans."

It is to be regretted that Mr. Lewis' appointment to discuss this subject more fully with the late David A. Wells was frustrated by Mr Wells' illness and death.

The question which should be uppermost in the minds of those whose business it is to determine what rate of interest should be used in the computation of premiums and reserves for the companies which they represent, is not so much the present average rate of interest on securities now held, but rather what rate of interest will they obtain on securities which they may have occasion to purchase hereafter.

As Mr. Macdonald says in his discussion of Mr. McCabe's paper entitled "Notes on the rate of interest for premiums and reserves in Life Companies," "another important feature which demands consideration in determining the rate of interest, and argues in favor of the adoption of a low rate of interest, is the necessity of guarding

against a decline in the rate of dividends allowable to policy-holders due to a change in the basis of valuation. It is important that dividends should, as far as possible, be on an increasing ratio from term to term as declared."

Further on Mr. Macdonald says "it appears to me that we must look for a still further lowering of the interest rate," and in the discussion of the same paper Mr. Hann says "a low rate of interest goes hand in hand with a prosperous condition of trade," all of which sentiments I most heartily approve.

Mr. Van Cise: While I am sure that we all feel indebted to Mr. Lewis for his scholarly paper in regard to rates of interest, I am compelled to differ with his conclusions. He says, in closing his paper: "On the whole, therefore, while it seems to be beyond reasonable doubt that the assumption of four per cent. as a rate of interest upon reserves is safe for at least a generation to come, a conservative provisional estimate of the permanent average yield to be expected hereafter from invested capital for many years would perhaps fix it at about five per cent."

About six or seven years ago I advised the managers of our company to adopt a three per cent. basis for new business, although the step was not actually taken until four years ago, and I have always thought that the Connecticut Mutual deserves great credit for taking the lead in that matter. In my judgment, no effort should be spared to render the business of life insurance absolutely safe; and, to that end, the rate of interest assumed in the calculation of premiums and reserves should be certainly low enough to permit the certain assurance of a margin in excess of the assumed rate for security.

Now, when the State of Massachusetts fixed four per cent. as the rate by which liabilities should be calculated, seven per cent. was the legal rate here in the East, and I do not suppose that there was a loan of any kind, unless it was Government bonds, made at less than six per cent. There was a margin of two or three per cent. for profit and safety. If that was the case then, should we not now make our calculations on new business at three per cent., for in the last year or two, for the first time, very large amounts of capital have been loaned by insurance companies, not only in the East, but in the West, on bond and mortgage, at the rate of four per cent., and there are many issues of railroad bonds bearing interest at from three and one-half to five per cent. which are selling at a premium. I think there are reasons which show that the rates in the future are not to be high. One is the large number of financial institutions that hold large amounts of loanable capital that did not exist in the periods to which Mr. Wells referred in his paper. It should be considered also that the amount of undeveloped country is growing less all the time. There is less and less capital required to develop unexplored regions. The portion of the world undeveloped is growing smaller all the time, and there will be less demand for capital on that account.

MR. MESSENGER: Since Mr. Lewis read his paper I have heard quite a number of financiers make the statement that they believed that the time had come when, as far as the rate of interest is concerned, the tide is turning. That is, that the rate has been declining for quite a number of years, until finally it has reached a point where it will again commence to rise. I am inclined to think that that is the opinion of the majority of those who have studied the subject, and am not much inclined to question it myself.

But in connection with this, there is another point to which I wish to call attention. I believe that, as Mr. Ogden stated it, as far as life insurance calculations are concerned, we must base them upon the rate to be earned in the future: that is, upon investments which it may be possible to make now and in the next few years, investments that will run for a long term. Nevertheless, if we admit (which I do not admit) that we should base those calculations upon the average rate of interest which we are actually earning from year to year on invested assets, I think, everything considered, that we have reliable facts before us that tend to show that at present we have had very little reason to think that the rate of interest is increasing. For this reason all of the companies possess large amounts of assets which were invested years ago at high rates of interest as compared with the present prevailing rates—five, six and seven per cent.

The terms of these investments are rapidly coming to an end. The funds must be reinvested; and the point to which I wish to particularly call your attention is this: Will not the loss in the rate of interest, due to the reinvestment of assets which have been earning interest at high rates, more than counterbalance any slight increase which we may be able to effect in the present prevailing rate of interest, as compared with what it was a year or so ago?

MR. W. C. WRIGHT: Mr. Lewis' general conclusion is that the average gross rate of interest throughout long periods of time is about five per cent., and that the rate to be earned is subject to very considerable fluctuations through long periods of time, running down, as he instances, below three per cent. Now, if that is true, and we have to deduct a fair allowance for the expenses which always must attach to the obtaining of interest upon invested funds, we find only such margin left as shows the none too sufficient safety of a fixed assumption such as Mr. Van Cise has recommended, and which is the lowest which the American companies have considered.

MR. WELLS: I have had occasion recently to look at the rate of interest earned by the life insurance companies whose statements are contained in the report of the Connecticut Insurance Department, exclusive of the credit to profit and loss. Taking the interest earned, less what the companies reported in the Gain and Loss Exhibit as investment expenses, I found an average rate of 4.36 per cent. upon admitted assets, at the beginning of the year. I

then examined the reports to ascertain the amounts reported as investment expenses. Of the six Connecticut companies five of them (all, save the Connecticut Mutual) reported only as investment expenses those which the law says shall be included, taxes upon real estate, repairs upon real estate, and all other expenses. One company reported only taxes and repairs. I found that seven other companies reported repairs, expenses and taxes. The other companies had reported something in addition to the sum of those items, more or less. But the actual expenses involved in the holding and handling of investments are certainly in excess of that item as reported by the companies in the Gain and Loss Exhibit. It may safely be put at a quarter of one per cent. beyond that reported, at least. In other words, I do not believe that the net earnings of the companies—the thirty-one companies reporting to the Connecticut Department last year on their admitted assets at the beginning of the year—were more than at a rate of 4.1 per cent.

Now, in regard to the matter of surplus. It is true that the earnings of the surplus go a long way toward helping out the interest rate. But at least a part of the earnings of such surplus ought to go elsewhere, in all justice, than to help out the interest earned upon the reserve. A part of that surplus represents money held for the benefit of individuals, or individual classes of policy-holders, and it is surplus that is accumulating for the benefit of individuals, not for the benefit of the company as a whole. Then, again, I think, perhaps, we are disposed to put too much stress upon the surplus which the companies hold. A great many companies hold surplus enough to offset the difference between the legal four per cent. reserve and the three per cent. reserve; but the holding of such surplus is not equivalent to putting the business upon a three per cent. basis, by a long way. A three per cent. basis involves not only the holding of present assets considerably in excess of the four per cent reserve, but it involves also a larger future premium income. If a company were to appropriate its surplus to-day, and place all its business upon a three per cent. basis, it could not increase the premiums that it is receiving upon its old business, which premiums are rather below what a company would feel obliged to charge if its business were upon a three per cent. basis.

MR. ST. JOHN: I wish to add a few words respecting that portion of my earlier remarks relating to the correlation between the rising prices of commodities and the rates of interest under short-term loans.

I do not think that the parallelism mentioned will hold good in respect of what are commonly known as "call loans," because the rates paid for money borrowed under them are subject to sudden and violent fluctuations of a narrowly local character, wholly disconnected with the market prices of what are usually termed commodities—products of industry fitted for immediate consumption. A large, but indeterminate, portion of these transactions are

conducted with a view to depressing or inflating the prices of titles to property—shares of railway, mining or so-called industrial corporations—hence the rates of interest included in this class of transactions form a disturbing element if introduced into the statistics relating to the correlation between interest rates and prices of commodities.

Mr. Macaulay: We are all interested in this question, because by our conclusions thereon we must form the policy of our companies. Apart, altogether, from the future rate of interest to be taken into account in fixing our reserves, if interest is likely to go up in the future, then certainly our companies ought to invest in securities of the shortest possible term, so as to be able to get the benefit of the higher rate of interest in the future. On the other hand, if interest is to go down, then naturally we would all advise our companies to invest in the long-term securities, so that, if interest does go down, these securities will rise to a premium. Viewed from that standpoint the question becomes one of great importance. Viewing it from the simple standpoint of supply and demand, I think that most of us will agree that during the past two years a wave of prosperity has been going over the whole world, and probably more over this continent than anywhere else, and that the demand for capital has been tremendously stimulated. I doubt if at any time in the last twenty-five years or more there has been such a demand for capital for new enterprises of all kinds as in the last couple of years. Now, what has been the result? Old investments—old securities of long standing—which have become thoroughly absorbed by the investing public, and are off the market, so to speak, have not been very greatly affected. But new securities, even if thoroughly safe, have had to be floated at higher rates of interest than would have been the case a few years ago. It is during times of prosperity that interest goes up, and when the reaction comes, and periods of depression follow, and confidence in investments ceases, then interest tumbles down. During these two years, while to a certain extent there has been a rise of the rate of interest, especially in regard to new investments, I am afraid that it has not been sufficient to justify us in thinking that that rise wi l continue when the pendulum swings to the other extreme, and when we find ourselves in the period of depression, which is just as sure to follow this period of expansion as night is to follow day. I agree with everything that Mr. Lewis says, and yet I am afraid that we must consider that in all probability, while it may not continue as rapidly in the future as in the past, yet the probability is that we will have lower rates in the future than at the present; and leaving out of account altogether, for the present, the question of what rate we should assume in fixing our reserves, I think that those companies which are wise enough to invest at this time in very long-term securities will find it to their advantage in the years to come.

SHOULD A THREE PER CENT. VALUATION BE MADE COMPULSORY?

WALTER S. NICHOLS.

PAGE 172, VOL. VI.

MR. IRELAND: At the last two meetings of the Actuarial Society there have been presented five original papers having more or less direct reference to the question of rates of interest, and comments by four members on these papers, thus giving articles by nine members of the Society, and the diversity of conclusions arrived at illustrates in a very striking way the uncertainties that figure so prominently in a consideration of the subject.

The papers of Mr. St. John and Mr. Lewis dealt largely with the philosophical and scholarly aspect of the subject; the former did not push his paper to any definite conclusion, except that he considered that the opinions expressed in a certain collection of letters written to the President of the Equitable Life Assurance Society were based on too small a foundation to make them valuable in determining far-reaching conclusions. Mr. Lewis thinks that 1897 was the turn of the tide in interest rates and showed minimum results, and that for a generation to come four per cent. may be assumed as a safe figure.

Mr. Bradshaw's paper was largely devoted to showing the effects of changing from a four and one-half to a three and one-half per cent. basis, but he expresses the opinion that such a change on *all* business, new and old, in seven years (as was proposed in Canada) would be too sudden, and that the change should be made more gradually. It may be noted that the law, as passed, was altered in this respect, making the rate on old business four and one-half per cent. for ten years to come, four per cent. for five years after that, and then three and one-half per cent.

Mr. McCabe's argument was in favor of lowering the assumed rate of interest, and hints at three per cent. as a proper rate to be used.

Mr. Messenger, in commenting on Mr. St. John's paper, expressed his "belief that we can look forward to interest rates becoming gradually lower and more stable." He also expresses the idea "that a gradual decline in interest rates usually accompanies an increase throughout civilized countries in the extent and degree of what in evolution is called homogeneity," an idea that seems to the present writer to be well founded and to have an important bearing on the subject.

In Mr. Macdonald's comments on Mr. McCabe's paper, he says that "everything indicates that even lower rates will obtain than at present," and he thinks that on new contracts the assumed rate should not exceed three and one-half per cent. and possibly should be still lower.

Mr. Macaulay, commenting on the same paper, puts forth the idea that the legal standard of valuation should only be what is required for solvency, and that companies may well make for

themselves a higher standard. He does not think that there is any justification for lowering the rate below four per cent. in making a net valuation as a basis of solvency. We had, at least in the United States, become so accustomed to thinking of the Actuaries' four per cent. standard (which is practically what the life companies have been held up to by the laws) as fulfilling the extremest needs of conservatism, that until these latter years it has not been deemed necessary to have any stricter or different standard for the use of the companies in their own affairs.

In Mr. Hann's comments on the same paper he gives it as his judgment that the question of the rates of interest to be used in making premiums and reserves "should be regulated by healthy competition, rather than by paternal government."

The subject assigned to the present writer for consideration was the paper of Mr. Nichols on the question, "Should a three per cent. valuation be made compulsory?" Mr. Nichols attaches some little weight to the opinions of the correspondents of President Alexander, already alluded to, but also recognizes the fact that what most strongly impresses us is what we have personally observed since 1870, which period covers but a single fluctuation of the historic rate. The two per cent. of Holland a hundred years ago, he says, was the inferior limit, and he asks whether we, too, may not be near *our* inferior limit. He hints in picturesque fashion at the radical changes in motive power, in machinery, in devices and inventions of all kinds that have taken place within the memory of those now living, and indicates some of the directions in which equally radical and surprising changes may take place in the future, all of which must be recognized as possible facts in the adjustment of commercial questions like the one we are considering. He thinks that any change of standard should be so shaped as not to injure vested interests, unless the danger is imminent.

On one point there will probably be no dissent by any of these or other writers, viz., that even three per cent. would not now leave as wide a margin between rates actually earned and the rate assumed, as there was when four and one-half or four per cent. was assumed as the standard by different ones of the United States. There is nothing in the published figures of actual earnings by American Companies to show that existing business may not safely be carried along on a four per cent. basis; but when first-class investments in well-secured bonds net not more than three and five-eighths to three and seven-eighths per cent., it is pretty evident that a proper conservatism will not allow an indefinite extension of business on a four per cent. basis. Money loaned on mortgage, it is true, may still be very safely secured on a four and one-half or five per cent. basis, and in some cases at five and one-half per cent.

It is also agreed, on all sides, that possible variations in future *interest* rates are much greater, and questions concerning them much more perplexing, than in the case of future *mortality* rates.

And not only is it impossible to tell what rates will be current 25 or 50 years hence, but it is with difficulty that we get at anything like accurate and reliable information as to the rate of interest earnings at the present time. The Connecticut Insurance Report for the present year gives a table of ratios of net interest to mean amount of admitted assets for each of the past ten years, for all the companies included (varying from 24 to 31 in number). The figures are :

1890 = 5.19%	1894 = 4.81%	1897 = 4.74%
1891 = 5.00%	1895 = 4.46%	1898 = 4.32%
1892 = 4.70%	1896 = 4.19%	1899 = 4.27%
1893 = 4.57%		

But for the last three of these years the figures are based on the Gain and Loss exhibits, and in these years, as perhaps in the others, the guesswork and other arbitrary methods of apportioning expenses, of including changes in market values, and so on, make the results worthy of being classed as rough approximations only.

A table published in the Insurance Year Book, in 1899, gives "the rate of interest earned on mean invested funds" for 27 life companies, with these resulting averages for five-year periods, viz.:

1879 to 1883 (inclusive)	= 5.57%
1884 to 1888 "	= 5.42%
1889 to 1893 "	= 5.20%
1894 to 1898 "	= 4.91%

which makes a showing that, if accepted, would indicate that rates now obtained are safely above present legal requirements, but are approaching them at a pace that calls for serious consideration.

With all these things in view, legislation fixing the standard rate for new business at three and one-half per cent. cannot be considered excessive in its caution, but it does not seem that three per cent. need be insisted on. Some companies have already adopted this latter rate, and it is quite possible that others may do so of their own accord. The adopting of such a rate as a legal standard by one State or government might seriously embarrass those companies that were subject to its laws, by putting them into competition with other companies not so restricted; and beyond the safety line it is not the province of governments to dictate. They should be cautious, and not oppressive, especially when legislating with reference to contracts already made.

Mr. Wells: The papers written by Mr. Nichols are always interesting and suggestive, and the one under discussion is particularly so. The major part of the paper is taken up with a discussion of the probability of a further reduction in the rate of interest. Mr. Nichols leaves us somewhat in question as to his own views in the matter. He says : "To me the problem of the future rate seems as insoluble as in the days of De Morgan"; and in this feeling I suspect that most of us sympathize. His references to the destruction

of fixed capital and the absorption of new by changed processes and improvements are exceedingly interesting and suggestive, and their force cannot but impress itself upon all of us. At the same time Mr. Nichols apparently believes, and in this, too, probably the majority of us agree with him, "that a lower rate of interest in this country should be anticipated in the future than has ruled in the past." He apparently believes it probable that we have not yet touched the lower limit of the interest rate; that the pendulum has not yet reached the lowest point of its arc, even though "We have simply been living while the pendulum was swinging downward."

Assuming, then, that while the interest rate of the future is an unknown quantity, we should at least be prepared for a still further decline, the question remains which forms the title of Mr. Nichols' paper, "Should a Three per cent. Valuation be made Compulsory?" Mr. Nichols does not answer the question, and, indeed, leaves us even more in doubt as to his own opinion than he does as regards his opinion in the matter of the interest rate. He says: "If it was a question de novo of computing premiums and reserves, I have little doubt that a 3 per cent. standard would be generally adopted," and such, apparently, would have been his own preference under such conditions; but as to whether under existing conditions he would advocate a change in the legal standard of valuation, I am unable to determine from his paper.

Mr. Nichols makes a bare reference to the distinction between contracts already entered into and future contracts, a distinction which I think should be made very broad. No change in legal requirements could change the terms and conditions of existing contracts. It could change neither the contract premium, the interest earned, nor the amount or date of payment by the company. In short, it could change neither the actual resources nor the actual liabilities of the company under the contract. The only thing that it could affect would be the payment of dividends. Directly or indirectly the law might probably step in to prevent payment of dividends to either policyholders or stockholders until a new and higher standard of reserve was reached and maintained. Is such a course wise or best? I doubt it. The savings from a favorable mortality and from the legal reserve on lapsed and surrendered policies will suffice, as regards existing policies, to offset any probable shrinkage of interest rates below 4 per cent. in case of most companies, or they can be trusted to retain such surplus as may be necessary to make their future condition secure. There may be companies whose optimistic views regarding future interest rates, and whose greed for business have led them into offering contracts in the past, and to a course in the present which may affect them seriously in the future, should interest rates continue to fall. There may be companies whose officials need to have their backbones stiffened by a legal enactment to compel them to do what they themselves believe to be the correct and safe thing, fearing lest they be outbid by some more sanguine or more reck-

less competitor. I am not prepared to say that I think the danger so serious as to make it desirable to take the control of the companies out of their hands.

As regards new business the case is different. A new rule of valuation does not interfere with any existing obligation, right, custom, or representation heretofore made. It can compel a sufficient rate of premium as well as a sufficient reserve. We cannot rely upon margins of profit from other sources to make up a deficiency in interest earnings on new business. It should be self-sustaining. It should not be dependent upon a present surplus. Competition has driven most of the companies to guaranteeing cash values which will leave practically no gain from surrendered policies. A gain from profit and loss cannot be assumed, and an appreciation in market values accompanying the falling of the interest rate occurs only in case of securities running for fixed terms and at fixed rates of interest. With the extension of the field, and the writing of annuities and of substandard risks, it is by no means certain that we can count upon as large a gain from a favorable mortality, as compared with standard tables, as heretofore, though this is probably the most reliable item of surplus. It seems to me, therefore, that it is the part of wisdom to assume a lower interest rate in all computations relating to such business. The only possible reason or justification for a legal valuation made by State officials upon a fixed basis is a purpose to thus make secure the ability of the companies to meet their contracts at maturity. A valuation which leaves such ability in doubt is a deception and a snare. It should be made upon a basis which makes certain, so far as anything human can be certain, the solvency of the companies. It should not be made a test of solvency in such wise as to compel the placing of a company in the hands of a receiver if it could not meet the test. The company might well be forbidden to solicit new business, but should be given the benefit of every doubt as regards its ability to meet its contracts before being driven into insolvency.

The adoption of a more conservative basis by a company, whether of its own motion or under compulsion, must not be deferred until the emergency arises. Should a company ever find itself in such a position that it could not carry out its old contracts upon the present basis, it would be too late to place such contracts upon a more conservative basis. There would be no resource from which to provide the necessary funds. The only possible salvation for the company would be the writing of new business at such a rate of premium and upon such a basis that it should be not only self-sustaining but furnish a surplus which could be appropriated to the maintaining of the earlier business; and this is a course which would probably be impracticable, and a necessity to be avoided.

Mr. E. P. Marshall: The rate of interest and its downward tendency in this country have naturally attracted much attention and

discussion at the last two meetings of the Society. The matter is of vital importance to the insurance interests of the country, the interest rate being one of the foundation stones upon which the whole structure of life insurance contracts rests.

When the new 2 per cent. Government bonds are held at a premium, and while we have been presented the spectacle of 4 per cent. U. S. bonds selling at a premium of 34 per cent. to 38 per cent., as Mr. Nichols in his paper well says, "it would be idle to dispute the indications now before us point unmistakably toward a falling rate of interest."

The truth of this statement none can question. The difficulty experienced in investing funds in first-class, unquestioned securities (the only sort in which a life insurance company may safely invest) is becoming every day greater. This class of securities is scarce and hard to obtain. First mortgages on real estate, in the opinion of many a form of security that cannot be equaled or surpassed, are hardly to be obtained in many of our Middle and Western States at a greater rate than 6 per cent., and many such loans have been placed at 5 per cent. and 5½ per cent. and some large lines at even lower rates. According to the last report of the Insurance Department of Connecticut the life insurance companies doing business in that State experienced a decline of net interest on the mean amount of admitted assets for the ten years ending December 31st, 1899, from 5.19 to 4.27 per cent., a fall during the decade of nearly 1.00 per cent. This decline, almost startling as it is to the managers of our American companies, has been gradually going on for several years past. Notwithstanding all this is true and the problem of safe investments one that will command the highest ability to solve, I am inclined to the opinion implied by Mr. Nichols and expressed by Mr. Lewis, in the summation of his paper on the Normal Rate of Interest:—"While it seems to be beyond reasonable doubt that the assumption of 4 per cent. as a rate of interest upon reserves is safe for at least a generation to come, a conservative provisional estimate of the permanent average yield to be expected hereafter from invested capital for many years to come would perhaps fix it at about 5 per cent."

Should a 3 per cent. valuation as a standard of solvency be made compulsory by law it would undoubtedly result disastrously to a number of new and struggling companies. I take it that it is the aim and purpose of the State to foster the growth of life insurance companies and kindred institutions that will be beneficial to the people. Therefore, to make the standard of solvency so severe, artificially increasing liabilities in such a manner, would be certainly unwise and hardly contemplated by the spirit of the law, at least. Such a change, as has been remarked, would also unmistakably affect the dividends to policy-holder and the cost of business. Might it not even furnish an excuse for further extravagance on the part of managers in securing new business, owing to the increase in the annual premium necessarily required upon the new

basis, if adopted? Granted that the indications at present point to a still lower rate of interest; upon the other hand, do not industrial enterprise, the opening up of new territories and large demands for capital in developing this new territory all indicate the destruction of old and absorption of new capital? As Mr. Nichols so aptly puts it, "this march of invention and improvement, which is wiping out millions of fixed capital and calling for new to take its place, makes such demands for capital that the pendulum will soon be on its upward swing."

The younger companies, as they gather strength with age, naturally will desire and voluntarily set aside such extra reserves as will make them safe beyond the bare requirements of the law, and I think with Mr. Nichols, the interest question can be safely left to the managers who are responsible for the success and know the needs of their companies. What is the safe rate of interest to assume must, however, be largely a matter of opinion and good judgment. What might be easy for an old company with an accumulated surplus might be ruinous for the young company. Any changes should only be made after the most ample notice.

If the expense account were more closely watched and greater energy expended in solving the problem of reducing the cost of obtaining new business, a higher standard of reserve might not seem so necessary.

Personally, I have long been of the opinion that the American table of mortality with 4 per cent. interest is amply safe as a standard of solvency for all American companies. Any other table or rate of interest calling for a higher standard than 4 per cent., if forced upon them, would in all probability work a serious hardship upon the new and struggling companies, if it did not eventually result in their ruin.

Mr. Van Cise: Mr. President, I might say a few words in regard to this question of accumulated surplus, which was mentioned as a reason for not needing a higher basis of reserve. Now, as the actuary of the company that has the largest surplus, I would say that about nine-tenths of that surplus is specifically promised to policy-holders at certain periods, and, as a matter of contract and good faith, must be paid. It cannot be relied upon to make up for any deficiency in interest. I would also say that we must all the time make a distinction here between new business and old business. I do not suppose that any of us think that a higher standard should be fixed by law, even if it could be legally done, for old business, but only for new business; and therefore I think that any margin of surplus that the companies have, over and above contract requirements, should be held for any emergency that may arise in connection with the old business.

Mr. Wells: I want to emphasize once more that we should make a distinction, as Mr. Van Cise has said, between the new and the old business. When we hear it said, as we do very often, that the requirement of a higher standard of valuation, or a lower

interest rate, would prove destructive to some companies, I cannot help thinking that the person who says it has in mind the requirement of a higher basis of valuation upon the old business. I cannot see why the requirement of a lower interest rate and a more conservative valuation upon new business should work any disadvantage to new companies, as compared with old companies, or to young companies, as compared with older companies, or to small companies, as compared with large companies. And I do not see wherein there can be anything but safety in the requirement of a more conservative valuation upon new business. If we are going to attempt to apply it to old business that is quite another matter.

MR. DE BOER: Did you intimate in your remarks that you favor the adoption of three per cent. for valuation purposes by the State Departments?

MR. WELLS: I do not want to lay down a law on this subject. I have not advocated that, as many people have, as a cure for all sorts of evils. Nevertheless, I do think that a State might fairly, reasonably and properly require a three per cent. reserve upon new business from a date long enough in the future to permit the companies to make their computations and their contracts in preparation for it. I would not make that basis a standard of solvency, in a sense that would throw a company into the hands of a receiver, if impaired by that standard. I think you must leave a broad margin between what should be required of a company as a condition of continuance and of allowing the people to put their faith in it as an active insurance company, and what should be required before we put a company into the hands of a receiver and wind it up. We should give the company the benefit of every doubt, in the one case, and we should give the public the benefit of the doubt, in the other case. As to new business, the company should be required to come up to a standard that should be absolutely safe, and as to putting it into the hands of a receiver, there should not be any question that the company was unable to continue to fulfill its contracts.

MR. NICHOLS: I had not intended that my paper should be presented as an original communication to the Society, but rather as a review of a previous paper, but on account of its peculiar character I found that it developed into quite a new and original paper. When it was prepared the general tendency of thought on this question among life insurance men seemed to be in one direction. That direction was largely indicated, I think, by a pamphlet embodying the opinions of men who were supposed to be in close touch with financial questions all over the country, and whose opinion seemed to voice but one sentiment, namely, a permanent fall in the rate of interest. It seemed to me that there was danger in looking simply on that side of the question. There was another reason also for the line of argument contained in my paper. This was the attitude of legislatures and of possible legislative interference on this question. I have no confidence in our legislatures; even if they are

well disposed, there is no guarantee of their ability at any given time to take the right and proper action. The country contains more than forty different States. In the majority of those States every important life insurance company is represented, and there is an ever-present danger in any one of the legislatures of those States commencing a course of action without the slightest intelligent idea of the subject on which they propose to legislate. Suppose any important State should undertake to pass a law that would force the companies, if not directly, at any rate indirectly, and to place the old business as well as new upon a three per cent. basis. No intelligent insurance man would agree with such a proposition, but the question is whether the companies can depend upon themselves to always have influence enough to educate these legislators to see the distinction between simply putting the new business and that of putting all the business upon a more stringent basis of valuation.

It seems to me that the position of the legislatures in these matters should be to follow, rather than to lead, the way on such questions as that, leaving the life companies themselves a large amount of responsibility. The latter are the experts; they are the parties that are best capable of judging these things, and legislation, after all, making a matter of this kind compulsory, does not take away the responsibility from the officers. The leaving of even a four per cent. reserve does not take away the responsibility, it seems to me, from the managers of any life insurance company, if they feel that that possibly is not adequate to make their position safe, and I question whether you are not in danger of jeopardizing the interests of life insurance more by advocating even a three per cent. reserve on new business than by letting things alone, unless it can be proved to be necessary, almost, for the protection of the business itself.

Abstract from the Minutes of the Annual Meeting of the Actuarial Society of America, held in New York, N. Y., on Thursday and Friday, May 17th and 18th, 1900.

FIRST SESSION.

New York Life Insurance Company's Building,
Thursday, May 17th, 1900, 11 A. M.

The meeting was called to order by the President, Mr. Thomas B. Macaulay, who, after the roll call, addressed the Society. (See page 310.)

MEMBERS PRESENT.

Barker,	Loveland,	St. John,
Brinkerhoff,	Lunger,	Salter,
Craig,	McCabe,	Sartelle,
De Boer,	Macaulay,	Smith,
Fackler,	Macdonald,	Starr,
Gore,	E. P. Marshall,	Tatlock,
Hall,	Messenger,	Torrey,
Hann,	Nichols,	Van Cise,
Holcombe,	Ogden,	Welch,
Huntington,	Peiler,	Wells,
Ireland,	Pierson,	Woodward,
Lee,	Plumley,	P. T. Wright,
Lewis,		W. C. Wright.

ASSOCIATES ATTENDING.

Hall,	Hutcheson,
Hitchins,	Linzmeyer.

The minutes of the meeting of October 19th and 20th, 1899, were approved as printed.

A summary of the business transacted by the Council was presented and approved.

The Treasurer presented his annual report, which, on motion of Mr. Woodward, seconded by Mr. Gore, was duly adopted.

The Secretary then read the following communication :

Office of
David Parks Fackler,
Consulting Actuary.

New York, May 16th, 1900.

To the Council of the Actuarial Society:

The Society is now beginning to have an appreciable number of Associates, among whom are men of considerable ability and promise. The Constitution of the Society debars them from actual participation in the meetings, but it is understood, I believe, that if one of them presents

a paper of merit through one of the Members of the Society, it will receive consideration and may be published in the Society's Papers. As it is desirable to encourage these future members to begin independent investigations and studies for the general benefit of the profession, I beg leave to offer a prize of one hundred and fifty dollars, to be known as the Associates' Triennial Prize, for the best paper on any proper actuarial topic that may be presented within the next three years by any one who is now or shall become an Associate of the Society within that period; the general conditions under which papers shall be offered to be determined by a committee of three members of the Council at this time, and the final award to be made by a committee appointed by the Council, and consisting of three or more members of the Society.

It is suggested that one of the conditions should be that early presentation will be one element of merit, so that if two papers appear to be of equal intrinsic merit, the award shall go to the one first presented.

The word "triennial" is used in the title of the prize, because it is intended to continue the offer for succeeding terms, if this offer meets with favor and is found to develop laudable effort on the part of the Associates. Indeed, if the latter is found to be the case, it is probable that the Society, either collectively or through individuals, will take care that the offer is continued indefinitely.

Very respectfully,

D. P. FACKLER.

Mr. Ireland, seconded by Mr. Hall, moved that the offer of Mr. Fackler be accepted, with many thanks on the part of the Society, and with special recognition of Mr. Fackler's continued interest in this direction; and furthermore, that a committee of four members of the Council be appointed by the President for the purpose of formulating rules for the competition for the offered prize. Motion carried.

The Society then proceeded to the election of officers for the ensuing year and of two members of the Council to fill the vacancies caused by the expiration of the terms of service of Messrs. E. P. Marshall and J. G. Van Cise.

The following were duly elected: President, Thomas B. Macaulay; 1st Vice-President, Oscar B. Ireland; 2d Vice-President, Israel C. Pierson; Secretary, John Tatlock, Jr.; Treasurer, John B. Lunger. Members of Council—John K. Gore and Archibald A. Welch.

On motion of Mr. Ireland, it was voted to send a message to Ex-President Bloomfield J. Miller, expressing sympathy with him in his illness and best wishes for a speedy recovery therefrom.

At 1 P. M. the Society adjourned until 2.30 P. M.

During the recess the members of the Society were entertained at lunch by the officers of the New York Life Insurance Co.

SECOND SESSION.

THURSDAY, May 17th, 1900, 2.30 P. M.

New papers were presented and read. (See pages 261-309.)

Mr. Pierson spoke of the plans for the Third International Congress of Actuaries, to meet in Paris in June, and announced that

the Council had named the following gentlemen as the delegates of the Society: President Macaulay and Messrs. McClintock, Pierson, Weeks, Lewis and Hann.

The Society then proceeded to the discussion of papers read at previous meetings. (See pages 322-346.)

The Society adjourned at 5.30 P. M.

THURSDAY, May 17th, 1900, 7.30 P. M.

The Society met together, for the annual dinner, at the Metropolitan Club. President Macaulay presided.

Speeches were made by Mr. Lewis, Mr. De Boer, Mr. Nichols and others.

THIRD SESSION.

FRIDAY, May 18th, 1900, 10.15 A. M.

President Macaulay in the chair.

The discussion of papers was continued.

The President announced that he appointed, as the committee to formulate the rules for the conduct of the competition for the Associates' Triennial Prize, the following gentlemen: Messrs. Fackler, Nichols, Pierson and Tatlock.

The following resolution, offered by Mr. Ireland, was unanimously adopted:

"*Resolved*, That the thanks of the Society, and an expression of hearty appreciation, be conveyed to the officers of the New York Life Insurance Co. in recognition of their hospitality in providing accommodations for the present meeting of the Actuarial Society, and in ministering in so many ways to the comfort and convenience of its members."

On motion of Mr. Pierson, seconded by Mr. Barker, it was voted to extend an invitation to the Permanent Committee of the International Congress of Actuaries to hold the next meeting in New York in September, 1903; also voted that the following communication, embodying this invitation, be presented at the meeting of the Congress in Paris, in June, 1900, and be read in English, French and German:

To the President and Members of the Third International Congress of Actuaries, assembling in Paris, June 25th-30th, 1900:

The Actuarial Society of America, convened in annual meeting on this 18th day of May, 1900, sends cordial greetings and the expression of its best wishes that the meetings and deliberations of the Congress may be fraught with interest to all its members and great benefit to the actuarial profession.

In the name of the Actuarial Society of America, the subscribers hereby extend a cordial invitation to the officers and members of the Congress to meet in the City of New York in September, 1903. On behalf of the Society and of others interested in the work of the Congress, the subscribers assure to all the members thereof a hearty wel-

come to the shores of America, and the cordial extension of all coöperation and facilities necessary and proper for the attainment of the objects of the Congress.

It is earnestly hoped that it may be the pleasure of the Congress to accept this invitation, and that it may be the privilege of the Actuarial Society of America to receive in their own land as guests those whose hospitality many of its members have heretofore enjoyed.

With the assurance of the highest respect,

THOMAS B. MACAULAY,
President.

JOHN TATLOCK, JR.,
Secretary.

The members of the Society were again invited to take lunch with the officers of the New York Life Insurance Co., and at 12.30 P. M. the Society adjourned.

JOHN TATLOCK, JR.,
Secretary,
32 Nassau Street,
New York.

Constitution and By-Laws of the Actuarial Society of America, as Amended October 19th, 1899.

Constitution.

Art. I.—This organization shall be called "The Actuarial Society of America."

Art. II.—*Object.*

The Actuarial Society of America has for its object the promotion of actuarial science by personal intercourse, presentation of appropriate papers, discussion, and such other methods as may be found desirable.

Art. III.—*Membership.*

The membership of the society shall consist of the subscribers and of such other persons, connected with actuarial pursuits, as shall be duly elected or admitted.

Art. IV.—*Officers.*

The officers shall consist of a President, a first and a second Vice-President, a Secretary and a Treasurer.

Art. V.—*Council.*

The officers, with ex-presidents and six other members shall constitute the Council. At the first election of members of Council, after the adoption of this article, two members shall be elected to serve for three years, two for two years and two for one year; and annually thereafter two members shall be elected for three years. No retiring member of the Council shall be eligible for re-election at the same meeting.

Art. VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices. The duties of the Council shall be in general to manage the affairs of the Society in the intervals between its meetings, to call meetings and to make arrangements for the same, to decide upon papers offered for reading and discussion, and to pass upon candidates to be presented to the Society for election.

Art. VII.—*Election of Officers and Council.*

The officers shall be elected by ballot at the annual meeting for the term of one year. The President and Vice-Presidents shall

not be eligible for the same office for more than two consecutive years.

ART. VIII.—*Meetings.*

The annual meetings of the Society shall be held on the first Thursday after the fourteenth day of May in each year. Other meetings may be called by the Council from time to time, and shall be called by the President at any time upon the written request of ten members; twenty days' notice of such meetings shall be given by the Secretary.

ART. IX.—*Associates.*

Any one twenty-one years of age, pursuing actuarial studies and looking forward to future membership, may, on the conditions hereinafter stated, be enrolled as an associate, and in such capacity may be present, without participation, at the meetings of the Society, and receive its publications. The conditions are, that he shall be nominated to the Council by at least two members of the Society; that notice of such nomination shall be given by the Secretary to each member of the Society; that, not less than twenty days after such notice has been given, his nomination shall be approved by the Council, with not more than one negative vote, and that he shall subsequently pay such examination fee and pass such preliminary examination as may be prescribed by the Council. When the Council finds that a candidate has passed an equivalent examination required by another recognized society of actuaries it may waive the examination hereby required.

ART. X.—*Admission to Membership.*

Any associate, who has been such for at least one year, and is twenty-five years of age, may apply to the Council for membership. Every member of the Council shall be notified confidentially of such application at least twenty days before the meeting at which it shall be acted upon. If his application be approved by the Council he shall become a member on paying such examination fee and passing such final examination as shall be prescribed by the Council. Otherwise, no one shall be admitted as a member unless by the unanimous recommendation of the Council, followed by a unanimous ballot of the Society.

ART. XI.—*Quorum.*

Four members of the Council shall constitute a quorum. Ten members of the Society at any duly called meeting shall constitute a quorum.

ART. XII. No resolution expressive of opinion shall be entertained at any meeting.

ART. XIII.—*Amendments.*

This Constitution may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of such proposed amendment shall have been sent to each member by the Secretary.

BY-LAWS.

ART. I.—*Order of Business.*

At the meeting of the Society the following order of business shall be observed, unless the members vote otherwise for the time being :

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. The minutes of the last meeting read and passed.
5. Summary of the business transacted by the Council since the last meeting of the Society presented for its information and approval.
6. Candidates for membership considered and voted upon.
7. Reports of Committees and Officers.
8. At annual meeting, election of Officers and Council.
9. Unfinished business ; new business.
10. Reading of papers prepared for presentation to the Society.
11. Discussion of papers presented.
12. Reading of a summary of the business just transacted at the meeting for correction if necessary.
13. The Council meetings shall proceed in this order as far as it may apply to them.

ART. II.—Meetings of the Council shall be called whenever the President or four members of the Council so request, but not without sending notice to each member of the Council ten or more days before the time appointed ; such notice shall state the subjects intended to be brought before the meeting, and should

other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ART. III.—*Duties of Officers.*

The President, or, in his absence, the next Vice-President in order, shall preside at meetings of the Society and of the Council. At the Society meetings he shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The duties of the Secretary shall consist in keeping a full and accurate record of the proceedings at the meetings of the Society and of the Council, sending out calls for the said meetings, and, with the approval of the President and Council, carrying on the correspondence of the Society.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Society or the Council; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a Committee of the Council.

ART. IV.—*Dues.*

Each member shall pay ten dollars, and each associate five dollars, upon entrance, and a like amount at each annual meeting thereafter, except in the case of members not residing on this continent, who shall pay five dollars at the times stated.

It shall be the duty of the Treasurer to notify by mail any member or associate whose dues may be six months in arrear, and to accompany such notice by a copy of this article. If such member or associate shall fail to pay his dues within three months from the date of mailing such notice, the Treasurer shall report the delinquent to the Secretary, by whom his name shall be stricken from the rolls, and he shall thereupon cease to be a member or associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ART. V.—*Amendments.*

These By-Laws may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of the proposed amendment shall have been sent to each member by the Secretary.

Information Regarding Admission

to the

Actuarial Society of America.

The Constitution of the Actuarial Society states that its object is "the promotion of actuarial science by personal intercourse, presentation of appropriate papers, discussion and such other methods as may be found desirable."

There are two grades in the Society, viz. : Members and Associates, and, practically speaking, no one can become a Member without having passed through the grade of Associate. Anyone, 21 years of age, who has been pursuing actuarial studies with the intention of entering the actuarial profession and is favorably known to two Members of the Society, may make application to the Council, and, if his application is approved by the Council and he can then pass the examination prescribed by the Council, he will be admitted as an Associate.

The examination for admission as an Associate has the following scope :

1. Arithmetic, Algebra and Plane Geometry, including the theory and use of Logarithms.

2. The Principles of Double Entry Bookkeeping ; the Elements of Compound Interest, including Annuities Certain.

3. The Doctrine of Probabilities ; the Elements of Interpolation.

4. The Application of Mathematics to Life Contingencies, including the Theory and Use of Commutation Tables, the Computation of Premiums for Term, Ordinary and Limited Payment Life and Endowment Policies, and also for Policies with return of Premium ; and the different modes of computing Reserves ; all the above for single and for joint lives.

5. Practical Examples in all the foregoing subjects.

6. General Nature of Life Insurance Contracts ; the outlines of the History of Life Insurance, and the source and character of the principal Mortality Tables.

Any Associate, who has been such for at least one year and is 25 years of age, may apply to the Council for Membership, and if

his application is approved and he passes a further examination, he will then become a Member.

The examination for admission as a Member has the following range of subjects :

1. Methods of Constructing and Graduating Mortality Tables and the use of the Formulas of Gompertz and Makeham.

2. Methods of Loading Premiums to provide for Expenses and Contingencies.

3. Valuation of the Liabilities and Assets of Life Insurance Companies.

4. The Assessment of Expenses and the Distribution of Surplus.

5. Practical Treatment of cases of Alteration or Surrender of Life Insurance Contracts.

6. Application of the Calculus of Finite Differences and of the Differential and Integral Calculus to Life Contingencies.

7. Laws of the United States and Canada relating to Life Insurance.

8. Insurance of Under-Average Lives and Extra Premiums for Special Hazards.

The Council have directed that examinations shall take place on the first Thursday in May in each year, and the custom has been to hold them in several places and in such localities as will be convenient to two or more candidates. Examinations thus far have been exclusively in writing.

An application for admission to examination must be submitted to the Council, which cannot act thereon until at least twenty days after the Secretary has given notice of such candidacy to all the Members of the Society ; and, as the Council meets only at long intervals, any person who wishes to apply for admission to the examination should do so some months ahead of the time for the examination, if possible ; otherwise it may not be practicable to give the prescribed notice to the Members twenty days before some Council meeting preceding the examination day, and his nomination may, therefore, have to be held over until after the examination, so that he would have to wait until next year.

The fee for admission to examination is ten dollars. The annual dues of Associates are five dollars, and of Members ten dollars (except for Members not residing on this continent, who pay five dollars only).

If an approved candidate for admission to examination as Associate is found to have passed an equivalent examination required by another recognized society of actuaries, the Council may waive the examination which is prescribed, and in such case no examination fee will be required, although the regular form of application must be made.

Applications for admission should be made on the Society's blank form, which may be obtained by enclosing a stamped and directed envelope to the Secretary of the Society.

JOHN TATLOCK, JR.,
Secretary,
32 Nassau Street,
New York.

Examinations of the Society, Held on June 7, 1900.

Examination Committee: D. P. Fackler, Walter S. Nichols, John K. Gore.

Examination for Admission as Associate.

1. $x^2+y^2=7+xy$, $x^3+y^3=6xy-1$. Find the values of x and y.

2. In the triangle A B C the angle C is bisected by a line which cuts A B at D. Show that C D^2=A C. B C.—A D. B D.

3. When the nominal rate of interest is 4 per cent, what is the effective rate of interest when convertible quarterly?

4. $x^2+y^2+x+y=a$, $xy=b$. Find x and y.

5. A lottery contains 30 tickets numbered from one to thirty
   (*a*) What is the chance that the first ticket drawn will be a multiple of 5 or 11?
   (*b*) What is the chance that it will be a multiple of 3 or 5?
   (*c*) Explain the difference between these two cases.

6. A can hit a target 4 times out of 5, B, 3 times out of 4 and C, 2 times out of 3. They all fire at once, and two balls strike the target. What is the probability that C is the one who missed?

7. Find the probability that at least one of the lives (x), (y) and (z), will fail between the nth and $(n+m)$th year.

8. Assuming interest at 5 per cent., the present value of an annuity of \$1, payable at the end of the year on a given life, is found to be \$11.66. What is the present value of an insurance of \$1 on the same life?

9. An annuity is payable once in four months each year, and the next installment is just due. What is the value of the annuity on the understanding (1st) that no proportional part is payable after death; (2d) that a proportional part is payable after death; *i. e.*, if death occurs one month after a payment was due one-fourth of an installment will be paid to the heirs?

10. By a provision in a will a chair in a university is endowed with a payment of \$5,000 annually during the life of the present incumbent, aged 60, and to be continued during the life of his successor. It is desired to compound by the payment of a sum down. The age of the successor at the death of the incumbent is assumed to be 40. What sum should be paid the university for a release from the obligation, assuming one payment has just been made?

11. At age 33 the net annual premiums for endowment insurance are as follows, per \$1,000: Twenty-year, \$38.41; twenty-five-

year, $30.04; thirty-year, $24.98. Find by interpolation the premium for a policy maturing at death, or age 55.

12. Prove that $\log.\ (1+x)^{1+x}(1-x)^{1-x}=2\ m\ (\frac{x^2}{1.2}+\frac{x^4}{3.4}+\frac{x^6}{5.6}+\cdots)$ Where m is the modulus of the system.

13. Deduce the formula for joint life insurance

$$l_{xy}=v\,(1+a_{xy})-a_{xy}.$$

14. An estate is left to two sons—A, aged 20, and B, aged 30—for life, the income to be divided equally between them during their joint lives, and then to go to the survivor. They agree to release their interest for $10,000; how should that amount be divided between them?

15. By a certain mortality table the value of an annuity of $100, first payment immediate, on a life aged 35 is $1,640.998, and the single premium for insurance of $1,000 for life is $293.35. Find the rate of interest used.

16. Give the formula for the thirty-payment annual premium, at age 45, for a policy under which the insurance shall begin at $1,000 the first year and increase thereafter by $40 yearly for fifteen years, until it becomes $1,600, and then diminish at the same rate to the end of life.

17. Give the formula for the gross annual premium for a twenty-year simple endowment policy which will bear 10 per cent. expenses and provide that all premiums paid shall be returned in case death occurs after ten years—and before maturity.

18. Show that the value of a life annuity at any age x is less than that of an annuity certain for the term of the curtate expectation of life at the same age.

19. A company publishes a table of net annuity values for the computation of reduction of premiums, stating that the same is based on its mortality experience and 4 per cent. interest. How could you find what the death rate had been at age 40?

20. A party aged x applies for a whole-life policy of $10,000, the annual office premium of which is $300, but he can afford to pay annually only $250, and wishes the insurance reduced during the first twenty years to such an extent as will enable the company thereafter to insure for the full $10,000; he paying $250 annually each year from the first. The premiums are loaded $33\frac{1}{3}$ per cent.: (1) What fixed amount would be insured during the twenty years? (2) What fixed amount would be insured during the first year, if the insurance is to be increased by equal amounts annually during the twenty years?

Examination for Admission to Membership.

1. State and deduce Gompertz's formula for expressing the law of human mortality.

2. (*a*) What is the common principle involved in formulas like that of Gompertz and Makeham, and wherein do they differ from ordinary methods of graduation? (*b*) Is a table of mean duration of life calculated from a series of ungraduated observations likely to be more reliable or less so than a similar table computed from such observations after graduation, and why?

3. Express $\triangle^n u_0$ in terms of the successive values of the function u, and show that if the law holds good for $\triangle^n u_0$, it will also hold good for $\triangle^{n+1} u_0$.

4. What are the relative advantages claimed for a net and a gross valuation? When may a company technically solvent be practically insolvent, and the reverse, where either method is employed?

5. The income of $10,000 is left by will to two persons aged 30 and 31, with reversion to the survivor. Given $a_{30.31}$, and the mortality table corresponding—(*a*) Find the approximate single net premium for insuring the elder against loss of the reversion. (*b*) Under what condition as to survivorship would this be the true premium?

6. x' and x'' represent the numbers embraced in the first and second ten years of a company's experience respectively. It is proposed to combine the two. Two formulas are suggested, first,

$$\tfrac{1}{2}\left(\frac{dx'}{lx'}+\frac{dx''}{lx''}\right), \quad \text{second,} \quad \frac{dx'+dx''}{lx'+lx''}.$$

(1) Which is the more nearly correct, and why? (2) What advantages, if any, has such a disintegrated experience over the simple aggregate of the whole in judging of the mortality to be expected?

7. Is the granting of new insurances on old lives undesirable? And does any objection that may be made apply to the continuance of insurance originally granted on young lives, after old age is reached?

8. An ordinary life policy was issued six years ago at age 35, and the holder now desires to change it to a twenty-payment life policy from the original date before making the seventh payment. What charge should be made for the change, if no commission is to be paid any agent, and the policy is on the annual dividend plan? What if the policy is on the five-year dividend plan?

9. Having given the following ungraded table of the number living at ages 20 to 36, inclusive, find the number living at ages 27,

28 and 29, graduated according to Woolhouse's formula for graduation, or any formula based for graduation on central differences :—

Age.	No. Living.	Age.	No. Living.
20	8,216	29	7,727
21	8,168	30	7,666
22	8,114	31	7,601
23	8,066	32	7,543
24	8,002	33	7,476
25	7,942	34	7,414
26	7,901	35	7,347
27	7,844	36	7,284
28	7,790		

10. What method would you recommend a company to adopt for insuring under-average lives? Give your reasons.

11. What peculiarities are generally connected with the extra premiums charged for continuous installment provisions for ordinary life policies and how are the special reserves affected thereby?

12. What general principles should regulate the contributions of members for expenses? State briefly your views regarding the cost of new business, and how it should be treated.

13. State briefly the prevailing doctrines of the courts as to insurable interest in case of assignment; as to the interest of the individual policy-holder in the reserve; as to the rights of creditor as assignee.

14. State definitely a system of loading for ordinary and limited payment premiums—both whole life and endowment insurance; also, for term policies; it being understood that agents' commissions will be percentages on the premiums, both first year and renewal.

15. What is meant by the *force of mortality*, and wherein does it differ from the rate of mortality? Why can it not be exactly computed from ordinary mortality tables, and from what class of tables can it be? Under what condition does the central death rate at any age equal the force of mortality at an age six months greater?

16. A company changes its premium rates from a 4 per cent. to a 3 per cent. basis, the percentage of loading remaining the same. Discuss the effect upon the dividends of new policy-holders as compared with those of old policy-holders, assuming that reserves on the new business only are affected by the change.

17. What is the law as to the title of life policies, and how does it differ in the case of ordinary companies and beneficial orders? (Candidates will answer as to laws of their place of residence.)

18. Given the following data, find by central differences the number living at age 30 :—

Age.	No. Living.
20	93,268
24	90,529
28	87,726
32	84,831
36	81,814
40	78,653

19. Find by Lubbock's formula of approximate summation the value of an annuity, payable at the end of the year, on a life aged 40, the rate of interest being 4 per cent., and the following data being given :—

Log. $v^{10}=\bar{1}.82967$ *Log.* $l_{40}=4.89572$
Log. $v^{20}=\bar{1}.65933$ *Log.* $l_{50}=4.84209$
Log. $v^{30}=\bar{1}.48900$ *Log.* $l_{60}=4.74798$
Log. $v^{40}=\bar{1}.31867$ *Log.* $l_{70}=4.55433$
Log. $v^{50}=\bar{1}.14833$ *Log.* $l_{80}=4.12353$
Log. $l_{90}=3.12024$

(A table of logarithms will be furnished for this problem.)

20. When the rate of mortality is considered to vary only according to the current age of the insured, a plain curve is used to illustrate the relations between the rates at different ages : How could we illustrate to the eye by a single geometrical figure the different rates of mortality at each age, varying according to the time since medical examination?

21. Given a table of the theoretical reversionary values of $1 according to a certain company's experience and a safe rate of interest—How could you frame directly therefrom a table for practical use that would give the company a margin of 10 per cent. and be consistent and reasonable at every age?

VOL. VI. NO. 24.

PAPERS AND TRANSACTIONS.

OCT. 25TH AND 26TH, 1900.

ON THE VALUATION OF INDUSTRIAL LIFE POLICIES.

W. S. SMITH.

WHEN the system of Industrial Insurance was adopted and put into effect in this country there was no provision in the laws of any of the States for any different treatment in the valuation of this kind of business than of the usual and ordinary sorts. It was all Life Insurance in the view of the laws, and the reserve liability was expected to be estimated by the rules applicable to the usual kinds.

But from the infinite detail implied by the nature of Industrial Insurance, it was at once apparent that no system of seriatim or individual treatment, such as was applied to the ordinary plans, could be practicable and that some method of grouping must be employed.

The first plan tried was that of grouping by weeks of issue, and the sum of all the policies issued at the same age in any one week was counted as one policy, and a list showing the amount insured at each of the ages during that week was made out and placed by itself. The same process was followed for each of the weeks of the year, and as the result there were fifty-two separate lists needed in order to represent the business of a year.

But as these policies were all on the weekly premium plan they were liable to lapse in any one of the fifty-two weeks, so that by the end of the year these original schedules would not by any means represent the number or amount of insurance then in force, and every item would need to be resorted and corrected for every week of issue to determine its then condition. This called for an

immense amount of detail even for a single year, and what it would become as the business proceeded from year to year indicated an amount of labor and expense that would be almost prohibitory of the business. It was then attempted to condense it a little further into months of issue, but substantially the same difficulty remained.

After a good deal of study and experiment by all concerned, it was concluded by the companies that, although it might probably result in charging them a considerably excessive reserve, the only practicable way open was to refer the whole work to a method of grouping by years, assuming the middle day of the year as the average day of issue and reporting all policies and amounts issued at the various ages during the year as if bearing date July 1st. This reduced the fifty-two original weekly lists to a single list for each year and the amounts reported were those in force on Dec. 31st, the date fixed by the laws for computation of reserves for the annual statements.

This plan greatly relieved and simplified the complications and brought the valuation of industrial policies into much more practical shape. And if the insurances under these policies, like those of the ordinary business, were to be paid for by annual premiums in advance, they would have required a method no different from the ordinary methods for their valuation.

From the fact, however, that these premiums were not yearly in advance, but were payable weekly throughout the year, it must result that on Dec. 31st there remained yet to be paid of the yearly premium an average of at least one-half of the yearly total, and as in the ordinary tabular valuations the reserve is charged as if the whole had been paid in advance, there must be allowed as a credit to offset the overcharge an amount equal to the unpaid portion for the full year. As the valuation is on the *net* premium method the overcharge to be credited would be the sum of the *net* deferred instalments. This same thing occurs also in the valuation of the ordinary business where premiums are paid quarterly or semi-annually, and the adjustment or balance is made in the manner above indicated, by crediting in assets the amount thus overcharged, but it is a much simpler matter than in the case of the industrials from the fact that the amount thus to be considered is relatively of much slighter importance, and also that as the loading on the ordinary is approximately estimated at a percentage of the gross deferred amount, the correction is readily made. In the case of the

industrials, however, this matter became of very great importance and difficulty, both from the large relative proportion of the deferred amount, and especially as from the nature of the industrial premium no average percentage could be assumed as the loading that would be safe or fair for any one year, to say nothing of the constant changes and variations as the years went on. To meet this difficulty it became clear that some plan radically different from that applied to the ordinary must be devised.

The premium of the industrial, like the ordinary, is made up of three distinct parts, for three distinct purposes—Expenses, Cost of Insurance and Reserve. If the premium is payable in weekly instalments, each of the fifty-two parts must contribute its quota to each of these purposes. The expense part goes out at once to pay current charges. The death claims are also paid immediately—within a day or two—so the cost of insurance part also goes out at once. Thus these two parts of the premium promptly go to their purposes, and, in a sense, may hardly be said to reach the company's office at all. There remains to be considered then only the part going to the reserve.

As each of these weekly instalments must pay its share, it is clear that on the 31st of December of first year there would be chargeable to the policy as many fifty-second parts of the whole terminal reserve as there have been weekly payments made, and, if the policy is dated July 1st, then on the 31st of December, the end of the first half year, there would be one-half the first year's terminal reserve, by the usual tables, chargeable on account of the policy.

By this plan the whole matter of deferred premiums is eliminated from the reserve calculations, and need not be considered either as a charge or a credit, and would not appear at all in any of the accounts. It simplifies the whole case down to a consideration of what the premium has contributed to the reserve, pure and simple. Thus, on Dec. 31st of first year there would be charged one-half the first terminal reserve; Dec. 31st of second year, the whole of first terminal plus one-half the difference between the first and second terminals; Dec. 31st of third year, the second terminal plus one-half the difference between the second and third terminals, etc.

As the industrial policies are written with age at next birthday, it follows that the ages on the average are one-half year too old, and if the valuations were to be made from schedules made up at

these policy ages the reserve charge would be correspondingly excessive. To correct this the usual terminal tables are adjusted to the actual ages by taking the half sum of the two nearest terminals; thus, for example, the value for the first year of a policy issued as for age 20 would be based on half the sum of the terminals for policies issued at ages 19 and 20, making the charge as if the policy were issued at actual age of 19½.

In the evolution and progress of the industrial business to its present magnitude, there have developed, as might be expected, proprieties for modifying in more or less important particulars the exact application of the above-described method. But no change of the established principle has been found either proper or desirable. The working is simple and convenient and in ultimate results accords with that in the ordinary business.

An American Method of allotting Surplus to Deferred Bonus Policies.*

RUFUS W. WEEKS.

IT is a privilege to be allowed to express the gratitude which I feel, and which we all must feel, to Mr. Ryan. He has gathered and condensed the facts for us, in a form compact, yet clear and vivid, on a subject which is of great interest and importance to us all. To accumulate the same facts would be possible for any one of us only at the cost of great labor and research, and even then we could not feel sure of our perspective as we can of Mr. Ryan's.

Among the admirable qualities of Mr. Ryan's paper I especially note its non-dogmatic tone, so thoroughly in place on this subject; for surely of all the aspects of life insurance this is the one where the insistence upon any one set of principles is least justified. Absolute equity is not possible here; the most we can hope for is to avoid opposite extremes of inequity, and to come somewhere fairly between these extremes. There may be several systems of distribution of surplus, producing diverse results, yet all equally equitable, and between such systems choice must therefore be made on other grounds than that of a supposed mathematical equity. It may be said that the system which produces the healthiest results is the best, and by this I mean the system which makes it easiest to induce the largest number of persons to insure and to induce the largest number of persons to remain insured.

Mr. Ryan does not mention the subject of the length of the bonus periods, *i. e.*, the intervals between the successive bonuses. It would be interesting to know whether there have been any changes in the practice of the British companies which indicate a trend toward what would be in effect annual bonuses, or, on the other hand, toward longer bonus intervals than formerly prevailed, or whether the average practice in this respect remains the same as thirty years ago.

*Paper read at the International Congress of Actuaries, Paris, 1900; following Mr. Gerald H. Ryan's paper on the Methods of Surplus Distribution practiced by British Companies in the Past and at Present.

In the United States the principal issue in the matter of the distribution of surplus is this question of the length of the bonus periods. There are two typical systems, which include by far the greater part of the policies in force, namely, the annual dividend system, under which a bonus is declared every year for every policy in force, and the deferred dividend system, under which no bonus is declared until a policy has been in force for a considerable term of years—generally twenty years.

The latter system, the long-period, deferred-bonus system, is at present the popular one, and I think officers of life insurance companies which have practiced this system for many years will substantially agree with me that it is working well and producing a healthy expansion and stability in life insurance.

If I may take so much of your time, I should like to indicate, in outline, a method of distributing surplus among long-period, deferred-bonus policies which is workable and which will secure equitable results—not exclusively equitable results, for no such are attainable. In order to work this method, it is necessary to keep the policies of the company in group registers, or valuation books, *i. e.*, books where the policies issued in the same calendar year, on the same plan of insurance and at the same age of the insured are grouped together, and where the history of each group is entered year by year. This system of recording policies is so useful in other respects and for all kinds of tabulation, that it would seem desirable for any company to adopt it, even aside from its possible use in the calculation of bonuses.

The method which I am about to outline is in thorough accordance with the contribution formula, but avoids the difficulties which Mr. Ryan speaks of as arising in the application of that formula, difficulties in the way of finding a working basis for the assigning to each policy of its share of the surplus derived from each of the various sources, *i. e.*, loading, interest and mortality provision.

The system consists in constructing a hypothetical history of assumed groups of policies, based on the average facts of the company's history during the years of the period elapsed, generally 10, 15 or 20 years. The aim may be stated as follows; having imagined a separate company to have been formed twenty years ago, containing 10,000 policies all of the same plan, amount and age of insured, to imagine that company to have had an experience year by year parallel with the experience of the actual company

taken at large, and then to see what the entire assets of the imaginary company would now be, and how many policies there would now be in force. The share of each policy in such hypothetical assets is then taken as a standard or typical gross value, and after deducting the policy reserve, made on whatever basis is then deemed best, the remainder is surplus and may be paid as bonus.

Such a process as the above was first used, I believe, by the Equitable Life Assurance Society of New York, which in 1867 published examples of groups of policies worked out on certain assumptions for twenty years in advance. The method I am now explaining is the same in outline as that then used by the Equitable, but introduces certain differentiations in the ratios used, which are necessary in order to produce true results. The ratios required fall under the following heads :

(1) Rates of mortality.
(2) Rates of discontinuance.
(3) Rates of expense.
(4) Rates of interest.
(5) Rates of surrender allowances.

First, then, as to rates of mortality. In order to secure justice as between the groups of the different periods it is necessary to use a table of mortality in two dimensions, *i. e.*, a compound table like that devised by Dr. Sprague, showing separate mortality rates for successive years of duration of the insurances. This mortality table must closely represent the actual experience of the company, taken on the amounts insured (a mortality experience on lives insured will not answer the purpose).

Second, as to rates of discontinuance. From the summations of the group valuation books may be readily derived the actual scale of lapses for the successive policy years, which may then be graduated without interference with its practical effect. It is possible at no great expense of labor to construct differing scales for the different plans of insurance, and even to refine so far as to have separate scales for typical ages of insured.

Third, as to rates of expense. Here, in order to do relative justice to the groups of different bonus periods, it is necessary to differentiate between the expenses of the first year of insurance and those of subsequent years. Various methods of making this distinction have been suggested, and after testing many of them on

the actual outlay of a company for the various kinds of expenses, I have reached the conclusion that a fair ratio between the rate of expense to be levied on the first year's premiums and that on subsequent premiums is the ratio of 6 to 1.

It is also necessary to decide upon some relation between the rates of expense assessable on the various plans of insurance; for example, as between the Ordinary Life policy and the Endowment Assurance policy. This question has been much discussed in various forms, and it is most likely that if a rule of mathematical equity were possible a fair consensus would have been already reached. That there is no such consensus is evidence that no such rule is possible. What we have to do is to avoid the extremes of inequity, and at the same time to pay attention to the principle which so often determines methods of taxation, viz., the relative ease of bearing the burden. The extremes of inequity are, on the one hand, the levying of equal percentages of the premium on low and high premium policies, and on the other hand, the levying of the same expense charge in dollars and cents on low and high premiums. A fair mean consists in determining a standard ratio on Ordinary Life premiums, and then in the case of policies having higher premiums than Ordinary Life charging such standard ratio on so much of the premium as equals the Ordinary Life premium, and charging half that ratio on the excess of the premium. This method will be found to give satisfactory results, *i. e.*, results which will appear to the average business man, fair as between Life and Endowment Assurance policies. Having decided upon these two principles, it is no difficult task to ascertain from the premium receipts and the expenses of the company during each of the calendar years of the period elapsed, the typical Ordinary Life expense ratios, first year's and renewal, corresponding to the experience of the company for each of the calendar years. These ratios should then be graduated for the period of years, so that any tendency toward greater or less economy may appear in a gradual change in the percentages.

Fourth. The rates of interest actually realized by the company during the successive years of the period should also be graduated so as not to show abrupt changes from year to year.

Fifth. The average rates of surrender allowances can easily be made up from the group valuation books, if the surrender value granted by the company on every policy discontinued has been entered in them.

It may seem at first sight that the foregoing process would be complicated and laborious, but in practice it is not found to be either one or the other. Only a few typical groups need be worked out, on the standard plans only, such as Ordinary Life, 10-Payment Life, 20-Payment Life, 10-Year Endowment Assurance and 20-Year Endowment Assurance, and only for typical ages at issue—25, 35, 45, 55, and, on some plans, 65.* From the results found on these few groups it is easy to construct by interpolation tables of standard values for all the plans and all the ages.

The method is a safe one, because it deals with each policy as if the entire company had been composed of similar policies; it is equitable because it follows the facts, and because each individual assumption as to the weight of the several facts is a fair one. It has the great advantage of remaining equally safe and equitable when the rates of premiums are changed, or when the basis of the reserve is changed, each group being credited with premiums at the actual rate it has paid and being charged with the reserve on the basis held to be necessary at the time when its period ends.

The method has the further great advantage of remaining equitable even when great variations take place in the annual output of new insurance. The annual dividend system, when coupled with the requirement of a net-level-premium reserve (which appears to be a necessity in these days, though it be an artificial necessity) makes it impossible for a company to do a new business increasing rapidly in amount relatively to the business already in force without injustice to the old policyholders. Such a large volume of new business carries with it a large outlay in first year's commissions, and that even though the rate of commission be not increased. A material increase, therefore, takes place in the ratio of entire expenses to entire premiums, and there follows a reduction of the dividends to the old policyholders, although the expenses relating to the old policies have not increased, and thus a manifest injustice is done. The deferred-dividend system, handled by the above method, obviates this difficulty, and by the expedient of charging each generation of policyholders with its own first year's expenses and its own expenses of successive renewal years, it does justice between the successive generations, though the new business of some years should be greater than that of preceding years, and even though it should be done at a greater rate of expense.

* The process is illustrated in the table on page 372.

Specimen-Calculation of Standard Gross Value of Deferred-Bonus Policy:

ILLUSTRATING THE PROCESS OUTLINED IN THE FOREGOING PAPER.

Ten-Year Endowment Assurance, with Ten-Year Deferred-Bonus Period; Issued in 1890, at Age 45; Annual Premium, $111.30.

(The corresponding ordinary life premium being $39.10, and the excess of the actual premium over the ordinary life rate being $72.20.)

	m=1890 n=1	m=1891 n=2	m=1892 n=3	m=1893 n=4	m=1894 n=5	m=1895 n=6	m=1896 n=7	m=1897 n=8	m=1898 n=9	m=1899 n=10
(1) Rate of Mortality, n of $45+n-1$	.00640	.00783	.00956	.01116	.01258	.01370	.01440	.01502	.01593	.01675
(2) Rate of Discontinuance	.081	.033	.148	.045	.031	.019	.011	.004		
(3) Number paying, out of 10,000 issued	10,000	9,131	8,761	7,393	6,981	6,679	6,462	6,299	6,179	6,081
(4) Number dying in policy year "n"	64	71	84	83	88	92	93	95	98	102
(5) Number discontinued at end of policy year "n"	805	299	1,284	329	214	125	70	25		
(6) Ordinary life expense ratios (6 to 1) for year "m," on first premiums and on renewal premiums	75% (& 12½%)	(74% &) 12⅔%	(73% &) 12⅚%	(72% &) 12%	(71% &) 11⅚%	(70% &) 11⅔%	(69% &) 11½%	(68% &) 11⅓%	(67% &) 11⅙%	(66% &) 11%
(7) Expense charge on the portion of the premium ($39.10) equal to ordinary life	$29.32	$4.82	$4.76	$4.69	$4.63	$4.56	$4.50	$4.43	$4.37	$4.30
(8) Expense charge (half rate) on the remainder ($72.20)	27.08	4.45	4.39	4.33	4.27	4.21	4.15	4.09	4.03	3.97
(9) Entire expense charge	56.40	9.27	9.15	9.02	8.90	8.77	8.65	8.52	8.40	8.27
(10) Effective premium	54.90	102.03	102.15	102.28	102.40	102.53	102.65	102.78	102.90	103.03
(11) Effective net rate of interest	.0470	.0465	.0460	.0455	.0450	.0445	.0440	.0435	.0435	.0435
(12) Effective premiums received: (3) × (10)	$549,000	$931,636	$894,936	$756,156	$714,854	$684,798	$663,324	$647,411	$635,819	$626,525
(13) Death payments: (4) × $1,000	64,000	71,000	84,000	83,000	88,000	92,000	93,000	95,000	98,000	102,000
(14) Death payments, adding 4 months' interest	65,003	72,100	85,288	84,259	89,320	93,364	94,364	96,378	99,421	103,479
(15) Average value allowed on policies discontinued at end of policy year "n"	0	0	$173.02	$279.03	$413.42	$514.78	$623.48	$730.13		
(16) Surrender values allowed: (5) × 15)			$222,158	$91,801	$88,472	$64,348	$43,644	$18,253		
(17) Fund in hand at end of year (n—1) plus effective premiums of year "n": $(21)_{n-1}+(12)_n$	$549,000	$1,441,436	$2,331,299	$2,887,249	$3,557,413	$4,224,503	$4,918,105	$5,643,905	$6,410,603	$7,216,568
(18) One year's interest on same: (11) × (17)	25,803	67,027	107,240	131,370	160,084	187,990	216,397	245,510	278,861	313,921
(19) Sum of (17) and (18)	574,803	1,508,463	2,438,539	3,018,619	3,717,497	4,412,493	5,134,502	5,889,415	6,689,464	7,530,489
(20) Disbursements for death payments and surrender values: (14)+(16)	65,003	72,100	307,446	176,060	177,792	157,712	138,008	114,631	99,421	103,479
(21) Fund in hand at end of policy year "n": (19)—(20)	509,800	1,436,363	2,131,093	2,842,559	3,539,705	4,254,781	4,996,494	5,774,784	6,590,043	7,427,010

Number remaining at end of period, 5,979. Share of each, $1,242.18.

On the Objects to be attained in Future Investigations of Mortality and Death Loss.*

EMORY McCLINTOCK.

ONE of the most useful functions of the actuary of a company, or of a number of actuaries working in concert, is to investigate the losses which have been incurred on special classes of policies. Among the various classes which may be taken separately for investigation, those arranged according to the occupations of the insured are naturally among the first to be considered, and it is also customary and useful to make similar examinations concerning business transacted in special localities or under particular plans of insurance, or having regard to the individual characteristics or the family records of the lives insured, or in fact concerning any fraction of the experience which may be regarded as likely to afford information available in future in the selection of new risks. The chief object of this paper is to suggest that actuaries will in all probability find it of increasing importance to devote attention to such special investigations, by which the actual results on a given fraction of business are compared with an ideal standard, and of less importance relatively to frame new life tables. I shall take occasion incidentally to urge that when tables are to be framed for life insurance purposes, attention should be paid to the experience of life companies with respect to money losses as well as with respect to the number of deaths.

In the past, whenever a large company or an assemblage of companies has made a mortality investigation, it has been usual to bring the results together so as to form a new mortality table illustrative of the aggregate experience which has been investigated. It is possible that fifty such tables may exist. If so, it may be questioned whether more than one-fifth of them has possessed any value beyond the gratifying of casual interest; whether the other four-fifths of the tables produced would not have been more useful had they been confined to special classes of business, and whether even the few investigations which have resulted in useful general

*English version of paper presented at the Third International Congress of Actuaries, Paris, June 28th, 1900.

tables would not have been of greater benefit had the same opportunities been utilized also for specialized examinations. It is more important for the future interests of life insurance to learn how fishermen compare with farmers, how physicians compare with clergymen, how brewers compare with manufacturers of soda-water, and the like, than it is to gather together all these heterogeneous materials into one grand average in the form of a new life table. Much useful work has already been done in this way, chiefly in private, and in some cases the results have been published, to the great advantage of actuarial science.

As regards life tables in general, no two can ever possibly be alike, because in every instance they are obtained by massing together the good and the bad in proportions which can never be duplicated elsewhere or at another time. There are classes of business which are permanently good, and there are other classes which are persistently bad. There are classes which to the end of life exhibit an annual mortality of not more than two-thirds of that expected by the customary tables, and other classes which to the end of life exhibit an annual mortality fully four-thirds of the expected. Between these extremes there may be found many gradations. For example, with a sufficient volume of business, we shall find that of those insured at a certain age, the persons who have both parents living at the time of insurance are on the average better risks than those of whom one parent has already died, and that the latter are on the average better risks than those who have neither parent living at the time of insurance. For example, again, we shall find that persons of disproportionately heavy weight are on the average not good risks, with the exception, perhaps, of such portion of them as have an unusually good family record. There is almost no end to the interesting and valuable facts which can be developed in this way. Work of this kind is particularly useful when carried on by a combination of different companies, or by actuarial societies, because in such cases there is no hindrance to the publication of the results. It is not usually desirable to make public the experience of a single office concerning any special class of risks which, judged by the results, ought never to have been accepted. This objection does not arise when a number of offices combine to supply their materials indistinguishably to some central authority trusted by all concerned. As time goes on it will become more and more the duty of every actuarial society to gather materials for the investigation of special

classes of risks varying according to occupation or otherwise, and to abstain more and more from the mere piling together of heterogeneous materials toward the formation of another conglomerate table.

The object for which these special investigations are needed is to assist companies in the initial selection of their risks, by the increase of their knowledge of the various circumstances which indicate or affect longevity. This object is radically different, not only from the formation of new conglomerate tables, but also from the inquiry which every company needs constantly to make, whether its current losses are keeping within the expected bounds. To carry out such special investigations properly it is necessary to segregate a certain class of risks and to determine whether it is on the whole a class of good risks, and to do this it is necessary to compare the mortality and loss experience upon the selected class with some chosen standard which represents the experience of an ideal class of good risks. This standard is not to be found in any table customarily used for the calculation of premiums and reserves, because those tables are known to include a certain proportion of bad risks, as it is desirable that they should, in order to be on the safe side. Before suggesting for consideration the table which I think might be used as a first or provisional standard for representing the ideal class of good risks, I will mention that in the private investigations of special classes of risks which I have made from time to time in recent years, I have assumed as an ideal standard, for insurances more than four years in force, an annual mortality seven-eighths of that of the American table at each age ; for insurances more than one year and less than five years in force, seven-tenths, and for insurances less than one year in force four-tenths, of the mortality according to the same table. I have been accustomed to speak of the expected loss according to these assumptions as the "normal loss," and in each special investigation I have compared the actual loss with the "normal loss." The classes of risks upon which the actual loss is less than the normal loss are regarded as good risks, on the average, and those upon which it is greater are regarded as doubtful or bad risks, on the average, and therefore as requiring unusual caution in selection. I mention this only as an illustration of what has been done in practice. The ideal standard which I have used is obviously not perfect. There is no reason to suppose that the class of good risks must have a mortality at each age which is a

uniform fraction of the general mortality shown by a customary table which includes a certain proportion of bad risks. There is no reason to suppose that the benefit of selection is uniform during the second, third and fourth years of insurance. Nevertheless, these are minor blemishes, and I have no doubt that the results obtained do not differ greatly from those which might have been obtained by the use of a more perfect standard.

In the entire absence of tables based upon life insurance experience of the best nature, including only the more desirable classes of risks, it would appear that at present we cannot do better, in the search for a model or ideal standard, than to make use of a table of mortality which appears to be the only one existing which has expressly been founded upon the statistics of a very large number of the better class of lives. This is the Healthy English Male Table of Dr. Farr, which embodies the deaths which occurred during five years in sixty-three selected healthy districts of England, having a total male population of about half a million. This table was published in the Philosophical Transactions of the Royal Society for 1860, and was reprinted in Vol. IX. of the Journal of the Institute of Actuaries. In 1893, Mr. McKenzie, of the Government Life Insurance Department of New Zealand, prepared for the Actuarial Society of America a number of columns belonging to this life table, which were printed in full in Vol. III. of the Society's Papers and Transactions. From this publication I quote the following percentages of annual mortality, together with those found by taking seven-eighths of the corresponding percentages according to the American Table :

Age.	Healthy English.	Seven-eighths American.
25	.00775	.00706
35	.00865	.00783
45	.01076	.00977
55	.01656	.01625
65	.03680	.03511
75	.08400	.08527
85	.18018	.20611

It is possible that some objection may occur to others which has not occurred to me, but in default of any good reason to the contrary it would appear that the Healthy English Male Table may advantageously be taken as the model, or standard of good risks

for comparative purposes, both in private investigations of individual companies and in those more important examinations of special classes of risks which will undoubtedly be made from time to time by bodies of actuaries. In the use of this table, or of any table, as a standard for comparisons, it is important to observe that due allowance must be made for the effects of medical selection. It is known that the expected loss is much less during the first year of insurance, and to a diminishing extent during a few years thereafter, owing to medical selection. It is proper therefore to reduce the expected loss accordingly, and I would suggest for consideration, in connection with the Healthy English Male standard, that for the first year of insurance one-half of the annual mortality might be taken for the normal or expected mortality or death loss, for the second year two-thirds, for the third year four-fifths, and for the fourth year nine-tenths, these proportions being of course subject to alteration at discretion. It is unnecessary to point out that if this precaution be omitted, and if in the total expected loss there is included the full tabular expectation for the earlier years of insurance, the result for any single class will be too favorable, and as between different classes will be most favorable to that which contains the greater proportion of recent entrants.

In process of time, without doubt, there will be available large masses of statistics relating to insured lives from which the more undesirable classes of risks shall have been excluded. When any such volume of experience is available, a table can be constructed which may take the place of the Healthy English Male Table, or any other table first adopted as the model or ideal standard for the class of good risks.

Let us suppose that such a volume of experience, excluding undesirable risks, has become available. Let us suppose it to be feasible to separate the lives under consideration into two groups equal in number, such that each life included in the first group shall have been insured for a greater amount than any life included in the second group. We shall thus have two large groups of equal numbers, the first representing the experience on large amounts of insurance and the second on small amounts. Let these two experiences be tabulated separately, so that for each experience we have a table of numbers living and dying in the usual form of mortality tables, whether adjusted or unadjusted. It is certain that these two tables must differ throughout. They are, however, in this shape of no special use to us, and we therefore

proceed to combine them. How shall we combine them? The answer to this question depends on the answer to another. Shall we regard all the lives in both groups as of equal weight, or shall we give greater weight to the lives in the first group, which includes those persons who have taken and paid for the larger amounts of insurance? This is a momentous question. It is to be remembered that the business of life insurance deals only with money. The business consists in paying money for death losses, in consideration of the previous receipt of money in premiums, and in the holding and investing of money in the reserve funds. The premiums are calculated for the purpose of meeting money losses. If the premium according to the table derived from group No. 1 is larger than that derived from group No. 2, or smaller, and it certainly must be either larger or smaller, it is certain that the lives in these two groups must be weighed as well as counted in order to produce a table from which to calculate a correct premium on the assumption that the experience of the future is to be judged by the experience of the past. Is this assumption true? On the whole, it is much nearer the truth than the other assumption usually made, that the average losses with respect to amounts insured must correspond to the average number of deaths with respect to the number of lives insured, at each age. The more this matter is examined, the more clearly will the fact appear that there is no other way of giving weight to those lives insured for the larger amounts than by framing the table which is the object of the investigation by the help of two separate examinations, one having regard to the number of deaths with respect to the lives insured, and the other having regard to the losses in money with respect to the amounts insured. With a sufficiently large volume of experience, the materials relating to amounts will on the whole be of much greater importance as a guide for the future than the materials relating to lives. My own judgment is clear that a table based upon the experience of money losses, rectified at all points which may appear exceptional or doubtful, particularly in youth or extreme old age, by careful consideration of the experience relating to the number of deaths, will supply a final table much superior, as a financial guide for the future, to a table based solely upon the number of deaths. This remark applies, of course, both to special tables relating to good risks and to those more general tables which it is safer to use for computing premiums and reserves.

The construction of tables of numbers living and dying from

statistics of money losses was initiated on the 19th of November, 1874, at a meeting held in New York by a committee of the Chamber of Life Insurance, appointed for collecting the mortality experience of the American life insurance companies, where I proposed that the committee should frame a table based upon money losses rather than upon lives, urging that actuarial calculations dealt only with money, that the lives insured should be weighed as well as counted, and that the pecuniary experience of the past was the best guide for the future, assuming due caution to be exercised with regard to those ages of the table for which the loss experienced might be inadequate. The proposition was carried into effect.*

The Institute of Actuaries has been collecting a great mass of statistics referring both to lives and to amounts insured. My belief is that the most useful table which can be framed from these statistics will be one which shall exclude the first few years after entrance, and which shall follow the experience relating to money losses, corrected at discretion by reference to the experience based on lives.

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* The only previous reference to such a table was made sarcastically in 1853 by Mr. William Spens, in a paper published in Vol. IV. of the *Journal of the Institute of Actuaries*. He attacked the Experience of the 17 Offices, because it had been based upon policies instead of lives, and intimated that if it had been taken upon amounts instead of upon policies, the results would have been closer to the true mortality, or if not, would at least have represented something, namely, "the financial mortality." In his own words: "Had the Experience data been made out in this way, I believe they would have represented the truth with much more accuracy—at all events, it is obvious they would have represented, so to speak, the financial mortality."

In Mr. Meikle's report of 1869 on the experience of the Scottish offices, Table No. 96, published as Table G, contained for each age the total amount insured, the actual losses by death, and their ratio, the unadjusted probability of loss. Mr. Meikle did not suggest that it might be well to frame a table of numbers living and dying from these or any similar summaries of money loss. Retrospective investigations of money loss had also been made by other actuaries, including Mr. J. J. Downes.

Should Life Companies discriminate against Women?

JOHN K. GORE.

A LARGE part of what follows was read before the Society at the spring meeting in 1899 in connection with some comments suggested by the address of the President, Mr. Miller, at the previous fall meeting. The Society requested at the time that the comments be presented later in the form of a paper.

It has been demonstrated that the mortality of women at ages 20 to 45, as compared with that of men is greater among insured lives than among the general population. This fact was referred to by Mr. James Meikle in his "Observations on the Rate of Mortality of Assured Lives, as expressed by Ten Assurance Companies in Scotland from 1815 to 1863," and Mr. Asa S. Wing says in his comments on tables Nos. 14 and 15 of the mortality experience of the Provident Life and Trust Company: "This is in accordance with most observations on assured female lives, notwithstanding that statistics show a higher death rate for men than for women in the general population."

For the convenience of those who may wish to pursue this subject further, I have prepared a list of all the published tables known to me, showing the relative mortality of the sexes. These tables naturally divide themselves into two groups, the first having to do with the general population and annuitants, and the second including only insured lives. In the former group the mortality of women is in every case either lower than that of men at the ages in question, or if higher, only slightly so—on an average considerably less than two higher per thousand exposed to risk:

1746—Observations on the Mortality of the Nominees of the French Tontines of 1695 and 1740. Des Parcieux.

1769—Reversionary Payments, etc. Dr. Price. (Comments on the Northampton and other Tables.)

1783—Reversionary Payments, etc., Fourth Edition. Dr. Price. (Mortality Tables for the Kingdom of Sweden, for Chester and for Stockholm.)

1815—Swedish Population Tables, 1765-1796. Mr. Joshua Milne.

1829—Report on Government Life Annuitants. Mr. John Finlaison.

1829—Tables of the Probability and Expectation of Male and Female Lives in Glasgow. Mr. James J. Duncan.

1840—Second Report of Registrar-General. (Mortality by sexes of five large cities in England contrasted with the whole of England and Wales.)

1843—English Life Table No. 1. Dr. Farr.

1850—Massachusetts and Maryland Censuses.

1855, 1856, 1858 and 1862—Reports of the Scottish Registrar-General.

1853 and 1859—English Life Table No. 2. Dr. Farr.

1860—Report on the Mortality of the Government Life Annuitants. Mr. A. G. Finlaison.

1861—On the Rate of Mortality prevailing among the Females of the Peerage during the Nineteenth Century. Mr. A. H. Bailey and Mr. Archibald Day.

1864—English Life Table No. 3. Dr. Farr.

1872—Sixth Registration Report, Michigan. Dr. Henry B. Baker.

1884—Report on Mortality of Government Life Annuitants. Mr. A. J. Finlaison.

1885—English Life Table No. 4. Dr. William Ogle.

1885 (?)—Mortality of Germany. "Year Book of German Insurance Companies, 1900."

1885 (?)—Mortality of Prussia. "Year Book of German Insurance Companies, 1900."

1886—United States Census of 1880. Dr. Billings. (Including especially vital statistics of Baltimore, Boston, Brooklyn, Philadelphia and New York, and of the States of Massachusetts and New Jersey.)

1893—Brighton (England) Life Table. Dr. Arthur Newsholme.

1893—Mortality of New South Wales. T. A. Coghlan. (Based on the Census of 1891.)

1894—A New Life Table for Glasgow, Scotland. Dr. Arch. K. Chalmers.

1894—Mortality of Manchester, England, 1881-1890.

1895—English Life Table No. 5. Dr. John Tatham.

1895—Mortality of Liverpool, England, 1881-1890.

1895—Mortality of London, England, 1881-1890.

1895—Mortality of New Zealand. Mr. George Leslie.

1896—Report on the Health of Portsmouth, England, for the year 1895. Dr. B. H. Mumby.

1896—United States Census of 1890. Dr. Billings. (Including especially vital statistics of Baltimore, Boston, Brooklyn, Philadelphia and New York, and of the States of Massachusetts and New Jersey.)

1897—English Life Table No. 5, Selected Healthy Districts. Dr. John Tatham.

1898—Report on the Health of Oldham, England. Mr. Charles H. Tattersall.

1898—The Vital Statistics of Massachusetts for 1897, with a Life Table. Dr. Samuel W. Abbott.

1899—*Combined Experience of Life Annuitants (1863-1893). Unadjusted Data. Institute of Actuaries and Faculty of Actuaries.

The second group of tables includes only insured lives. In this group the mortality of women is always higher than that of men for the general period of life covered by ages 20 to 45, the differences averaging more than three higher per thousand exposed to risk.

1843—Seventeen Life Offices (Actuaries' Table).

1847—Mortality Experience of Royal Prussian Widows' Fund, 1776-1845. Brune.

1854 (?)—Mortality Experience of Royal Prussian Widows' Fund, 1776-1852. Brune-Fischer.

1854—On the Rates of Mortality prevailing amongst the Male and Female Lives insured in the Eagle Assurance Company during the 44 years ending 31st December, 1851. Mr. Charles Jellicoe.

1855—On the Results of the Operations of the Gotha Life Assurance Bank for the first twenty-five years of its existence, particularly with respect to the mortality amongst the Lives Assured. Contributed to the *Assurance Magazine* by Dr. R. G. Hopf.

1869—Institute of Actuaries' Experience Table.

1872—Observations on the Rate of Mortality of Assured Lives, as experienced by Ten Assurance Companies in Scotland from 1815 to 1863. Mr. James Meikle.

1877 (?)—Mortality Experience of Twenty-three German Life Companies to 1875. (Healthy Lives.) Dr. Zillmer.

*These data having come to us in unadjusted form the final figures are not yet available. I find by a rough adjustment, however, that at ages 20 to 45 the female rate of mortality is less than the male at each age by an average of about 2.71 per thousand exposed.

1881—Systems and Tables of Life Insurance. Mr. L. W. Meech.

1884—Mortality Experience of the John Hancock Mutual Life Insurance Company. Mr. George B. Woodward.

1884—Mortality Experience of the Connecticut Mutual Life Insurance Company. Mr. D. H. Wells.

1886—Mortality Experience of the Provident Life and Trust Company. Mr. Asa S. Wing.

In the first group of tables are four on annuitants. Among these annuitants the mortality of women is decidedly lower than that of men at the ages 20 to 45. In the general population the two sexes experience an almost equal mortality during this period, that of women being slightly lower on the average. Among insured lives, however, the death rate of women exceeds that of men at these ages by approximately the same differences that exist in favor of women among annuitants.

It seems clear, therefore, that there is an adverse selection, either conscious or unconscious, on the part of women among both annuitants and insured lives.

Among insured lives women form so small a percentage that it is extremely difficult, on account of insufficient data, to learn the real causes of their less favorable mortality as compared with that of men. The reason generally assigned for the heavier death rate of insured women is the fact that it is practically impossible to secure satisfactory medical examinations It is the belief of medical examiners that women do not answer all the questions put to them in the course of an examination as frankly as do men. Women do not, as a rule, remove their corsets for their physical examination, and this fact alone would often make easy a selection against the company. One of our members with whom I have corresponded on this general subject says: "I wonder that the medical directors do not require women to present themselves for examination *without corsets*, so that there may be a reasonable chance of hearing the respiration and heart sounds and ascertaining approximately the actual figure of the woman." While no data exist showing that the ordinary insufficient medical examination of women for insurance is the chief cause of their less favorable mortality, there can be no doubt that it is in a large measure responsible.

Many who have considered the subject of the heavier mortality of female lives at the child-bearing ages find one cause in the psychological reason that women, having finer sensibilities and instincts than men, are frequently able in their own cases to presage the

results of infirmities existing or illnesses to be, and that thus they are able to select against the companies.

In attempting to explain the relatively higher mortality of women, Dr. Hopf, in his paper on the Gotha experience, before referred to, says in this connection:

"I think we must seek the principal cause of it in the circumstance that women, from the greater bashfulness peculiar to their sex, frequently do not communicate all their bodily infirmities and irregularities to their physicians, much less to others, and feel themselves, therefore, much less under obligation to give notice to the assurance office of what they consider their own secret respecting the condition of their body. They think even a question about it an indiscretion and a violation of their delicacy, which might entitle them by itself to avoid the truth in the answer. But the finer constitution of their nerves undoubtedly enables them to feel earlier, and before they are perceptible outwardly, those ailments and changes which may be going on in their system; and, however indistinct such a feeling may be in them, yet they are more impelled by it than men to take precautions against the threatening evil. Allusion is often made to a faculty of presaging peculiar to single persons, and attributed in a higher degree to women than to men. If such a faculty of presaging really exists, it may result from that natural cause; but it forms at the same time a sort of influence against which life assurance companies must be on their guard. At all events, these offices cannot be sufficiently cautious in assuring women under 40 years of age, after the observations mentioned above; and they are perfectly entitled to demand a higher premium from women than from men for assurances at this age."

Dr. Hopf notes that at ages under 40 females insured in the Gotha have a much higher mortality than the females in the general population of England, France, Belgium and Saxony, and says further:

"There is, therefore, no doubt that a greater proportion of females who assure their lives at the younger ages die earlier. The deviation is too significant and too constant to be considered accidental. We are not able to explain it by any other supposition than by the circumstance that women feel internal hidden infirmities and defects in a higher degree than men, and have a presentiment of approaching danger in consequence of them, which impels them to assure their lives; or that they understand better and more

skillfully than men how to hide the true state of their health, and to deceive by it even their medical men. This is facilitated by the proceeding of many physicians, who think themselves obliged to act with greater delicacy and respect in the medical examination of females."

However much we may be inclined to agree with the opinions of Dr. Hopf, as to some women's power of divination, and instances are not wanting that seem to give them color, we can neither prove nor disprove them. There can be no doubt that the companies suffer from this kind of selection, but whether or not the selection has been materially greater on the part of women than of men cannot be known.

Another reason for the higher mortality of insured women in the case of some companies is probably the fact that they insure a larger proportion of wives than exists in the general population at the ages referred to. The information I have indicates that in the companies which accept only those married women who are self-supporting or who are in receipt of an income which will cease at death, the excess of female over male mortality is less than in the companies which are not so conservative.

The following tables have been calculated for me from the United States Census of 1890:

Comparative Mortality per 1,000 of Single Males and Females.

Ages.	Males.	Females.
15 to 19	4.45	4.60
20 to 24	6.04	5.52
25 to 34	8.30	7.30
35 to 44	12.15	9.92
45 to 54	17.91	13.76
55 to 64	27.39	24.24
65 and upward	72.10	68.22

In the above table at all ages over 19 the mortality of single women is less than the mortality of single men.

COMPARATIVE MORTALITY PER 1,000 OF MARRIED MALES AND FEMALES.

Ages.	Males.	Females.
15 to 19	11.31	6.89
20 to 24	5.19	6.73
25 to 34	5.33	7.39
35 to 44	7.21	8.58
45 to 54	10.76	10.19
55 to 64	19.17	17.27
65 and upward	50.78	46.54

In the last table the mortality of married women is in excess of the mortality of married men at ages 20 to 44. At all other ages the female mortality is less.

A comparison of the two tables shows that at ages 20 to 34 the mortality of married women is greater than that of single women. When, moreover, we take into consideration the inevitable selection against the companies under wives' policies, the relatively heavy mortality of insured women may be partly accounted for, in some companies at least.

Dr. John Munn, in "A Contribution to the Medical Statistics of Life Assurance with Hints on the Selection of Lives," said :

"Almost all the cases of female life assurance are made upon the lives of wives. Apart from the moral objection to this kind of insurance we see there is also the commercial objection, partly from their occupation being indoors, partly from their greater liability to death from some causes which are not known to the other sex, such as parturition ; partly from the greater mortality of parturition and its immediate consequences, upon that class of females who are generally included amongst female insurers. Partly, perhaps, from the greater sensitiveness and exposure of the married female to coldness, neglect, anxiety or ill-usage on the part of the husband, and the larger share which falls to her lot in the labor of watching over infantile diseases, from loss of sleep and from various sufferings connected with the *res angusta domi* (the straitened circumstances of domestic life)

"But whatever be the causes, there can be no doubt of the fact, that not only the mortality of this class from phthisis is high, but that it occurs at an earlier period of life than in the other sex, *so that it is very doubtful whether joint assurance upon the lives of husband and wife should be at all considered.*"

Mr. James Meikle, in his "Observations, etc.," previously referred to, after calling attention to the greater mortality of women in the Scottish companies at ages below 45, says: "This may perhaps be owing to the circumstance that a larger proportion would be married in the assured females than in the population."

In its industrial department the Prudential Insurance Company formerly issued a special $500 policy on industrial lives. An investigation of the mortality experience of these policies under an exposure of about 400,000 policies for one year showed that at ages 20 to 44 the average loss per thousand women's policies exposed was about one less than the loss on men's policies. A more recent investigation of our experience under about 11,000 $500 policies issued through our ordinary department on the lives of what might be termed semi-industrial classes, at rates somewhat higher than the regular ordinary rates, discloses the same difference between the sexes of about one in a thousand exposed to risk, the difference being in favor of the women.

One reason for the more favorable mortality of women as compared with men in the two classes of policies just referred to may be that in these cases we do not insure wives only, but women in general, as we find upon an investigation of the conjugal condition of the women insured under our special industrial policies that the ratio of married to single is less than in the general population of the country for the corresponding ages according to the United States Census of 1890. Another reason may be found in the peculiar conditions of the industrial classes. Here the woman is, to some extent at least, a breadwinner. Her services are needed for economical reasons. Her loss would be felt financially far more than that of the woman whose surroundings are more favorable. Among the women of the industrial classes below the age of 48 the element of selection against the companies is almost zero.

There is certainly a selection against the old line companies as far as their women's policies are concerned. The most probable sources of this selection are found in the unsatisfactory medical examinations of women, in a prophetic instinct peculiar to the sex, in the tendency toward an undue proportion of insured *wives* and in the fact that generally the women come from those classes of society where their services do not have as distinct a money value as in the lower classes—where, in other words, there may more easily enter some element of speculation.

The fact remains that the regular ordinary companies which

have investigated their mortality by sexes have found the female death rate greater than the male at the general period of life included between ages 20 and 45. What should be the attitude of the companies toward women applicants at the ages in question?

In order to learn the practice of the companies of the United States and Canada I have corresponded with the actuaries of thirty-nine companies. Of these companies, thirteen charge women at child-bearing ages an extra premium, at least on life policies. Twenty companies do not charge an extra premium on women. Two companies do not insure women at all. Four actuaries did not tell me their companies' practice. Of the twenty companies not charging an extra premium seven had formerly done so, but had decided from their experience that they could safely omit the charge. Four companies not charging an extra premium place women's policies in a special dividend class.

Among American life companies, judging from mortality experiences thus far published comparing the sexes, the death rate of women at the ages in question has exceeded that of men by approximately three per thousand exposed to risk. The average difference at the same ages between the annual whole life premium on participating and on non-participating policies is about five dollars. On other forms of policies the differences are, of course, greater. In other words, the excess of regular participating premiums over regular non-participating premiums without any extra would apparently be sufficient to provide for the extra mortality under women's policies, and also to furnish a considerable dividend.

To charge an arbitrary extra premium much in excess of that warranted by experience does not seem fair to the ever-increasing number of women needing life insurance. As a result of my investigations, therefore, I have recently recommended in the case of my own company that it accept women at the same rates as men on participating policies, placing them, however, in a special dividend class. The plan is certainly both equitable and safe, provided the companies adopting it follow a conservative course in accepting women.

The Legal Rule of Valuation in case of Preliminary Term combined with Whole-Life Policies.

WALTER S. NICHOLS.

IN their recent reports the Insurance Commissioners of Massachusetts and Vermont have discussed at some length the question of valuation in case of preliminary term contracts, and the latter have suggested the propriety of its consideration by this society. It is mainly for the purpose of bringing the subject in a formal manner before the members that the present paper has been prepared. So far as possible, I shall seek to discuss it from a purely academic standpoint, ignoring all questions regarding the wisdom or propriety of such contracts, or the proper rule for their valuation. I think it is obvious to every actuary that these policies are but special cases of a general group embracing a wide variety of combinations which may be made, and whose distinguishing feature is the embodiment of essentially complete and separate contracts in a single instrument, and that the fundamental principles which apply to the class as a whole must apply to these.

The rule of government valuation is prescribed in each State by statute. It will be sufficient for our present purpose to quote the language of one of the most recent enactments as illustrative of the others. The General Insurance Act of Alabama, of 1897, provides that "the insurance commissioner shall each year compute the net value on the 31st of December, of the preceding year, of all outstanding policies, upon the basis of the Combined Experience or Actuaries' table or the American Experience table rate of mortality with the interest at 4 per cent. per annum, and the aggregate net value so ascertained shall be deemed its liability," etc. It will be noted that the statute is mandatory. Nothing is left to the discretion of the Commissioner. He is required to compute the net value. The whole discussion centres on the meaning of this expression. That meaning is a question of law. The construction of the statute in case of doubt rests with the Courts, and the Commissioner is bound by the decision. Two universally accepted legal rules would seem to govern its construction. Technical words like these must be taken according to their approved and known use in the business or profession where used, and also in a

sense, if possible, which will accord with the intention of the statute.

Strange to say, I have been unable to find in all the insurance literature which has been published, what might be called a complete, explicit and authoritative definition of the term net value, and a prominent member of this society has recently told me that such has been his experience. But for an implicit definition we have not far to seek. In the Text Book of the British Institute of Actuaries, a recognized standard authority on all such points, occurs the following definition of the generic term, "value of the policy," which, so far as I know, is in accordance with the views of all actuaries everywhere: "the value of the policy is thus seen to be the sum which the assurance office must have on hand to provide for that portion of the liability under the contract which the future premiums will not cover; and the only source from which it can be derived is the accumulations of the balance of past premiums not absorbed by the risk already incurred."

I call special attention to the two parts of this statement. The first is the definition proper. The value of the policy is the sum needed to make good the deficiencies in future premiums. The source from which alone it can be derived is the accumulations of past premiums. This is the real logical relation between the prospective and the retrospective methods. The first embodies the true principle of valuation. The second is correct because it reaches the same result by a different process.

With this generic definition before us, it is obvious the net value must be the similar deficiency of the future *net* premiums to meet the liability under the contract, and we are at once remanded to the meaning of the words "net premium" as employed and understood in actuarial science. Again to quote from the Text Book, "the net premium being that premium which according to the mortality table and rate of interest adopted, will exactly provide the benefit contracted for. In all practical operations, an addition usually called loading is made to the net premium. The net premium increased by the loading is called the office premium." On the correctness of this statement, too, the actuarial world is in substantial accord. Net premiums and policy values relate solely to those premiums and values which ignore the loading and deal with the pure mathematical cost of the risk.

I have gone into this extended discussion to show that the policy value contemplated by the statute must be based by the

Commissioner solely on the future mathematical premiums receivable according to the mortality and interest table presented, regardless of the office premiums. This is the generally understood and approved sense in which the language is used in the insurance profession and should determine the construction by the Courts.

In the case of a single, simple contract there is no dispute as to what constitutes the mathematical premium or the benefit promised. But in the case of the compound contracts here referred to, both are in issue because of the combination, and the question resolves itself into another, how for the purpose of valuation shall the contract be construed?

The language employed differs in different policies. In some it has been so veiled as to afford ground for the charge of misleading the insured into the belief that the contract is one of ordinary whole life. But I am talking simply of such as clearly express the intention of the parties, that the insurance is for a limited term, to be followed by a contract for the remainder of life, and the premium rates are separately charged for each—contracts whose language is so clear and explicit as to rule out all claim on the part of the insured to bring in extrinsic evidence to aid in their definition, in which it cannot be said that the terms of the promise admit of more senses than one, and should be performed in the sense in which the promisor apprehended that the promisee received it. When a policy thus clearly expresses that it is for a limited term, to be followed by a whole-life contract, and the whole contract is expressed in the policy, a court has no option but to so construe it, and the State Department is legally bound to accept such construction as the basis of its valuation.

Now, what is legally the net value of such combined contracts? It is the sum required to make good the deficiency of future net premiums receivable. When the policy is first issued no other insurance is in force than for a limited term. The parties have distinctly so agreed. The net premium can be no other than that required for the term in question. There can be no deficiency at the end of the term, for the premium is bound to be adequate. It follows that the net premium for the balance of such a contract must be the premium which will then just meet the expected future mortality at the assumed interest rate. The same fact will be apparent if we consider the principle involved in the computation of net premiums on this general class of policies. When two inde-

pendently complete contracts are included in a single instrument the only general rule by which the premiums are determined is to treat them as separate. The premium even of an ordinary endowment is computed by adding the premium of a term policy to that of a pure endowment, although the two are not made separately complete in the instrument. The premium on a policy to run for a specified term and to begin n years thereafter to run for another specified term, conditional on the payment of premium, is at the start the premium required for the initial term, to which is added the premium for that which follows. This is the actuary's recognized method of computation. Is the legal character of such a contract altered if clearly expressed, by reducing n to zero? Or suppose two separate policies are issued, the one for a preliminary term, the other to begin at its expiration, and in each is stated that the acceptance of the other is part of the consideration, must they not be separately valued according to their terms? Is the legal rule of valuation altered when these two contracts, still maintaining their separate entirety, are written and executed in a single instrument? In a word, does not the law require the Commissioners to construe them as a distinct preliminary, and subsequent contracts separately chargeable with the net premiums appropriate to each? Has he any power under the law to alter the contract made by the parties when it clearly expresses the intention that the preliminary premium is paid only for a term insurance? I answer no.

Three strong objections are made to this conclusion. First, that it is an evasion of the spirit, if not of the letter, of the net valuation law. Second, it is inconsistent in imposing different values on contracts substantially the same. Third, contracts so construed would discriminate between parties of similar age and expectation of life, and therefore be illegal.

The answer to the first must be sought in the character of the net valuation law itself, which is so generally misunderstood. That it neither does nor can insure solvency or even guarantee an adequate insurance fund when complied with, every actuary knows. Yet such is the popular belief. The design of the law, as explained by its author, Elizur Wright, was to protect the equities of the policyholder by preventing the expenditure of moneys paid in trust for the creation of a reserve fund, in the expectation of supplying the deficiency from future premium receipts. By compelling fidelity to the trust it would guard against insolvency through its violation. The law assumed as its basis the actuarial method

of computing the pure premiums that would equal the benefit according to a given mortality and interest rate, and adding a loading for expenses and contingencies to establish the office charge. This assumption was implicitly read into the contract by the statute, and the company was required to live up to it in disposing of its funds. The very foundation of the law is this assumed contract between the company and its members.

That such is the real purport of the law is evidenced by the statutory method frequently prescribed for dealing with technical deficiencies. The companies are not to be treated as actual insolvents, but simply to cease new business until they have complied with its requirements. There is apparently nothing in all this to prevent a company from contracting with a member for a preliminary term insurance which calls for no reserve in which no trust is created for its reservation. It must simply reserve according to the terms of its contract. If the premium actually charged is less than the net, an annuity must also be charged that will make good the future deficiency. This means that the law has no concern with the expenses of management or contingencies of any kind, nor with the actual premium agreed on, except that the full future mathematical premiums involved must be provided for either by the contract with the assured or by the company from its own funds; an anomalous condition if commercial solvency is the purport of the law.

Again the law was originally formed to meet conditions as they existed forty years ago. Those conditions have greatly changed. It is no longer practicable to do business on the same terms. What then might have seemed a mere attempt to evade the statute can to-day be pleaded at least as an apparent business necessity. It is a rule of statutory construction that the law if possible should follow the reason for the law. A construction which might have applied forty years ago need not of necessity apply to-day.

But it is said that the benefit granted under a preliminary term contract is in all essentials the same as under a continuous whole-life policy, therefore the value must be the same. The fallacy of this argument also lies in the assumption that the reserve is the measure of actual solvency, whereas, it is simply the sum required to be laid by under the assumed contract with the insured. The law does not read into the preliminary term policy, the same contract with the insured as to his net premium payments as in the simple whole-life policy. If a simultaneous

term insurance be granted to A, and a whole-life policy to B, to begin at the end of the term, and an immediate whole-life policy to C, all of equal age, no department in the country would assign a value to the first two equal to that of the third, though the benefits are equal, because the implied contract conditions as to net premium payments differ. It is no sufficient reply to say that in practice such deferred policies are not written. One could be legally issued to-day. It is a familiar law in many corporations that the directors can neither borrow nor be interested in contracts. But when those same directors have become members of a separate organization they may indirectly through its agency do either. In both cases the benefit may be the same, but the form of contract or of obligation is different.

The third, and perhaps most weighty, objection is the alleged violation of the anti-discrimination law. That law forbids discrimination between individuals of the same class as to the amount of premium charged. To be of the same class there must be an essential sameness of the contract, not simply of the amount of benefit. A policy stipulating for mere term insurance is not the same as another which couples with it an obligation to grant whole-life insurance at the end of the term. So long as every individual insurable on equal terms can buy either of these two contract forms at the price fixed by the company, there is no discrimination in the sense of the statute. The company is not forbidden to fix its own price, however arbitrary, on any contract form which it may issue. It has been asked why, if the principle is sound in case of preliminary contracts for a single year, the term may not be extended and the creation of a reserve be postponed indefinitely. I answer that it can until a point is reached where the premiums charged will no longer equal the mathematical for the remaining period. This, if you choose to call it so, is the weakness of the law.

I am not seeking in this argument either to defend or condemn the statute, but simply to show what I believe to be its correct interpretation. I believe that the real quarrel of those who object must be with the statute itself. If it fails of its assumed purpose, the remedy is a change of the law, not a forced construction of its requirements. Despite the explanations of its author, and its well-understood character among actuaries, the conviction has been widespread that its application furnishes a test of commercial solvency, and that the liability which it measures represents not

simply assumed contract obligations to existing policy holders, but an actual deficiency of their future payments, hence that the law must be construed, so far as possible, to effectuate that object. That one company may meet its requirements with every policy paid up on its books and not a dollar for future expenses, while another may fall short with a future premium income ten times its obligations, and that two companies may issue similar policies on the same lives and be charged with the same obligations, however different their premium rates, proves that the operation of the law is limited to maintaining the fund called for by the mathematical premium of the contract. A future premium, adequate to future contingencies and expenses, or its equivalent in the possession of an adequate annuity fund in hand, is essential to any law which would prescribe conditions of commercial solvency.

There is another phase of the question to which these arguments do not apply. I refer to contracts so drawn as to deceive the insured into the belief that they are simple whole-life policies in which the term feature, with its consequent surrender of claims to a reserve, is so carefully disguised that none but an expert could discover the hidden meaning. To such the legal maxim already quoted applies, that the instrument must be construed in the sense that the promisor apprehended that the promisee received it. Apart from this I can find nothing in the law as it stands which does not compel the valuation to be made like that of the renewable term policy, according to the term of the instrument. The question is simply one of sound practice. Failure of laws to accomplish what was expected from their enactment and methods of avoiding the operation of laws that hinder business competition are among the most familiar features in the history of legislation. The sale of his paper by the note broker at a discount below the legal rate is as much an evasion of the law as the issue of a preliminary term contract exempting the reserve, but nobody will question the legality of the note broker's act.

Note on Accuracy in Numerical Tables.

OSCAR B. IRELAND.

THE tables of numbers that are used by actuaries and other mathematicians are generally, of necessity, but approximations to the exact truth. If a logarithm could be carried out to so many decimal places as to show its precise value, the result would not be of practical use; if the probability of a given life's continuing through a year were absolutely indicated by a line of twenty decimals, they would not be employed in office work.

But the getting of tables that are as accurate as they profess to be, and that are carried out far enough to serve the best practical ends, is a serious and difficult matter. In the first place the calculations, both preliminary and final, must be extended enough to give, when the tables are used, results that are as near to the exact truth as the limits within which they must be expressed will allow; and secondly, the work should be so done that the errors of computer, copyist and printer may all be eliminated.

The paper presented in May by Miss Cushman calls attention to the insufficiency of eight-decimal place values of "v" as given by Chisholm, Meech, and other standard authors, and provides for all conceivable needs, for three different rates of interest, by giving these values carried out to twenty-five places of decimals. It is believed that only in the table prepared by Mr. Joseph H. Sprague, ten years ago, and presented to the Society, are some of these values given at greater length.

On the other hand, the standard set by so important a work as the Text Book of the Institute of Actuaries (1887) calls for much less minuteness of statement; logarithms of the D, N, and M columns are given to five places only, and the D columns contain, for the most part, but five figures. In the H^M tables published by the Institute in 1872, six-place logarithms were given, and the significant figures in the D columns were six, which same limits were observed in Meech's "System and Tables of Life Insurance."

In the American 3 per cent. tables published by the Connecticut Mutual in 1890, and in the elaborate volume on the same basis that the Mutual Benefit has just put out with such a generous hand,

seven-figure logarithms and seven-place D's are used. It seems to the writer that this last standard is the best, for while more abbreviated tables would in very many cases give the same practical results, there are times (for example, when work is being done by different computers, or in different ways) at which it is highly unsatisfactory to have results that are only "nearly" right, with no ready means of determining whether the computer is at fault or not. And it may be a matter of practical importance to know that a given result is correct to the last cent. In calculating the term of extended insurance under the contracts offered by so many companies, a very trifling difference in reserves will sometimes make a difference of a day or two in the extension, and this day or two might be the turning point as to whether the policy were a claim or not.

As to accuracy of workmanship, who has seen a perfect book? and who does not prize, with a great respect, the book that is most nearly infallible? Before De Morgan wrote his descriptive preface to the logarithm tables of Dr. Schrön he had twenty-four pages of them critically examined, with the result that the only faults discovered were an 8 upside down in one place and a 0 upside down in another; upon which he remarks: "that twenty-four pages selected at hazard should be found entirely free from error is what would seldom occur in a newly published table or book of any kind." Alas! who of us has not had the aggravation of trying to use a book where scarcely four pages could be found without an error, let alone the other twenty?

If the patience of German workers has produced remarkably good results in this direction, the perseverance of the English has also accomplished a good deal. Take for example Oakes's Reciprocals, with its two hundred and more pages: the work was carefully done by logarithms, with occasional arithmetical checks; it was all done in duplicate, and proofs made from one manuscript were read with the other; second and third proofs were examined by differences; and finally the whole was examined, and virtually recomputed, by use of the arithmometer. And yet mistakes crept into the book, for in two cases there is an error in the printing of the consecutive numbers, in the first column, that constitute the argument.

As to the best method of doing work of this kind, expert opinions are divided between logarithms, calculating machines, and plain, schoolboy arithmetic. Probably the majority favor the logarithmic

method, and this has the advantages that each step of the work is put on record and can be tested; and that a series of logarithms, once obtained, can often be made of use in several processes thereafter. In getting up joint life commutation columns, for instance, the same set of logarithms can be used over and over again. But when extreme accuracy is essential, one encounters such facts as this: in the Mutual Benefit tables N_{24} is given as 98407.723, with a logarithm of 4.9930292; but if one enters a seven-place table of logarithms, Schrön's, for example, with this logarithm he will find as the corresponding number 98407.727, and the same logarithm corresponds to all the numbers from 98407.716 to 98407.738.

To get the best results from the arithmometer it must be handled by one who knows a great deal more than how to turn the crank; he must have an intelligent understanding of the work that he is doing, and that combination of mathematical ingenuity and mechanical instincts that has enabled some of our own members to achieve such excellent results in this field.

In his preface to "Valuation Tables" based on the H^M table (1873), Mr. R. P. Hardy says that the "tables exhibit, it is believed, the first systematic attempt to utilize, upon a large scale, the arithmometer"; and he gives it as his decided opinion "that the arithmometer is an invaluable aid, in point both of accuracy and economy of time." The machine has the drawback that it leaves no tracks; no intermediate steps are recorded, and after a result has been copied from it, and another operation begun, no evidence can ever prove, to a certainty, just what the machine said.

There are still some computers, however, who maintain that for accurate results when dealing with large numbers the only satisfactory helps are the lead pencil and the multiplication table, and who bring to bear so much ingenuity of method, and so much practical skill, that the results they accomplish are surprising.

In general it may be said that all work of the kind under discussion should be done by two different computers, or by two different methods, or both; and that a simple calling back of proof sheets with manuscript is no guarantee of correctness in the printing.

The record of the accuracy of American publications is not a wholly satisfactory one: books whose authors have had the backing of substantial companies are in general the best, and often show creditable work; those prepared by individuals and put upon the market are sometimes sadly lacking in correctness. The large book

of Wright's tables, a monument to the industry of its compiler, has very many errors in it; and at least three volumes, in later years, have come from the American press, marred by a very damaging number of mistakes.

Mr. Starr's little book of annuities and Mr. Meech's extensive work have been admired for their accuracy, and when a collection of tables like the Connecticut Mutual's can be set right, after a ten-years' trial, by a table of errata with only five items in it, the compiler may well be congratulated.

This note is presented with the hope that other members will give to the Society their experience in work of the kind described, with such hints and suggestions as may be of practical use to other travelers along the same path.

Actuarial Science in Canada.*

THOMAS B. MACAULAY.

A SKETCH of the history of actuarial science in Canada must necessarily be brief. The Dominion is but in its youth, and its oldest Life office is little more than fifty years of age. The country, moreover, is so closely united to the motherland by political ties, and to its great neighbor to the south by geographical position, that it has very naturally been deeply influenced by the views and practices prevailing in these centres, and its actuarial history is indissolubly associated with theirs.

Certain Scottish offices established agencies in Canada about the year 1840, and some of the American companies shortly afterwards followed their example, but this introduction of life insurance as a business can hardly be said to mark the beginning of the study of life contingencies as a science. The first native company to be established was the Canada Life Assurance Company, which dates from 1847, and the actuarial history of the Dominion may probably be said to begin with that event. The founder and first president and manager of the new institution was Mr. Hugh C. Baker, a gentleman who had previously been a banker, and who, in addition to his financial experience, possessed a sound judgment and good education. He was by no means a mere man of affairs, but made it his duty to understand the mathematical principles on which the success of his company depended. To him may fairly be awarded the honor of being the first Canadian actuary.

In 1868 a second Canadian company opened its doors, and from that time onward the number has steadily increased, until there are now eighteen such institutions in active operation.

By the year 1875 the business had assumed sufficient importance to justify the legislature in establishing a Government Insurance Department and appointing a superintendent with supervisory powers over all the companies represented in the country. The first report issued by this official contains statements which had been furnished to him by the companies, and which stretch back to the year 1869. The development of the business from these humble beginnings has been very marked, and the progress of the Canadian companies is particularly noteworthy.

* Read before the International Congress of Actuaries, Paris, June 30th, 1900.

NEW ASSURANCE EFFECTED IN CANADA.

Year.	Canadian Companies.	Per Cent. of Total.	British Companies.	Per Cent. of Total.	American Companies.	Per Cent. of Total.
1870	$1,584,456	13.0	$1,657,493	13.6	$8,952,747	73.4
1885	14,881,695	54.8	3,950,647	14.5	8,332,646	30.7
1898	35,043,182	64.0	3,323,107	6.1	16,398,384	29.9

PREMIUM INCOME FROM CANADIAN POLICIES.

Year.	Canadian Companies.	Per Cent. of Total.	British Companies.	Per Cent. of Total.	American Companies.	Per Cent. of Total.
1870	$203,992	13.9	$531,250	36.3	$729,175	49.8
1885	2,092,986	45.3	803,980	17.4	1,723,012	37.3
1898	7,107,073	59.3	1,210,601	10.1	3,676,490	30.6

ASSURANCE IN FORCE IN CANADA.

Year.	Canadian Companies.	Per Cent. of Total.	British Companies.	Per Cent. of Total.	American Companies.	Per Cent. of Total.
1870	$6,404,438	15.0	$17,391,932	40.7	$18,898,353	44.3
1885	74,591,139	49.7	25,930,272	17.3	49,440,735	33.0
1898	226,209,636	61.5	36,606,195	9.7	105,708,154	28.8

It will be seen that the home companies have made great strides forward in the last quarter of a century, and that they now secure by far the greatest portion of the assurances written in the Dominion. The advances made by them are even greater than the foregoing comparison would indicate, for five of the leading offices now transact business abroad, and the results of their operations beyond the boundaries of Canada are not included above. Besides increasing the volume of their transactions, this broadening of the field has had a tendency to broaden also the scope of actuarial thought and investigation. In almost every portion of the British Empire, in the United States, in France, in Belgium, in Chili, in the open ports of China, and elsewhere, some one or more of our companies are represented, and the problems which the managers and actuaries have to solve are therefore of a very varied and cosmopolitan character. Canadian actuaries of to-day require to be posted to an extent which would hardly have been dreamed of a few decades ago, on the mortality prevailing not merely in their own and other

northern countries, but in the tropical and sub-tropical sections of almost the entire world. And it is by no means merely to such climatic questions that they have paid special attention. To Canada, for example, may fairly be given the credit of having introduced to the American Continent the system of assuring under-average lives by policies subject to liens or contingent debts. The European system of making an addition to the age of the life has never been practiced to any great extent in either Canada or the United States, but the contingent debt plan bids fair to become one of the features of American life assurance. The same remarks apply, though with less force, to the Australian non-forfeiture system, which has likewise found its way into the Continent through Canada. Reference might also be made to several other features peculiar to the life assurance practice of the country, as, for example, the "minimum" system of one of our companies, by which premiums are reduced by discounting a portion of the probable future profits. I mention these points merely to emphasize the fact that although in life assurance Canada has unquestionably been very deeply influenced by the United States, it nevertheless has a decided individuality of its own, and has in its turn not been without influence on its great neighbor.

Until comparatively recent times, Canadian actuaries were cut off by the barrier of distance from intercourse with their professional brethren abroad. Moreover, the head offices of the native companies themselves are in several cases hundreds of miles away from their nearest neighbor. Each actuary therefore took his own independent course, qualifying himself by reading the text-books and standard authorities, and following more or less closely the Journal of the Institute of Actuaries, but knowing comparatively little of the work being done by his confrères on his own side of the Atlantic. Desultory study of this kind is apt to be fragmentary and unsatisfactory, only such subjects being taken up as for the moment are interesting and important. Now, however, all this has been changed by the introduction of two new factors into the case. The first of these is the establishment by the Institute of Actuaries of centres at Montreal and Toronto at which candidates may present themselves for the yearly examinations of that body. The printed papers are sent out to the local supervisors under seal in the usual way, the answers being returned to London to be passed on by the chief examiners there. This privilege has been much appreciated, and many of those connected with the actuarial departments of the companies have taken advantage of it. A number of candidates have even

presented themselves from the United States. These examinations supply just the stimulus that was needed to induce our young men to take up the methodical and thorough study of the science in all its branches, and the results have been most encouraging. Already the standard of knowledge of the clerks employed on the various actuarial staffs is much higher than it was ten years ago, and I have the impression that intellectual equipment of these rank and file men of the Canadian companies will now compare very favorably indeed with that of those occupying similar positions in British or American offices. For all this we Canadians owe a deep debt of gratitude to the Institute of Actuaries, to whose broad and liberal policy these results are due.

But while these examinations have done so much for the juniors, and promise to raise up in years to come a large number of thoroughly qualified men, it is also true that the establishment of the Actuarial Society of America has had an even more important influence on the profession. In fact, it is difficult to overestimate the effect which has been produced by this latter society. When it was founded in April, 1889, it was, by the courtesy of the United States members, who were of course in an overwhelming majority, placed on an international basis. Even the name, which was at first proposed, "The American Actuarial Association," was changed on this account to "The Actuarial Society of America," the noun "America" being continental in its significance, and therefore preferable to the adjective "American," which is usually applied only to the United States. Of the thirty-eight charter members of this society, four were Canadians. The total membership has now grown to one hundred and five, exclusive of the Associates. Of this number seventy-two reside in America, nine of whom are Canadians. Of these latter, five are also Fellows of the Institute of Actuaries, three of them by examination.

While it has been perhaps necessary to give these details regarding the membership of the Actuarial Society, I have yet done so with some hesitation, for in the Society we all meet on a common ground. Americans and Canadians alike feel that the Society is theirs, in it take an equal pride, and from it reap equal benefits. Two of the eleven movable autumnal meetings have been held in Canada, the first in Toronto, the second in Montreal. The relations between the members are so cordial that the question of nationality occupies a very subordinate place, and is rarely heard of, except perhaps when the American members desire in some

practical manner to exhibit their friendship, or to emphasize the international character of the Society, by picking out some Canadian for special honor. The semi-annual meetings, lasting always for two days, have proved themselves to be bonds of union of incalculable value, removing the harshness of competition, and establishing friendships between persons who but for them would not have known each other. But I would be sorry indeed to leave the impression that the social effects are the most important work of the Society. It has put new life into the profession. The interest now taken in actuarial matters is very different from that of ten years ago. The numerous and valuable papers which have been read, the instructive and interesting discussions which have taken place, and the personal intercourse which the meetings have made possible, have, I think I may safely say, educated the mind and enlarged the views of every actuary on the continent. The profession on both sides of the line now occupies a recognized and honored position. The achievements of the Society have begotten in its members a sense of present self-respect, and at the same time high ideals and great expectations for the future. One of the most recent developments has been the establishment of a system of examinations which are held at various centres throughout the United States and Canada, and we have every reason to hope that these will exert a great and steadily increasing influence. Already the class of subordinate members, known as Associates, shows signs of rapid growth.

I had intended originally to enlarge this sketch by including a brief statement of the circumstances under which each of the Canadian companies came to be founded, and some facts regarding its founders. I have, however, been reluctantly compelled to abandon this idea for the present, partly because of the difficulty of obtaining full details regarding some matters now long past, and partly because of my own lack of time. I cannot, however, close without at least mentioning the names of a few of the pioneers who have preceded us. Of those who are yet in active service I will say nothing, but will speak only of those who have passed from us, or who are spending the evening of life in well-deserved rest from the worries of business. Of Mr. Hugh C. Baker I need add nothing. Another name, however, deserves special mention. On the establishment of the Government Insurance Department in 1875, Professor J. B. Cherriman, M.A., F.I.A., was appointed the first superintendent. The choice was a happy one, for Professor

Cherriman was a mathematician and astronomer of a very high order, and his scholarship, dignity and rectitude, and the non-political nature of the appointment, gave a character to the new department which it has never lost. He has been for many years living in retirement in England, and though now of advanced age, is yet, I am glad to hear, in good health. Reference must also be made to the veteran president and actuary of the Canada Life Assurance Company, Mr. A. G. Ramsay, F.I.A. Trained up in the head office of the Scottish Amicable Life, Mr. Ramsay succeeded to the position formerly held by Mr. Baker on the death of the latter. He successfully occupied the managerial chair for about thirty-five years, and has only recently retired on a well-earned pension. Another name is that of Mr. William Hendry, manager from its inception of the Ontario Mutual Life, and since his withdrawal from active work, a member of its board of directors. Mr. Hendry is a splendid specimen of the self-taught actuary and manager, and there are few men more highly respected or more deservedly honored than he.

But now I must close. Although our history in the Dominion is as yet but brief as regards actuarial work, the rapid increase in the number of our companies and consequently of our actuaries gives us every reason to hope that when a sketch similar to this shall be written for an international Congress to be held twenty-five years hence, that sketch will be longer and more important than the brief record which I have now had the pleasure to present.

[Presented by John B. Lunger.]

On the Mortality during First Policy Year under Endowment Insurances. Institute-Faculty Experience.

ARTHUR HUNTER.

WITHIN the last few weeks the data regarding Endowment Insurances, collected by the Institute of Actuaries, and by the Faculty of Actuaries (Scotland), have been published. In the publication are given the "Exposed to Risk" and "Deaths," according to policy years on lives insured in the various British Companies from the year 1863 to 1893, under the heading of "New" insurances. Statistical information is also given for "Old" insurances effected prior to 1st January, 1863, and brought under observation at their policy anniversaries in that year. No graduation or adjustment of the mortality among either "Old" or "New" insurances is given, with the exception of the grouping of the data for each ten years of age. That graduation is somewhat rough, and it is hard to make proper comparisons with other mortality experiences. I have accordingly made a graduation of the first year's mortality among "New" Endowment Insurances, and present it herewith. The work has been done by the Graphic Method as evolved by Dr. Sprague. The table on the following page shows the adjusted death rate per thousand, together with first differences.

The mortality rate is surprisingly low. In order to measure it with the mortality of a standard table I have calculated the expected number of deaths by the Actuaries' Table. The Expected deaths by that table would be 1,114.7, whereas the Actual deaths were only 340, or 30.5 per cent. of the Expected according to the Actuaries' Experience. As there were 128,128 "Exposed to Risk" in the first policy year, the material is large enough to give fairly reliable results. While it was expected that the first year's mortality among Endowment Insurances would be low, there are several reasons for its being so moderate, among which are the following:

(1) The practice of dating back policies in Britain is very much more common than in this country. This is due to the system of

FIRST YEAR'S MORTALITY—ADJUSTED.

INSTITUTE-FACULTY EXPERIENCE, 1863-1893—ENDOWMENT INSURANCES.

Age.	Death Rate per 1,000.	First Difference.	Age.	Death Rate per 1,000.	First Difference.	Age.	Death Rate per 1,000.	First Difference.
20	1.93	3	34	2.85	13	48	5.83	34
21	1.96	3	35	2.98	14	49	6.17	36
22	1.99	3	36	3.12	15	50	6.53	38
23	2.02	3	37	3.27	16	51	6.91	41
24	2.05	4	38	3.43	17	52	7.32	43
25	2.09	5	39	3.60	18	53	7.75	47
26	2.14	6	40	3.78	20	54	8.22	53
27	2.20	7	41	3.98	22	55	8.75	59
28	2.27	8	42	4.20	23	56	9.34	65
29	2.35	8	43	4.43	24	57	9.99	73
30	2.43	9	44	4.67	26	58	10.72	84
31	2.52	10	45	4.93	28	59	11.56	95
32	2.62	11	46	5.21	30	60	12.51	
33	2.73	12	47	5.51	32			

distributing profits every five or seven years. If the books close at the end of December, an applicant who is admitted at that time gets one year's more bonus addition than he who enters in, and has his policy dated January of the next year. The books are accordingly held open for two or three months after the date on which the financial year is supposed to close, and most policies issued during that period are dated back. It cannot be said, therefore, that there is a full year's experience during the first year after the date of the policy.

(2) The element of fraud is more largely eliminated in Great Britain than in America. This is due, in a great measure, to the custom of getting business from local agents who are very often the lawyers, bankers or postmasters of the town or village, and who, therefore, are not solely dependent upon their income from insurance. These men do not push for new business, but take such as naturally comes to them in the pursuit of their main business or profession. They are required to make a signed report on each risk, giving full information regarding habits, health and financial condition of the applicant. As the insurance is only a side issue, there is no especial incentive to gloss over certain imperfections in the applicant. In addition to the agent's report,

statements are required from two personal friends of the insured on similar lines to those required from the agent.

(3) So far as can be judged from the experience of several companies, the rate of mortality from suicide is not so great among insured lives in Great Britain as in the United States.

For the above reasons the mortality should not be taken as an indication of that which should prevail in this country ; in fact, I believe that the average mortality for the first year in all companies in the United States is about 50 per cent. higher.

The success of the graduation may be partly judged from the following table :

Ages.	Exposed to Risk.	Actual Deaths.	Expected Deaths.	Ratio.
20 to 29	67,472	138	143.1	104 per cent.
30 to 39	47,500	138	135	98 per cent.
40 to 60	13,156	64	62	97 per cent.
	128,128	340	340.1	100 per cent.

It would be exceedingly interesting to obtain the mortality rate according to Amounts (rate of financial loss), and compare it with the mortality rate according to Lives, but the statistics for such comparison have not yet been published. If we may judge from previous investigations into the respective mortalities according to Lives and according to Amounts in Great Britain, the latter will not be larger than the former.

In the same volume are given the data for Whole-Life Insurances with limited number of premiums. Applying my graduated mortality for Endowment Insurances to these Limited-Payment policies there are found to be Expected losses of 95, whereas the Actual losses are only 77. It therefore appears that in the Institute-Faculty Experience the mortality during the first year among Limited-Payment policies was actually less than among Endowment Insurances. It would, however, be unwise to lay too much stress on the first year's mortality on Limited-Payment policies, as the number "Exposed to Risk" is not large.

In order to determine whether the second year's mortality among Endowment Insurance and Limited-Payment Insurances was not inordinately high to compensate for the very low mortality of the first year, I tested the data by the Graduated Mortality rate

of the first year. On applying my Graduated q_x of the first year to the second year's data, the Expected deaths among Endowment Insurances were found to be 281, and the Actual 412, a ratio of 146 per cent., which is higher than would naturally be anticipated. The Expected deaths among Limited-Payment policies were 87, according to the Graduated Mortality rate of the first year, whereas the Actual deaths were 118, a ratio of 135 per cent. In the Institute of Actuaries' tables, published in 1869 (H^M), I find that the ratio of second to first year's mortality is 136 per cent. approximately. It therefore appears that the death rate of the second year among either Endowment Insurances or Limited-Payment Insurances was not *inordinately* high, in comparison with the first year's mortality.

The lowness of the Mortality may be best appreciated by showing that the Actual deaths among Endowment Insurances in the second year were only 44.8 per cent. of the Expected deaths, according to the Actuaries' Table. It will be remembered that the percentage of Actual to Expected for the first year by the same standard (Actuaries') was 30.5 per cent.

[Presented by James M. Craig.]

CHILD'S ENDOWMENTS.

A. C. WASHBURNE.

THE following formulæ may be considered as supplementary to the paper entitled, "Simple Endowments and Insurances involving a Return of Premiums with Simple or with Compound Interest," presented by Mr. D. P. Fackler to this Society, at its meeting in May, 1893.*

The title, "Child's Endowments," is here used because these formulæ have been extensively used for that class of policies; however, they are applicable to the higher ages.

To find the net single premium for an endowment, the premium to be returned with compound interest at the tabular rate per cent., in the event of the nominee dying before the age at which the endowment is payable.

Let Π = the net single premium,
$g\ \Pi$ = the gross single premium,
and i = the tabular rate of interest.

Here the benefit consists of two portions, first, the endowment payable if the life survives "n" years, and second, an insurance of $(1+i)g\ \Pi$ during the first year, $(1+i)^2 g\ \Pi$ during the second year, etc.

Since $1+i=\frac{1}{v}$, $(1+i)^2=\frac{1}{v^2}$, etc.

$$\text{Benefit side} = \mathsf{D}_{x+n}+g\ \Pi\left(\frac{\mathsf{C}_x}{v}+\frac{\mathsf{C}_{x+1}}{v^2}+\text{ etc. }+\frac{\mathsf{C}_{x+n-1}}{v^n}\right)$$

$$\text{Payment side} = \Pi\ \mathsf{D}_x$$

$$\frac{\mathsf{C}_x}{v}+\frac{\mathsf{C}_{x+1}}{v^2}+\text{ etc. }+\frac{\mathsf{C}_{x+n-1}}{v^n}=\frac{v^{x+1}d_x}{v}+\frac{v^{x+2}d_{x+1}}{v^2}+\text{ etc. }+\frac{v^{x+n}d_{x+n-1}}{v^n}$$

$$=v^x(d_x+d_{x+1}+\text{ etc. }+d_{x+n-1})$$

$$=v^x(l_x-l_{x+n})$$

$$=\mathsf{D}_x-\frac{\mathsf{D}_{x+n}}{v^n}$$

Hence, Benefit side $=\mathsf{D}_{x+n}+g\ \Pi\left(\mathsf{D}_x-\frac{\mathsf{D}_{x+n}}{v^n}\right)$

* See Volume III., Transactions, pp. 9—13.

$$\text{whence, } \Pi = \frac{D_{x+n}}{D_x - g\left(D_x - \frac{D_{x+n}}{v^n}\right)}$$

$$= \frac{D_{x+n}}{g\,\frac{D_{x+n}}{v^n} - (g-1)\,D_x}$$

$$= \frac{1}{g\,(1+i)^n - (g-1)\,\frac{D_x}{D_{x+n}}} \qquad (1)$$

The reserve for the above form of policy at the end of "z" years after issue is the single premium to secure $(1+i)^{z+1}\,g\,\Pi$, in event of death during the $z+1$st year after issue, $(1+i)^{z+2}g\,\Pi$, during the $z+2$d year, etc., and 1 if alive at the end of the nth year.

Let ${}_zV_x$ = the required reserve,

$$\text{then } {}_zV_x = \frac{D_{x+n} + g\,\Pi\left(\frac{C_{x+z}}{v^{z+1}} + \frac{C_{x+z+1}}{v^{z+2}} + \text{etc.} + \frac{C_{x+n-1}}{v^n}\right)}{D_{x+z}}$$

The expression, $\frac{C_{x+z}}{v^{z+1}} + \frac{C_{x+z+1}}{v^{z+2}} + \text{etc.} + \frac{C_{x+n-1}}{v^n}$, by conformity with a similar expression in the above development,

$$= D_x - \frac{D_{x+n}}{v^n} - \left(D_x - \frac{D_{x+z}}{v^z}\right) = \frac{D_{x+z}}{v^z} - \frac{D_{x+n}}{v^n}$$

$$\text{Hence, } {}_zV_x = \frac{D_{x+n} + g\,\Pi\left(\frac{D_{x+z}}{v^z} - \frac{D_{x+n}}{v^n}\right)}{D_{x+z}}$$

$$= \frac{D_{x+n}}{D_{x+z}} + g\,\Pi\,\frac{1}{v^z} - g\,\Pi\,\frac{1}{v^n}\,\frac{D_{x+n}}{D_{x+z}}$$

$$= g\,\Pi\,(1+i)^z - [g\,\Pi\,(1+i)^n - 1]\,\frac{D_{x+n}}{D_{x+z}} \qquad (2)$$

In formula (2) a different table of mortality than that used in formula (1) may be employed. That is, the quantity $g\,\Pi$ in formula (2) is the same amount as calculated by formula (1), but its factors, g and Π, may differ.

To find the net annual premium for an endowment, all premiums to be returned with compound interest at the tabular rate per cent. in the event of the nominee dying before the age at which the endowment is payable.

Let π = the net annual premium,

$g\,\pi$ = the gross annual premium,

and $F_n =$ the amount of 1 per annum at the end of "n" years at the required rate of interest,

$$F_n = (1+i) + (1+i)^2 + \text{etc.} + (1+i)^n, = \frac{\frac{1}{v^n} - 1}{1-v}$$

$$= \frac{1 + v + v^2 + \text{etc.} + v^{n-1}}{v^n}$$

The benefit now consists, first, of the endowment payable if the life survives "n" years, and second, of an insurance of $F_1 g\,\pi$ during the first year after issue, $F_2 g\,\pi$ during the second year, etc., therefore,

Benefit side $= \mathsf{D}_{x+n} + g\,\pi\,(F_1\,\mathsf{C}_x + F_2\,\mathsf{C}_{x+1} + \text{etc.} + F_n\,\mathsf{C}_{x+n-1})$

Payment side $= \pi\,(N_x - N_{x+n})$

$$F_1\,\mathsf{C}_x = \frac{1}{v}\,v^{x+1}\,d_x \qquad = v^x\,d_x$$

$$F_2\,\mathsf{C}_{x+1} = \frac{1+v}{v^2}\,v^{x+2}\,d_{x+1} \qquad = v^x\,d_{x+1} + v^{x+1}\,d_{x+1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \qquad \cdots\cdots\cdots\cdots\cdots$$

$$F_n\,\mathsf{C}_{x+n-1} = \frac{1 + v + v^2 + \ldots + v^{n-1}}{v^n}\,.\,v^{x+n}\,d_{x+n-1}$$

$$= v^x\,d_{x+n-1} + v^{x+1}\,d_{x+n-1} + \ldots + v^{x+n-1}\,d_{x+n-1}.$$

Taking the last expressions in column, the sum of first column

$$= v^x\,(d_x + d_{x+1} + \text{etc.} + d_{x+n-1}) = v^x\,(l_x - l_{x+n}) = \mathsf{D}_x - \frac{\mathsf{D}_{x+n}}{v^n}$$

second column

$$= v^{x+1}\,(d_{x+1} + \text{etc.} + d_{x+n-1}) = v^{x+1}\,(l_{x+1} - l_{x+n}) = \mathsf{D}_{x+1} - \frac{\mathsf{D}_{x+n}}{v^{n-1}}$$

nth column

$$= v^{x+n-1}\,d_{x+n-1} \qquad = v^{x+n-1}\,(l_{x+n-1} - l_{x+n}) = \mathsf{D}_{x+n-1} - \frac{\mathsf{D}_{x+n}}{v}$$

Hence, $F_1\,\mathsf{C}_x + F_2\,\mathsf{C}_{x+1} + \text{etc.} + F_n\,\mathsf{C}_{x+n-1} = \mathsf{D}_x + \mathsf{D}_{x+1} + \text{etc.}$

$$+ \mathsf{D}_{x+n-1} - \left(\frac{1}{v} + \frac{1}{v^2} + \text{etc.} + \frac{1}{v^n}\right)\mathsf{D}_{x+n}$$

$$= N_x - N_{x+n} - F_n\,\mathsf{D}_{x+n}$$

and Benefit side $= \mathsf{D}_{x+n} + g\,\pi\,(N_x - N_{x+n} - F_n\,\mathsf{D}_{x+n})$

whence
$$\pi = \frac{\mathsf{D}_{x+n}}{N_x - N_{x+n} - g\,(N_x - N_{x+n} - F_n\,\mathsf{D}_{x+n})}$$

$$= \frac{\mathsf{D}_{x+n}}{g\,F_n\,\mathsf{D}_{x+n} - (g-1)\,(N_x - N_{x+n})}$$

$$= \frac{1}{g\,F_n - (g-1)\,\dfrac{N_x - N_{x+n}}{\mathsf{D}_{x+n}}} \qquad (3)$$

The reserve for the above form of policy, at the end of "z" years after issue, is the single premium to secure an endowment of 1 at the end of "n" years, and an insurance of $F_{z+1}g\pi$ during the $z+1$st year, $F_{z+2}g\pi$ during the $z+2$d year, etc., less the present value of the future net premiums.

$$\therefore \quad {}_zV_x = \frac{\mathsf{D}_{x+n} + g\pi\,(F_{z+1}\,\mathsf{C}_{x+z} + F_{z+2}\,\mathsf{C}_{x+z+1} + \text{etc.} + F_n\,\mathsf{C}_{x+n-1})}{\mathsf{D}_{x+z}} - \pi\,\frac{N_{x+z} - N_{x+n}}{\mathsf{D}_{x+z}},$$

which, by conformity to the preceding development,

$$= \frac{\mathsf{D}_{x+n} + g\pi\,[N_x - N_{x+n} - F_n\,D_{x+n} - (N_x - N_{x+z} - F_z\,\mathsf{D}_{x+z})]}{\mathsf{D}_{x+z}} - \pi\,\frac{N_{x+z} - N_{x+n}}{\mathsf{D}_{x+z}}$$

$$= \frac{\mathsf{D}_{x+n} + g\pi\,(N_{x+z} - N_{x+n} + F_z\mathsf{D}_{x+z} - F_n\mathsf{D}_{x+n}) - \pi\,(N_{x+z} - N_{x+n})}{\mathsf{D}_{x+z}}$$

$$= \frac{\mathsf{D}_{x+n}}{\mathsf{D}_{x+z}} + (g\pi - \pi)\,\frac{N_{x+z} - N_{x+n}}{\mathsf{D}_{x+z}} + g\pi\,F_z - g\pi\,F_n\,\frac{D_{x+n}}{D_{x+z}}$$

$$= g\pi\,F_z + (g-1)\,\pi\,.\,(1 + a_{x+z\,.\,\overline{n-z-1}|}) - (g\pi\,F_n - 1)\,\frac{\mathsf{D}_{x+n}}{\mathsf{D}_{x+z}}. \qquad (4)$$

The accumulation formulæ are also applicable to these forms.

When $g = 1$, Formulæ (1) and (3) reduce to v^n and $\frac{1}{F_n}$ respectively, which, without loading, are the required single and annual premiums to produce 1 at the end of "n" years.

Combinations of these forms with those of various regular policies may be made. Such combinations especially apply to policies to be issued under the age of 21, where there are legal limitations as to the amount of insurance to be carried. They may provide for a return of premiums with interest in event of death prior to age 21, but may also enter upon the benefits of a regular contract at that age, the premium remaining level.

Address of the President,

Mr. Thomas B. Macaulay,

in opening the Fall Meeting, Oct. 25th, 1900.

Members of the Actuarial Society of America:

It is not often that a subject so full of interest and of pleasant associations presents itself as the text for a presidential address as that which is the natural theme for this occasion. During the past summer there was held in the city of Paris another of those remarkable international gatherings which will go down to history as milestones in the progress of actuarial science. It is an attractive task to attempt to convey to our brethren on this side of the Atlantic some idea of these meetings, but the difficulty, nevertheless, is great, for it is not possible to reduce to black and white the pleasure of the reminiscences with which our minds are stored. The peculiar charm which attaches to friendships which this Congress enabled your representatives to form or renew with persons interested in the same subjects as interest us, but viewing them from different national standpoints, and expressing their thoughts possibly in a different language, is something to which I cannot do justice.

The Third International Congress of Actuaries met on Monday, June 25th, 1900, in one of the large buildings of the Exhibition, known as the "Palais des Congrès," on the banks of the Seine. The members had been previously provided with passes which admitted them to all parts of the grounds. Those present had come from almost every corner of the civilized world. At the risk of wearying you I will mention the numbers from each country.

Third International Congress of Actuaries.

Country.	Subscribers.	Number Present.
Australia	13	1
Austria-Hungary	16	5
Belgium	25	5
Brazil	1	—
Canada	8	1
Cape Colony	3	—

THIRD INTERNATIONAL CONGRESS OF ACTUARIES.—Continued.

Country.	Subscribers.	Number Present.
Denmark	7	1
France	58	42
Germany	39	21
Great Britain, { England, 108 / Scotland, 15 }	123	{ 24 / 3 } 27
Holland	30	13
Italy	7	3
Japan	28	4
Luxemburg	2	—
Norway	9	1
Russia	13	5
Spain	5	1
Sweden	11	2
Switzerland	10	7
United States	64	8
	472	147

To have succeeded in bringing together such a large and representative gathering is not only an achievement of which our French actuarial friends may well be proud, but one which shows also the depth of the interest felt in the movement.

The meeting was called to order by M. Omer Lepreux, of Brussels, chairman of the Permanent Committee under the management of which the Congresses are held. M. Lepreux referred happily to the progress which actuarial science has made throughout the world in recent years, and even since the first Congress was held in Brussels in 1895. One of the most noticeable features had been the adoption, or at least the consideration, by the legislatures of various countries of measures dealing with actuarial questions. Our profession occupies a higher position in the world than ever before, and it is, perhaps, not too much to claim that part at least of the credit is due to the influence of these international gatherings.

The election of officers was then proceeded with. According to the rules governing the meeting, the president and general secretary were to be elected by the members as a whole, while the delegates from each country which had not less than fifteen subscribing members, of whom five at least were actually present, were entitled to elect from among their number a vice-president and secretary. The countries thus entitled to representation were

(in order of numbers) France, Great Britain, Germany, Holland, the United States, Belgium and Austria-Hungary. The members of other nationalities formed a separate section, with the same privileges. Our progressive friends from Japan were sorely disappointed that by the absence of one of their number, they fell just short of the needful five. And I must here record an act of kindly courtesy, which I could not but deeply appreciate. The members from the United States requested that the representatives of the Actuarial Society of America be not divided, and that the delegations from that country and from Canada be considered as one. This permission being at once given, they further did the Dominion the honor of electing its member vice-president for the two countries, the position of secretary being very deservedly given to Dr. I. C. Pierson. The complete list of officers was as follows :

HONORARY PRESIDENT, M. Millerand, Minister of Commerce, Posts and Telegraphs.

PRESIDENT, M. Paul Guieysse.

GENERAL SECRETARY, M. Léon Marie.

COUNTRY.	VICE-PRESIDENTS.	SECRETARIES.
	Messrs.	Messrs.
Austria-Hungary, .	Chas. Deperis,	J. Altenburger.
Belgium, . .	Amedée Begault,	F. Hankar.
France, . . .	H. Laurent,	Jules Cohen.
Germany, . .	Karl Samwur,	A. Gerkrath.
Great Britain, .	C. D. Higham,	Ernest Woods.
Netherlands, . .	M. C. Paraira,	C. L. Landré.
United States, .	T. B. Macaulay,	I. C. Pierson.
Other Nations, .	S. de Savitch (Russia),	Sven-Palme (Sweden).

M. Paul Guieysse, the newly-elected President, and M. Léon Marie, the General Secretary, were the unanimous choice of the meeting. M. Guieysse is a member of the French Chamber of Deputies, a former Minister, President of the Institut des Actuaires Français, a member of the national committees on railroads, on mutual benefit societies, and on workmen's accidents, a corresponding member of the British Institute of Actuaries, and, I have the pleasure to add, a member of our own Society. But besides possessing these social and scientific qualifications, M. Guieysse was otherwise a model presiding officer, being dignified, courteous and tactful. M. Marie, the popular and hardworking Secretary of

the French Institute is so well known to our members as the result of the two preceding Congresses that he needs no introduction at my hands. M. Guieysse, in his opening address, extended a cordial welcome to the visitors, and then gave a brief outline of actuarial history. He pointed out that although this can hardly be said to date back much earlier than the beginning of this century, it has already conquered for itself a very important place, and the coming century will see it grow, flourish and bear fruit as never before.

M. Bégault, on behalf of M. Smet de Nayer, honorary president of the first Congress, and one of the honorary vice-presidents of this one, expressed regret at his inability to be present. M. Smet de Nayer is now Prime Minister of Belgium, and honorary president of the Belgian actuarial society.

M. Deperis and Mr. Higham returned thanks for the welcome which had been so heartily extended. On behalf of the English-speaking members, Mr. Higham expressed their admiration of the Exhibition, and hoped that the new century would inaugurate an era of peace, harmony and prosperity for all the countries represented. If the congress helped to even a small extent in bringing about this result, the members of the Organizing Committee would have cause to congratulate themselves.

The first question on the order paper was then taken up. It was that of assurances against invalidity arising from disease, old age or accident. The interest which the actuaries of the continent exhibited in the matter was one of the revelations of the Congress. The governments of Germany, Austria, Holland, and of other countries have either undertaken to provide, or are considering the wisdom of doing so, by compulsory assurance, for the necessities of those members of the working classes who become incapacitated from work by illness or age. But while this would appear to have drawn particular attention to the subject, and while government measures were discussed somewhat, most of the interest centred round the work of friendly societies, and the possibilities which this field opens up for assurance companies. The discussions far exceeded the time allotted to them, and even then had to be left unfinished.

Mr. Gerecke opened with a paper on the position of this branch of assurance in Germany. Several of the life companies of that country issue policies which provide in the usual way for the payment of the sum assured at death, but which also stipulate that no

premiums shall become due for the period during which the life assured is incapacitated by accident or illness. Some of the offices go further, and pay an annuity equal to five or ten per cent. of the sum assured so long as the invalidity continues. Extra premiums are, of course, charged for these benefits. Only one young company issues sickness assurance contracts not in association with life policies. All the companies base their calculations on the experience of the German railways, as compiled by Behm, Zimmerman and Bentzien.

M. Maingie, of Brussels, followed with a paper, in which he pointed out the need of more reliable statistics and the necessity of proceeding on scientific lines, even though for a while the companies might have to proceed tentatively and experimentally.

Mr. Neison, the well-known English authority on this subject, then reviewed briefly the characteristics of the various British tables. He began with that of the Highland societies, covering observations from 1751 to 1820, and then referred to Mr. Ansell's, Mr. Neison's (Senior), the Oddfellows' (1848), Mr. Finlaison's English Friendly Societies', the Oddfellows' two later experiences (1860 and 1870), the Ancient Order of Foresters', investigated by himself, and finally to Mr. Sutton's English Friendly Societies' Experience. Some of these tables, the last in particular, are very valuable. Mr. Neison's conclusions are, however, worth quoting: "Sickness assurance is "essentially a business dependent on the efficiency of the local "supervision of the risk. Unless laxity in its administration pre- "supposes in the near future augmented taxation of the member, "it is not practicable to run the business. Assurance companies "have spent large sums of money in attempting to work it from "a central source, but in every instance, sooner or later, they have "been compelled to give it up. There would be a large fortune to "any one who could invent a method by which sickness assurance "could be run on similar lines to those which are now so success- "fully worked by the industrial assurance companies in respect of "death benefits."

Messrs. Landré and Janse, of Holland, then took up the question from their national standpoint. They presented the experience of a large industrial corporation, and mentioned that several Dutch companies have recently been established for the transaction of invalidity and accident assurance, but the results of their operations are not as yet important. They then gave some very interesting details regarding the proposed compulsory assurance law then

under consideration by the Netherlands Chamber of Deputies. The first Dutch table of sickness was constructed by Dr. Janse from the experiences of the "Caisse Générale des Malades" of Amsterdam.

Mr. Hamza, of Russia, closed with a review of the whole subject, and some details as to Russian forms of sickness assurance.

The discussion which followed was participated in by Messrs. Blaschke (Vienna), Hann (New York), Weber (Paris), Adler (London), Nicklaus (Paris), Altenburger (Trieste), Barrand (London), and others.

This question occupied the sittings of Monday afternoon, Tuesday morning and the greater part of Wednesday morning. On Tuesday afternoon a diversion was given in the form of an excursion from Paris to St. Germain. The trip down the winding Seine, which here describes an immense S, will not soon be forgotten. So tortuous is the river that nearly thirty miles have to be covered to reach the destination, which is only 11½ miles from the city by rail. We pass beautiful St. Cloud, and the distant Eiffel Tower can be seen now on the right hand, and shortly afterward in what seems a diametrically opposite position on the left. We finally arrive at St. Germain, with its far-famed terrace, its grand scenery and its historical associations. Here Louis XIV. was born, and here the exiled James II. of England lived, died and was buried. It was with regret that we turned away from these attractive scenes, and came back to Paris by train.

But I must return to business. The next subject taken up was that of methods of valuation and distribution of profits. This was opened by a very valuable paper by Mr. Gerald H. Ryan. It was in fact more than a paper—it was a small treatise, covering 36 pages, and to do justice to it by quotations or brief comments is impossible. Drs. Onnen and Peck, of the Hague, then submitted a paper explaining the manner in which the contribution idea is applied by the Dutch companies to the distribution of profits.

In the unavoidable absence of the President, on Wednesday afternoon, Mr. Higham, vice-president for Great Britain, occupied the chair. Considerable time was devoted to the discussion of the papers previously mentioned, and Messrs. Gunckel (Germany), Weeks, McClintock, Manly, Burridge, Van Schevichaven (Holland) and others took part.

The members then proceeded to the third question, that of mortality in different parts of the globe. Here again our British friends showed how heartily they supported the Congress, for the

opening paper was one by Mr. James Chatham. This article was voluminous and important, and included in its scope India, Ceylon, the British West Indies and Africa. This is probably the most useful grouping of the available statistics regarding these countries which has been published. Its value is further enhanced by a carefully prepared map of the world, in which the different sections are colored according to the rate of extra premium which, in the opinion of the author, should be charged.

On Thursday morning the speaker had the honor of being requested to preside, as M. Guieysse was unfortunately again absent. The first paper considered was one by Dr. Van Dorsten, which dealt principally with the mortality among Europeans residing in Java. This essay was the more important because, so far as I know, the statistics contained in it have not previously been available to English-speaking actuaries. The Dutch companies would not all agree to contribute their experience in the East Indies to a central committee, but, notwithstanding this, some valuable information has been obtained. Dr. Van Geer, in 1892, constructed a table showing the mortality among officers of the colonial army, their wives, widows and orphans. These results were based on the government records, to which he had access. They showed that even when deaths from wounds in war were excluded, the mortality among the officers was decidedly higher than that prevailing in Europe. The remarkable point was brought out, however, that the mortality among European women in the Dutch East Indies was apparently lower at all ages than among their sisters in northern countries. A few years later Dr. Van Geer made a much more extended investigation, covering officials of all classes in the Dutch East Indies, whether sent out by the home government or appointed in the colonies. The mortality shown in this table is more favorable than that of Finlaison's Indian Uncovenanted Service Experience, especially at the higher ages. As these later statistics of Dr. Van Geer do not include the wives, they furnished no means of checking his former results in reference to female lives. Neither are we told whether or not many native lives are included. Dr. Van Dorsten himself has worked out still another experience on the government officials sent out from Holland, excluding all those appointed in the East, and the results confirm Dr. Van Geer's figures. One noteworthy feature, however, is that of a death rate decreasing steadily from 14.21 per 1,000 at age 20, to a minimum of 8.15 at age 29, and thereafter again increasing.

M. Massé then contributed a paper dealing with mortality under "colonial risks." The French companies have agreed on a minimum tariff of extras covering all parts of the world, and this was given in full by M. Massé. He also furnished statistics regarding 1,576 French missionaries who went to tropical Asia, India, Cochin China, Annam, Japan or China. The observations extend from 1660 to 1888. Of these lives 772 had died, and 804 were still living.

In the discussion of this subject Messrs. Hann, Bourne and others took part.

The fourth question was that of the mortality in various professions. Dr. Samwur, of the Gotha Life, gave the experience of his company in regard to physicians, clergymen and teachers. The following comparison will give some idea of the results.

ANNUAL RATE OF MORTALITY PER 100, AGES 36–40.

Entire Company	Years	1829–1878	8.2
Physicians	"	1829–1885	11.0
Protestant clergymen	"	1829–1886	5.6
Teachers	"	1829–1890	6.1
Secondary professors	"	1829–1890	5.7

EXPECTATION OF LIFE AT AGE 35, AFTER ELIMINATING THE FIRST FIVE YEARS OF ASSURANCE.

Physicians	28.90	years.
Protestant clergymen	32.85	"
Teachers	32.03	"
Secondary professors	32.61	"

PERCENTAGE OF ACTUAL MORTALITY TO THAT EXPECTED ON THE BASIS OF THE SOCIETY'S GENERAL EXPERIENCE, 1829–1878.

	Ages 26–60.	Ages 61–90.	All Ages.
University professors, exclusive of professors of medicine	56.7	80.6	71.2
Secondary professors	77.4	90.2	83.5
Teachers	82.5	93.9	87.8
Protestant clergymen	73.6	94.0	85.9
Catholic clergymen	103.6	123.9	112.6
Physicians	116.2	105.1	111.0
Professors of medicine	91.6	139.5	113.8

Dr. Samwur also gave the mortality among the different professions by causes of death. His paper contains much food for thought.

Mr. McClintock presented a paper on the best way of making comparisons of death losses. In this he emphasized the importance of specialized mortality investigations. "It is more important "for the future interests of life insurance to learn how fishermen "compare with farmers, how physicians compare with clergymen, "how brewers compare with manufacturers of soda water, and the "like, than it is to gather together all the heterogeneous materials "into one grand average, in the form of a new life table. . . . . "There is almost no end to the interesting and valuable facts which "can be developed in this way. Work of this kind is particularly "useful when carried on by a combination of different companies "or by actuarial societies, because of the larger body of facts "secured, and also because in such cases there is no hindrance to "the publication of the results. It is not usually desirable to make "public the experience of a single office concerning any special "class of risks which, judged by the results, ought never to have "been accepted. This objection does not arise when a number of "offices combine to supply their materials indistinguishably to "some central authority trusted by all concerned. As time goes "on it will become more and more the duty of every actuarial "society to gather materials for the investigation of special classes "of risks varying according to occupation or otherwise."

I have pleasure in repeating this suggestion in full, because I believe it is one which will commend itself to our members, and I trust the time is not far distant when our Society will enter on this new and inviting path of usefulness. The idea has my most hearty endorsation.

Mr. Barrand's paper followed. It contained, among other matters, some new and striking facts regarding the mortality among publicans. I feel tempted to refer to it, but I have already spoken too long. The paper in any case is worthy of the careful study which I hope all our members will give to it when the published reports appear.

On Friday morning M. Guieysse welcomed to the platform Mr. T. E. Young, ex-President of the Institute of Actuaries, and of the Congress which met in London. M. Laurent, Vice-President of the French Institute, then presented a series of charts prepared by Dr. Bertillon, illustrating the mortality in various professions. The

ensuing discussion was participated in by Messrs. Weber, Laurent, Schooling and Blaschke.

The fifth question dealt with the proper method of valuing the negotiable securities owned by companies. Papers were submitted by Messrs. Emminghaus (Germany), Adan (Belgium), McClintock (United States), Hughes (Great Britain), and a joint paper by Messrs. Nierstrasz and Imminck (Holland). Messrs. McClintock, Ackland, Barrand, Hann, Quiquet (France), Bottini (Italy) and others took part in the discussion.

The Friday afternoon session was presided over by Dr. Samwur, Vice-President for Germany. Time will not permit me to do more than refer in the briefest manner to the proceedings. A long contribution on life assurance mathematics was presented by M. Bohlmann, of Göttingen. It was followed by papers on the undernoted subjects :

Valuations on the Preliminary Term Plan, by Mr. Lucius McAdam.

Some Suggested Rearrangements and Expansions of the Scheme of Universal Notation, by Dr. T. B. Sprague. This was referred to the Permanent Committee.

Life Assurance in Japan, by Mr. Awadzu.

A Table of Invalidity, based on the experience of the Berne Cantonal Sickness Assurance Society, by Dr. Moser.

Reflections on the General Theory of Assurance, by M. Nicklaus.

Considerable discussion then took place with regard to Government supervision, and in this Messrs. Lepreux (Belgium), Cheysson (France), Trefzer, Schartlin (both of Switzerland) and McClintock took part.

The Saturday morning gathering was the last business session. It was opened in person by M. Millerand, the Minister of Commerce. He made an excellent address, in which he pointed out the importance of the actuarial profession even from the standpoint of the legislature. Questions in which the statesman had to accept the actuary as his pilot were continually increasing both in number and in prominence. How important it was, then, that those pilots should be thoroughly informed! It is needless to say that M. Millerand's courtesy in opening the session and his kind remarks were much appreciated.

The papers for this last session had to be examined all too cursorily. They consisted of a most interesting series of reports on

the history of actuarial science throughout the world. They were as follows :

Australia, by Mr. Teece ;
Belgium, by M. Lefrancq ;
Canada, by the speaker ;
Denmark, by Dr. Gram ;
France, by M. Quiquet.
Germany, by Dr. Grosse ;
Great Britain, by Mr. Low ;
Holland, by the Manager of the Netherlands General Life Company ;
Japan, by Mr. Yano ;
Russia, by M. de Savitch ;
Spain, by M. Maluquer y Salvador ;
Sweden, by Dr. Tiselius ;
Switzerland, by the Director of the Federal Insurance Bureau ;
United States, by Mr. Pierson ;

These papers, I venture to say, will form a unique and valuable historical library.

The next item was one in which the members of this Society were particularly concerned. As president of our Society it was my privilege to extend a very cordial invitation in its name to the Congress to reassemble three years hence, in 1903, in the city of New York. Although we Canadians intend to do our share in entertaining our guests at that time, it was, of course, appropriate that a representative of the Society from New York itself should have a part in extending this invitation, and I therefore took the liberty of mentioning that as Mr. Pierson is now second vice-president of our Society, he would in all human probability be the president in 1903, when the fourth Congress would meet, and I referred to him the reading of the formal invitation, which I need hardly say he did in his usual happy manner, emphasizing it in such a way as to carry the meeting entirely with us. The invitation was accepted unanimously and with enthusiasm. It was surprising to learn afterwards, in private conversation, how large a proportion of those present intend to cross the Atlantic in 1903. The success of the Fourth International Congress of Actuaries rests now with ourselves. The German delegates gave notice that they intend at the next Congress to move that the one following be held in Berlin.

And now the time had come when these delightful gatherings had to end. The president, M. Guieysse, briefly thanked the members for their attendance, and for the assistance they had given in adding éclat to the tenth anniversary of the Institut des Actuaires Français. Mr. Higham acknowledged the great indebtedness of the visiting members for their cordial reception. He hoped that after having gone together thus far hand in hand for the benefit of humanity, they would continue to do so, and thus bring nearer the time of universal peace and brotherhood. He closed his very appropriate remarks by saying "Vive La France," a wish which the voices of the whole audience heartily echoed.

On Wednesday evening a brilliant reception had been given to the members of the Congress and their friends by M. Millerand, at the Ministry of Commerce. Next evening another reception was given by the members of the Committee of Organization in the Gallerie des Champs Elysées. Finally, on Saturday evening, a splendid closing banquet was held at the Hotel Continental, the attendance being limited to those who were members of the Congress. When this broke up we regretfully said good-bye to each other, hoping to meet again in New York.

Thus ended a series of meetings remarkable for the useful character of the work done, the genuine hospitality of the hosts, and the sociability and cordiality of the visitors. The ivy depicted on the silver medals which were distributed is a fitting emblem of the delightful memories clinging to this Third International Congress. If we on this side of the Atlantic are to be equally successful we will have to work hard.

How I wish I could close here! But there is another matter, very different in its nature, that I must refer to. How true it is, that in the midst of life we are in death. It is my sad duty to state that since we last met two of our members have gone from our midst, Mr. Augustus F. Harvey and Mr. Alexander J. Finlaison. The former was a personal friend of most of us, and his jovial, kindly face will be much missed. The latter was not so well known to us for, although he bore one of the most honored names in the actuarial world, he resided in England. To those of us, however, who had the privilege of his acquaintance, it came as a shock to think that he, too, with his courteous, and yet retiring personality, had gone. Truly the list of those whose memories alone are left to us grows apace.

Abstract of the Discussion of Papers read at previous Meetings.

Mortality Experience—The Northwestern Mutual Life Insurance Company. C. A. Loveland.

Page 261, Vol. VI.

Mr. Wing: Every contribution of actual mortality experience is valuable for comparison and study. We are under obligation to Mr. Loveland for furnishing in advance what he has given of the facts relating to the experience of The Northwestern Mutual Life Insurance Company for the 38 years, 1858 to 1895.

In the aggregate the experience is exceptionally favorable, as will appear by comparison with other published experiences on amount at risk, as follows:

		Ratio of Actual Loss by Death to Expected Amount by Combined Experience Table.
Metropolitan Life Assurance Society of London	1835 to 1864,	76.70 %
Connecticut Mutual Life Insurance Company	1846 to 1878,	*81.30 %
Mutual Benefit Life Insurance Company	1845 to 1879,	*82.70 %
John Hancock Life Insurance Company	1863 to 1883,	84.30 %
Washington Life Insurance Company.	1860 to 1885,	84.50 %
The Provident Life and Trust Company of Philadelphia	1866 to 1894,	61.91 %
The Northwestern Mutual Life Insurance Company	1858 to 1895,	68.71 %

* Estimated, being 92½ per cent. of ratios by American Mortality Table, as given in published Mortality Experiences of the companies referred to.

The calendar years covered by the experience of The Northwestern Mutual Life Insurance Company are more than those included in the experience of any other company in the list.

The arrangement of the results in 38 distinct groups, constituting so many separate mortality experiences, running through periods varying from 6 months to 37½ years, is unusual and interesting, but fails to give so many separate results which can properly be compared without large allowance for differing premises.

From the facts given in the paper under discussion, I incline to think that Mr. Loveland is hasty in his conclusion that the tontine form of insurance lowered the rate of mortality. That form of insurance undoubtedly increased the proportion of new business,

and in so far affected favorably the rate of mortality; but that is not the point made by Mr. Loveland.

In his groups of the 15 years, from 1881 to 1895, he sets aside the 5 years, 1891 to 1895, as "within the influence of medical selection," and then takes the experiences for the 10 years of issue, 1881 to 1890, as though they were not "within the influence of medical selection," and calls attention to the experience for these 10 years of issue as fully meeting the anticipation of the officers of the Company, that the tontine form of insurance "involving a selection in favor of the Company on the part of members accepting deferred dividend contracts would result in a decided improvement in the mortality rate."

Mr. Loveland appears to have ignored the fact that about one-half the exposures included in the 10-year group, 1881 to 1890, were as fully within the influence of medical selection as all the exposures in the 5-year group, 1891 to 1895. The gradation for percentage of actual to expected mortality would seem to be well accounted for in the different groups, by the effect of medical selection, if presented as follows:

For whole experience	68.71 %
For 10-year group, 1881-1890, where about one-half the exposures are influenced by medical selection	59.94 %
For 5-year group, 1891-1895, where all the exposures are influenced by medical selection	49.40 %

Or, if you arrange the years of issue in groups of five after 1875, you will get a result showing the gradual wearing away of the effect of medical selection as the first five years of exposure bear smaller and smaller proportion to all the exposures in each group.

Years of Issue.	Expected Loss.	Actual Loss.	Ratio of Actual to Probable.
1858–1875	$21,332,746	$17,240,368	80.82 %
1876–1880	5,002,883	3,608,471	72.13 %
1881–1885	8,491,722	5,527,174	65.09 %
1886–1890	11,163,923	6,254,340	56.02 %
1891–1895	5,315,518	2,626,120	49.40 %
Total, 1858-1895	$51,306,792	$35,256,473	68.71 %

Medical selection, and not selection by the applicant for insurance, would seem to be the controlling factor in this gradation of results.

In thus taking issue with Mr. Loveland, in the one conclusion he draws from the experience under discussion, I would like it to be understood that my argument is based upon the facts given in his paper. Fuller details of these facts may appear when the mortality experience of the Company is published, which will qualify what I have said, and possibly show better ground for Mr.

Loveland's conclusion than now appears. We shall await with interest the publication of the experience in detail.

An arrangement of the experience under years of exposure will add materially to its value. We assume that such an arrangement and much else of interest will be forthcoming shortly in connection with this important subject.

Mr. Wells: The principal value of Mr. Loveland's paper lies in the hope it suggests of good things to come. Aside from the fact that the mortality has been low as compared with the table, and the probability that the mortality under policies issued on the tontine dividend plan will be found to be less than that under otherwise similar policies, the results given are too general and indefinite to furnish a basis for any conclusions. Even the probability last mentioned is by no means demonstrated by the figures, for the recent selection must have exercised a large influence upon the tontine business. Mr. Loveland gives the experience to the end of 1895 of the policies issued in the years 1881 to 1890, inclusive, remarking that "the last five years, 1891-1895, were within the influence of medical examination at date assumed for this investigation." But while the business of the years 1881 to 1890 had been written from five to fifteen years prior to the investigation, 15 per cent. of the exposures on such business must have fallen within the first year, and 60 per cent. or more within five years after the medical examination, and it is hardly to be granted that the influence of the examination is limited to five years.

The suggestion of a decreasing mortality rate in later years seems to me to be based on too slight a foundation. Of the business of the eighteen years of issue 1858 to 1875, inclusive, ten show a lower rate of mortality to 1895 than to 1885, while eight show the reverse. The average of the percentages to 1895 is only $\frac{1}{20}$ of 1 per cent. below that to 1885. Even if a lower rate of mortality as compared with the table were shown in later years, our President, Mr. Macaulay, has conclusively shown that it may be due to other causes than a more favorable mortality.

But while I think the results furnished in Mr. Loveland's paper inconclusive, investigations of the mortality experience of substantial and well-managed companies are of great value, and I have no doubt but that the results of the present investigation will be of the highest interest to the Society, if we may hope to be favored with them more at length at some future time.

NOTE UPON ANNUITY CONTRACTS. CLAYTON C. HALL.

PAGE 266, VOL. VI.

Mr. Lewis: The decision discussed by Mr. Hall, rendered in January last by the Court of Appeals of Maryland, was a decision which, in its substance, would have been rendered by any other tribunal in the United States without the slightest hesitation, because the claim of the plaintiff in this case, which was argued very seriously, and which both the court below and the Court of

Appeals of Maryland tried its best to hear with a straight face, and to treat as if it were serious in its decision, was of so utterly flimsy and ridiculous a character as to deserve no standing in any court of justice.

In the first place, the claimant in the case had no standing in the court because he had no privity to the contract on which he made his claim; and if the claim had been a perfectly sound and good one, he had no right to present it, because it vested in somebody else.

In the second place, the claim which he made was founded on the assumption that an annuity contract in its nature is one which can only be created by a deed—that is, by executing it under seal, and not by a simple contract; whereas, there has never been a decision in any court of Great Britain or America, or anywhere else, to that effect, but simply an opinion expressed by certain textbooks that when an annuity is a hereditament—that is to say, when it belongs to somebody else than the life upon which it is granted, and therefore may become a matter of inheritance, then it is a proper subject for a deed. But when the annuity belongs to the life upon which it is granted, the possibility of an inheritance is excluded by the nature of the contract, and therefore there is no pretext whatever for calling it a hereditament, and every court of Christendom would agree that it is a simple contract for the payment of money.

In the third place, the claim was presented that a contract which had been completely executed—an annuity which had been paid until the death of the annuitant—was void from the beginning, and therefore the consideration money which had been paid for the original could be recovered by the administrator of the annuitant. This is a contradiction of what is familiar to us as a first principle of law—that when a contract is executed, no such remedy will lie. The only interest, therefore, which there is in this matter is the history of a seal, in its potency with reference to contracts.

In the Middle Ages, whither we look for the origin of the common law, very few formal contracts for future execution were made, except by persons of considerable importance, and in those times persons of importance in the community were very apt, like other persons, to be exempt from the necessity and destitute of the power to read and to write. They signified their assent by their "mark" upon the contracts; and as a mark in the form of a simple cross, or a scratch with the pen, has not the personal character and identifiability, so to speak, of a signature by one who can write, it was difficult to avoid forgeries and to guarantee the genuineness of the execution. The result was that every person of importance who had contracts to make had a seal, which, being in his own custody and carefully and jealously guarded, became, when it was used, the expression and signet of his individual will. This made the proof of the execution of contracts easy, and for this reason, and for this reason alone, a private individual seal became of the highest significance and importance in the execution of contracts.

Now that fact took root in the law, and became a first principle with the courts—that a seal in itself gave a validity and force to a contract which could not be given to it without the seal. Although the general diffusion of wealth extended the necessity of making contracts through the whole community—while in the general diffusion of education, people of importance and of property generally became able to write, and could by their signatures on papers do away with the necessity of the seal, and although the courts were the first to appreciate the fact that the individuality of a signature is often more clearly evidence of personal execution than the individuality of a seal itself, yet the conservatism of the courts retained, generation after generation, the disposition and tendency to attach peculiar importance to the use of a seal. And the old distinction of the common law, that a seal imports consideration and makes evidence of consideration unnecessary, while a mere parol contract, whether signed in writing or not, requires the consideration to be proved, and will not presume it—was adhered to by the courts for a series of generations. The very first serious attack upon it was made by the Supreme Court of the United States, a little more than a hundred years ago, when it said that the signing of a deed is the material part of the execution, and the seal is a mere form. From that day to this the tenor of movement in the courts of the United States and of the several States has been toward doing away with the great distinction between sealed and unsealed documents. But the tendency has gone on with different rapidity in different States. Great changes have been made in it, sometimes by statute law, and sometimes, gradually, by the operation of the courts themselves. But the distinction between the two kinds of instruments has been steadily fading away in the judicial and legal mind of the country.

It is not necessary to consider in detail the resulting condition of the law in the several States, inasmuch as the general tenor of this legislation is not revolutionary, but a mere recognition of the truth, already apparent in the decisions of the courts in other States as well as in those which have passed these enactments, that the technical question of the presence or the absence of a seal has almost universally lost its importance as bearing upon the validity of contracts. In a multitude of cases, for example, in which the seal is made necessary by the literal language of a statute or by a rule of law which the courts have not yet recognized as overruled and passed by, it is universally acknowledged that a court of equity, if it comes across such a contract, which is valid in all other respects and simply invalid for the want of the statutory requirement of the seal, will not say "That contract is invalid," but will compel the parties to reform it, and will treat it at once as if it had been reformed, and so enforce it.

That is the way in which courts of equity get around the difficulty, and that action on the part of courts of equity has practically done away, in the courts of our States, with the distinction. In Pennsylvania, Ohio and ten other States express legislation has pro-

vided, rather unnecessarily, that no seal shall be necessary to the validity of any policy of insurance. And the term "policy of insurance," you will note, is applied by the courts very generally to mean any contract made by a life insurance company within the province for which it is organized, so that it does not make any difference whether the policy of insurance is a policy to insure a man for life, or a policy of annuity. If it is a contract made by a life insurance company, within the province for which that company is organized, it needs no seal. And that principle, which has been embodied in the statute law of twelve States, is an undisputed principle in the law of all the States of the United States.

The reason for this is substantially and clearly this, which any layman can perfectly understand: policies of insurance and annuities are corporate obligations assumed by the corporations in pursuance of a purpose for which they are organized, and all contracts for these ends are especially favored by law. The old rule of common law was that a corporation can act only by its seal; but this rule is now wholly abandoned in England as well as in this country. The British courts, after a long period of transition upon this point, uniformly hold now that it has no application (this old rule of corporations) to trading or business companies—that is to say, it applies to none but municipal corporations. They, of course, cannot act in any way except through their seal. And, therefore, life insurance companies and all other corporations which are authorized by law to make trading contracts are bound by the acts and agreements of authorized officers and agents, whose authority to act and whose contracts may be proved by any satisfactory evidence, or even by implication from acts and records.

In the leading case of the Bank of Columbia against Patterson's Administrators (7 Cranch 299), Justice Story held that "wherever a corporation is acting within the scope of the legitimate purposes of its institution"—that is really the test of all these contracts—"wherever a corporation is acting within the scope of the legitimate purposes of its institution, all parol contracts (that is, not sealed) made by its authorized agents are express promises of the corporation, to be enforced by action, according to their terms."

I have found, in looking over the decisions of the highest courts of fourteen States, that they have expressly and in terms adopted that language of Justice Story as the law of their States. I have found a decision in only one State, where it is questioned and the case determined against it, and that is in the State of Kentucky, some twelve or fifteen years ago, and I find that since then the State of Kentucky, by her Supreme Court, has overruled that decision and adopted this language of Judge Story as the law. So that it is now the accepted law in every State in the Union.

MR. MACDONALD: The distinction is made in Great Britain, as between a municipal and a trading corporation, as mentioned by Mr. Lewis. The former can only act under seal, whereas, in regard to trading corporation the use of the seal of course is now recognized as unnecessary, except in that class of transactions in which

a natural person, or an individual, is required to use a seal. These cases, in which I will just read the decisions, bear clearly on this point.

The first case is that of Young & Co. *vs.* The Mayor of Leamington, House of Lords' Reports of 1883. The sum of six or seven thousand pounds was claimed by a firm of contractors for work done under the order of the engineer and surveyor of the corporation, which was not under the seal of the corporation. The judges were unanimous on the law of the case. Lord Justice Lindley, of the Court of Appeal, in delivering judgment, said: "It may be said that this is a hard and narrow view of the law, but my answer is that Parliament has thought it expedient to require this view to be taken, and it is not for this or any other Court to decline to give effect to a clearly expressed statute because it may lead to apparent hardship." Lord Bramwell said: "The Legislature has made provision for the protection of rate-payers, shareholders and others, who must act through the agency of a representative body, by requiring the observance of certain solemnities and formalities which involve preparation and reflection; thus the importance of the seal. It is idle to say that there is no magic in a wafer; it continually happens that carelessness and indifference on the one side, and the greed of gain on the other, cause a disregard of these safeguards, and improvident engagements are entered into."

This case, of course, only applies to the principle of a municipal corporation, where the seal was held to be necessary; and, so far as I know, no change has been made in the law of England since the time of that decision, which was in 1883.

The other case deals with that of a trading corporation, and applies, of course, to simple contracts similar to that which would be issued by an insurance company. The case is well known, and, I think, much quoted. It was the South of Ireland Colliery Company *vs.* Waddle, in which the corporation contracted with an engineer for the erection of a pumping engine and machinery for that purpose, and paid him part of the price. In the action brought by the corporation against the engineer for breach of contract, it was held in the Court of Common Pleas that the action was maintainable, though not under seal. This judgment was sustained on appeal to the Exchequer Chamber. Chief Justice Cockburn, in delivering judgment, said: "We are asked to overrule a long series of decisions in all the courts, which, in accordance with sound common sense, have held that the old rule as to corporations contracting only under seal does not apply to corporations or companies instituted for the purpose of trading, and we are invited to reintroduce a relic of barbarous iniquity. We are of the opinion that the judgment of the Court of Common Pleas ought to be affirmed. . . . . In earlier times, no doubt, corporations could only, subject to well-known exceptions, bind themselves by contracts under seal, but the contrary has since been laid down after a long series of cases, and may now be construed settled law."

In delivering judgment in the Court of Appeal, Mr. Montague Smith said: "The modern doctrine, as I understand it, is that a company which is established for the purpose of trading may make all such contracts as are of ordinary occurrence in that trade without the formality of a seal, and that the seal is required only in matters of unusual and extraordinary character which are not likely to arise in the ordinary course of business."

In Canada, where of course each province has the power of legislating with regard to contracts, some legislation has been introduced bearing upon insurance and annuity contracts, and dispensing with the use of a seal. In the Province of Ontario the Insurance Act, which was passed there some twenty years ago, and required the use of a seal, was to some extent uncertain. It was embodied in that act, in describing a "company," "company means and includes any corporation or any society or association incorporated or unincorporated, or any partnership or any underwriter that undertakes or effects for valuable consideration or agrees or offers so to undertake or effect in the Province any contract of insurance within the intent of this act."

It is further provided, "contract means and includes any contract or agreement, sealed, written or oral, the subject-matter of which is within the intent of sub-sec. 35 of this section." Section 35 reads as follows: "Insurance includes the following—whether the contract be one of primary insurance or of reinsurance, and whether the premium payable be a sum certain, or consists of sums uncertain or variable in time, number or amount. . . . . Sub-sec. G—contracts of endowment, assessment endowment, tontine, semi-tontine, lifetime benefits, annuities on lives or contracts of investment involving tontine or survivorship principles for the benefit of persisting members, or any contract of investment involving life contingencies."

Therefore under the laws of the Province of Ontario, as well as of the other provinces, the use of a seal is absolutely dispensed with by legislation.

MR. MACAULAY: What would be the position, Mr. Lewis, of a policy issued without a seal by a company whose charter said particularly that all its policies should be under seal?

MR. LEWIS: The company would be responsible for a violation of its charter, but would be relieved of no obligation by its own fault.

MR. MCCLINTOCK: Suppose a company whose charter requires the use of a seal issues a contract of annuity unsealed and receives the money, and three or four months after that the annuitant comes back and says he wants his money returned. What is his status?

MR. LEWIS: In strict law, he would have a claim. I take it, however, that where it is so universally settled as it is in the States of New York, Pennsylvania and Ohio, and in many other States, that a court of equity would intervene to enforce such a contract by reforming it, requiring the company to seal it. The question

is, has it given him a contract which *he* can enforce? In all those States he can enforce it by a proceeding in a court of equity, requiring the company to seal the contract and then enforce it as though it were sealed. But in such cases I should consider that the essential contract already exists.

MR. MACAULAY: In such a case, if the annuitant wanted the return of his money, on the ground that the contract was not binding upon the company, and the company volunteered to seal it and make it enforceable, would not that be sufficient?

MR. LEWIS: The company would never be relieved by the courts, but would be liable to all the penalties, of whatever nature they might be, which the legislature had provided or the courts could enforce, for a breach of its charter.

MR. IRELAND: I desire to inquire of Mr. Lewis as to what really and actually constitutes a seal.

MR LEWIS: In the old times, of course, it was an impression on wax, made by the seal, which was kept in the custody of the person. But since every person did not have one, it came to be admitted by the courts that one man could adopt the seal of another by giving satisfactory evidence that he had used it himself, and it was accepted as his seal. Then, when the principles of the law came to be applied in this country, and nobody was in the habit of having a seal, the State of Pennsylvania was the first to recognize the practice of a man making a seal with a pen, and the courts there acknowledged as a principle of law, that if a man made anything with a pen which he intended as a seal, that should be accepted as a seal. But it was not until about ten years ago that such practice became law in New York by recognition of the legislature.

MR. CRAIG: A law of the State of New York provides that any company can loan a policyholder up to the full reserve on his policy, notwithstanding anything to the contrary in its charter. Would not that principle apply to a seal? In other words, does not the law of the State override the individual charter, if there should be any difference?

MR. LEWIS: Unfortunately we have a statute in New York, which is called the "Statutory Interpretation Law," which provides that where any specific provision is made in the charter of any company, it shall not be overruled by any general provision of other statutes. The consequence is that a corporation has to regard its charter limitations, even if the general provisions of the law would release it from them.

DUTIES OF AN ACTUARY FROM A PRACTICAL STANDPOINT.

JOEL G. VAN CISE.

PAGE 272, VOL. VI.

MR. GORE: There is much valuable advice in this paper. The actuary is shown to have ever-increasing burdens and responsibilities. He must be, it is said, more than a mere mathematician,

and the experience of every man here will corroborate that statement. It is sometimes a temptation to an actuary to go too far in purely mathematical directions. Certainly it is the tendency of others at times to fail to give due weight to the value of mathematical principles and results. An important duty of an actuary, therefore, is to strive to maintain a proper balance between the purely theoretical and the so-called practical.

A point suggested by this paper is the pressure brought upon actuaries to acquiesce in the adoption, for competitive reasons, of features and conditions of the policies of other companies. In other words, we are urged continually because of competition to devise and adopt features that shall be more liberal than the features of the policies of other companies; and it is often difficult, when we really feel that our own company cannot excel every other company in every particular, to reconcile a desire to comply with a specific request that shall conquer competition with our sense of responsibility for the soundness of our recommendations. Personally, however, I have yet to learn of a policy issued by any company represented in our Society that has been made so liberal that it may be said to be unsafe.

MR. HANN: I desire merely to say this: if an actuary is true to himself, and true to his company, he is bound to be either a plus or a minus, but he certainly will not be a zero. Therefore, I say that we ought, in all earnestness, to stand up for what is true, for what is right in principle, and, above all things, to look to the sufficiency of the charges for the benefits promised in the contracts.

MR. NICHOLS: It appears to me that there is something abnormal in the relation, as I understand it, in which the American actuaries stand to their companies; that while on the shoulders of the actuary rests the responsibility of what is practically an executive department of the company, yet it is regarded as a mere clerical position. I think this a serious defect. In the military and naval services the line and the staff are distinct, and while the direct control and management are vested in the line, yet the authorities recognize the independence of the staff, in relation to matters for which it is primarily responsible, and its members are sustained in that position. I think in the ideal company the directors will recognize analogous independent relations between the actuary and the executive department.

NOTE ON REVERSIONARY ANNUITIES. J. H. NITCHIE.

PAGE 278, VOL. VI.

MR. FACKLER: I imagine that the formulæ given in Mr. Nitchie's note were intended to afford a solution of the problem involved in providing annuities for municipal employees retiring after a certain term of service, x representing the age of an employee, and y, the age of his wife; the terms of the annuity being that, in case of death, after a service of n years, the wife shall become entitled to the annuity. The annuity, we will say, will be $100, in case he

dies in the nth year; in case he dies in a year after that, it is to be increased by, say, $10, and the annuity of $110 continued during the wife's life. The longer the husband shall have served, and the greater the age of his wife, and possibly the greater her need of support, the larger shall be the amount of the annuity. I have examined this formula with some care, and think that, despite its length, Mr. Nitchie has reduced it to as small a number of terms as can be used under the circumstances.

PRESENT VALUES OF ONE DOLLAR FOR ANY NUMBER OF YEARS FROM ONE TO ONE HUNDRED, AT 3, 3½ AND 4 PER CENT. ANNUAL COMPOUND INTEREST. EMMA W. CUSHMAN.

PAGE 279, VOL. VI.

MR. TATLOCK: I thank you, Mr. President, for the opportunity to express the appreciation of the Society for, and its courteous recognition of, this first contribution to its Transactions from our only lady member. It is a pleasure, also, to call attention to this contribution of Miss Cushman's, as an example of that devotion to duty, deep sense of responsibility, and unfailing industry, which, as is well known to all who have the honor of Miss Cushman's acquaintance, she puts into all her work.

The general question of the limit of accuracy in numerical calculations, in almost every branch of the exact sciences, is a question that must necessarily be left to the individual judgment. It is probably true, also, that in most cases the individual judgment leans rather to the side of inferior accuracy; that is to say, it is satisfied with results not quite exact, or not of the degree of exactness that might be attained, on the ground that the results as deduced, with the amount of labor expended, are quite sufficient for the purpose in hand. It is therefore pertinent that an intimation should be given that a more than customary degree of accuracy in numerical calculations is worth striving for. It is true, also, that the development of increased facilities in methods and instruments furnishes a field for their use, although the direction in which such facilities are to be availed of may not always be apparent at the time. From this point of view I have no doubt but that in the future many of us will find a field in which we shall be very glad to avail ourselves of the facilities offered by Miss Cushman in this paper which she has contributed.

MASSACHUSETTS ANNUITIES. EDWARD J. SARTELLE.

PAGE 286, VOL. VI.

MR. MCCLINTOCK: The thanks of the Society are due to Mr. Sartelle for the pains and devotion to duty shown in preparing so extensive a set of tables as these printed in the last transactions. Every such contribution to our transactions gives us one more important fundamental set of tables upon which we may base such

future calculations as may be deemed advisable, and which, with others, may be made subject of choice for useful purposes.

The heading of the paper is "Massachusetts Annuities." The tables, however, are headed "Massachusetts Life Table," and I think that the heading, as printed, relating merely to annuities, tends to obscure the value of the document, particularly as there is nothing said to indicate why this table should be used for annuity purposes. All it says is that it suggests a possible basis for annuities. Now the ordinary belief is that annuity calculations should be made by companies, as far as possible, upon annuity experience, and I think that that understanding must remain as a maxim of the profession until some reason is given why miscellaneous information of this kind should be used for that purpose. There is no obvious connection between the mortality of the Massachusetts population and the business done by companies who issue annuities. It is, I think, true that the annuity figures, as developed by this table, would, if used for reserves upon annuities issued, or even for premiums for annuities, with the proper loading, be somewhat more rigid, and provide a larger reserve, than tables ordinarily used. But that fact of itself would not justify using it for annuity purposes. If tables based upon annuity experience are for any reason regarded as unsafe, or not quite safe enough, it would seem better to use those figures as a basis, and to diminish the mortality in the annuity experience, according to some rule which is presumed to be reasonable, so as to make it a safer basis for operations, rather than to take a strange table of this nature, even though the strange table happens to be safe.

I find nothing whatever to criticise in the paper, unless possibly a question might be raised as to the nature of the experience. The experience, as a whole, shows exceedingly low mortality. It would indicate, if the tables are correct, that people live longer in Massachusetts than they do elsewhere. I should be pleased to learn, if it could be learned, exactly how reliable the Massachusetts data may be The tendency of registrars of deaths is to fail to get in all the deaths. The tendency of people taking a census is to put down every name that they can muster. A census, therefore, is likely to err, if at all, in getting in too many persons living, while the list of deaths, from the registration, is likely to err, if at all, in not containing all the deaths. Consequently I am inclined to think that while the people of Massachusetts are undoubtedly long-lived, they are not quite so long-lived as these statistics would indicate.

MR. WEEKS: The value of the tables worked out by Mr. Sartelle depends upon two things—first, the reliability of the Life Table on which these values are based, as inferred from the statistics which are its sources and from the treatment which these statistics have received, and second, its safety as compared with other tables, for use in calculating annuity or insurance risks.

First, as to the sources and construction of the Massachusetts Life Table. The sources are fully indicated in Mr. Sartelle's paper (being recorded in a pamphlet by Dr. Samuel W. Abbott, Secretary of the State Board of Health of Massachusetts, on "The Vital Statistics of Massachusetts for 1897, with a Life Table based upon the experience of the Five-Year Period, 1893-97"). A reading of Dr. Abbott's pamphlet shows in a general way the methods which he used in deriving the Life Table from the statistics. Dr. Abbott expresses grave doubt of the accuracy of the census enumeration, the figures of which (with graduation) constitute the denominators which he has used in deriving the rate of mortality; he intimates that the census enumeration is probably short of the facts, and in fact thinks he has proved this for the lives under five years of age. The shortage which he claims to have shown is immense. I quote from him (page 19 of the pamphlet, "Vital Statistics"): "The whole number of persons living under five years of age in the State in 1895, as stated by the census, was 235,647, but the number calculated from the living births in these years was 294,604, or 58,957 more than the figures of the census would indicate." I have referred to the successive Registration Reports of births and deaths for Massachusetts for the years 1890 to 1895, inclusive, and show in the following tables the figures I have extracted from those reports, and the method by which I have derived from such figures the approximate number of children under five years of age living in the middle of the year 1895.

ITEMS TAKEN FROM THE MASSACHUSETTS ANNUAL REPORTS OF BIRTHS, MARRIAGES AND DEATHS, FOR 1890 TO 1895, INCLUSIVE.

	Born.	Died under 1 year.	Died 1-2.	Died 2-3.	Died 3-4.	Died 4-5.
Males						
1890	29,524	5,319				
1891	32,532	5,677	1,088			
1892	33,758	5,898	1,239	518		
1893	34,328	5,958	1,277	598	383	
1894	34,338	5,999	1,212	550	403	309
1895	34,623	5,919	1,201	539	367	282
Females						
1890	28,227	4,306				
1891	30,434	4,509	1,041			
1892	31,951	4,751	1,079	468		
1893	32,829	5,032	1,153	534	406	
1894	32,575	4,900	1,160	531	385	268
1895	32,905	4,645	1,117	513	359	260

Above items as modified for derivation of estimated number of children under five living June 30th, 1895.

Males						
1890	(½) 14,762	(⅔) 3,546				
1891	32,532	5,677	(½) 544			
1892	33,758	5,898	1,239	(½) 259		
1893	34,328	5,958	1,277	598	(½) 192	
1894	34,338	5,999	1,212	550	403	(½) 154
1895	(½) 17,312	(⅓) 1,973	(½) 600	(½) 270	(½) 183	(½) 141
Totals,	167,030	29,051	4,872	1,677	778	295
						778
						1,677
						4,872
Total Deaths,	36,673					29,051
	130,357				Total Deaths,	36,673
Females						
1890	(½) 14,114	(⅔) 2,871				
1891	30,434	4,509	(½) 520			
1892	31,951	4,751	1,079	(½) 234		
1893	32,829	5,032	1,153	534	(½) 203	
1894	32,575	4,900	1,160	531	385	(½) 134
1895	(½) 16,452	(⅓) 1,548	(½) 558	(½) 256	(½) 180	(½) 130
Totals,	158,355	23,611	4,470	1,555	768	264
						768
						1,555
						4,470
Total Deaths,	30,668					23,611
	127,687				Total Deaths,	30,668

Resulting estimate of number of children under five living June 30th, 1895:

Males,	130,357
Females,	127,687
Both sexes,	258,044

As against Dr. Abbott's estimate of the population in 1895 under five years of 294,604, my estimate, based on the Five-Year Period from July 1st, 1890, to June 30th, 1895, is, as shown by the tables above, 258,044. On referring to the Massachusetts State Census of 1895, Volume II., page 303, I find the number of children living up to the age of five years to be reported as follows:

Under 1	year	48,933
1	"	29,974
2	"	56,338
3	"	51,698
4	"	48,704
5	"	47,938
Total up to and including the age of five,		283,585

Omitting the number given at the age of five we have a total, up to and including age four, of 235,647, and this is the number which Dr. Abbott states in his pamphlet as given by the State Census for the population under five years. Evidently, in using the figures in this way, the assumption is involved that all the children whose age is given as five years are over five years old, a very improbable assumption, as parents are quite likely to speak of a child who is four and one-half or more as five years old. That this way of speaking has prevailed in the reports made is evident when we notice the small number of children reported at the age of one. The number living between the ages of one and two must surely have been greater than the number living between the ages of two and three, and yet the figures given in the Census are, at the age of one, 29,974; at the age of two, 56,338. We must, therefore, suppose that a large fraction of the 47,938 children reported in the Census as of the age of five were below that age; less than half would be enough to make a perfect agreement between the Census and the Registration Reports. My inference is that the discredit thrown by Dr. Abbott on one of his two sources is undeserved, which recalls the remark of Mr. Macaulay in his address as President last May, as to what is likely to happen when our friends the doctors undertake to handle statistics.

Taking the male lives, I have made a comparison of the original figures of deaths (drawn from the Registration Reports) with the figures of population contained in the census, by groups of ages, with sufficient closeness to prove the substantial accuracy of the mortality rates deduced by Dr. Abbott, throughout the table, beyond the age of five. I have no doubt that a similar comparison would show a like correctness in the mortality rates deduced on female lives; hence, I may say in general, that (barring the first five lines of each table) I have more confidence in Dr. Abbott's tables than he himself has.

As regards the second point—the possible usefulness of these tables in measuring risks—a comparison naturally suggests itself with other recent mortality tables. Mr. W. C. Wright has already

made this comparison, in admirable form, on the point of rate of mortality at quinquennial ages. In the following table I have compared the annuity values of the Massachusetts Life Table with those of the new English Life Healthy Districts Table (constructed by Dr. Tatham from statistics of the years 1881-1890, and published in 1897, in the "Supplement to the 55th Annual Report of the Registrar-General, Part II."), also with the annuity values of Mr. McClintock's "Annuitants' Mortality Tables." The quotations of annuity values of the last-named table I have begun at age 50, as the mortality experience on which it was based (that of Annuitants in American Life Insurance Companies) included very slight data below that age.

If the test of a mortality table for practical use be that it should be safe but not excessively safe, then it would appear from comparing the three columns of annuity values that the special usefulness of the Massachusetts Life Table, so far as concerns annuities, would be on male lives, at ages over 65.

THREE PER CENT. ANNUITY VALUES (a_x) ON THE FOLLOWING THREE MORTALITY TABLES:

English Life Healthy Districts, 1881-90—Tatham. Annuitants' Mortality—McClintock. Massachusetts Life Table, 1893-97—Abbott.

Age.	English Life Healthy Districts.	McClintock.	Massachusetts Life.
Males			
1	24.936	.....	23.357
5	25.657	.....	24.445
10	24.954	.....	23.794
15	23.910	.....	22.672
20	22 842	.....	21.615
25	21.741	.....	20.618
30	20.581	.....	19.582
35	19.302	.....	18.418
40	17.877	.....	17.083
45	16.304	.....	15.567
50	14.579	14.286	13.917
55	12.726	12.543	12.220
60	10.785	10.731	10.507
65	8.842	8.911	8.824
70	6.955	7.151	7.219
75	5.220	5.522	5.799
80	3.736	4.084	4.525
85	2.557	2.878	3.399
90	1.681	1.918	2.428
95	1.062	1.195	1.557
Females			
1	25.134	.....	23.587
5	25.748	.....	24.614
10	25.045	.....	24.013
15	24.073	.....	22.967
20	23 106	.....	22.016
25	22.098	.....	21.082
30	21.029	.....	20.073
35	19.850	.....	18.961
40	18.505	.....	17.710
45	16.955	.....	16.286
50	15.211	16.031	14.697
55	13.323	14.194	13.026
60	11.338	12.275	11.313
65	9.338	10.329	9.582
70	7.412	8.423	7.912
75	5.659	6.629	6.461
80	4.161	5.013	5.185
85	2.956	3.625	4.013
90	2.038	2.493	2.895
95	1.366	1.617	1.828

We are all interested in the question, What is the best mortality table covering the first quarter of a century of life, for use in calculating premiums and reserves for Pure Endowments and Deferred Annuities on the lives of children? The life insurance companies have had no experience which will give them any help on this point, and we therefore look with great interest on any new mass of experience published bearing on this question. For the sake of safety many of us have used the British Peerage Tables, based on the investigations of Mr. Bailey and Mr. Day, recorded in their paper read before the Institute of Actuaries in 1861, and published in The Journal, Volume IX., page 305 (see also paper by Mr. Berridge, J. I. A., Volume XII., page 220, and paper by Mr. von Baumhauer, J. I. A., Volume XVI., page 34). It is now very interesting to compare these old tables with the corresponding mortality rates of the new English Life Healthy Districts Table above mentioned, as well as with those of the Massachusetts Life Table. These three mortality rates are given in the table following, both for males and for females, from birth to age 25.

I have already given my reasons for thinking that the true mortality rates for ages one to four, inclusive, derived from the Massachusetts statistics, are materially higher than those given in the table. The change, if made, however, would not affect the standing of that table relatively to the two others referred to. An examination of the three mortality rates will suggest the use of the Healthy Districts Table rather than of the Massachusetts Life, in place of the old Peerage Table, certainly after the fifth or sixth year of life.

MORTALITY RATES ON YOUNG LIVES, ON THE FOLLOWING THREE TABLES: British Peerage; English Life Healthy Districts; Massachusetts Life.

Age.	British Peerage.	English Life Healthy Districts.	Massachusetts Life.
Males			
0	.07821	.11915	.17431
1	.01625	.03321	.04222
2	.00865	.01316	.02010
3	.00482	.00904	.01401
4	.00435	.00712	.01078
5	.00390	.00556	.00812
6	.00365	.00433	.00653
7	.00342	.00340	.00536
8	.00326	.00273	.00444
9	.00316	.00230	.00384
10	.00326	.00207	.00325
11	.00340	.00200	.00291
12	.00359	.00207	.00278
13	.00381	.00223	.00297
14	.00418	.00247	.00360
15	.00473	.00276	.00427
16	.00564	.00308	.00489
17	.00743	.00340	.00529
18	.00876	.00369	.00575
19	.00970	.00395	.00619
20	.01031	.00418	.00665
21	.01065	.00440	.00708
22	.01079	.00463	.00749
23	.01076	.00487	.00788
24	.01061	.00514	.00821
25	.01037	.00541	.00847
Females			
0	.05971	.09350	.15061
1	.01635	.02984	.04005
2	.00702	.01257	.01915
3	.00517	.00863	.01393
4	.00480	.00666	.00999
5	.00475	.00516	.00832
6	.00485	.00406	.00661
7	.00491	.00329	.00548
8	.00503	.00279	.00454
9	.00519	.00249	.00394
10	.00535	.00237	.00348
11	.00560	.00237	.00305
12	.00586	.00249	.00307
13	.00615	.00270	.00335
14	.00652	.00298	.00399
15	.00686	.00330	.00472
16	.00723	.00365	.00536
17	.00755	.00397	.00571
18	.00780	.00424	.00608
19	.00798	.00446	.00644
20	.00809	.00464	.00680
21	.00814	.00480	.00714
22	.00816	.00497	.00732
23	.00819	.00516	.00751
24	.00823	.00538	.00770
25	.00828	.00562	.00788

MR. W. C. WRIGHT: Mr. Sartelle has made a most valuable contribution to the papers and transactions of this Society by his present paper. I was already aware that the Registration Reports and State Censuses of Massachusetts contained gratifying evidence of the possibility of longevity under social conditions favorable to the aged; but I was not aware, till Mr. Sartelle drew attention to the fact at our last meeting, that the table of mortality derived by Dr. Abbott from the Census of 1895, and the Registration Reports of 1893-97, actually produced higher annuity values at most high ages than any table previously published. The Census of Massachusetts for 1895 records 7,283 males and 11,227 females as upwards of eighty years of age, and 6 males and 29 females as upwards of ninety-nine years, in a population of 2,500,183, the oldest male being 111, and the two oldest females 108 years of age. The death rate of the quinquennial period selected by Dr. Abbott, 19.23, is very near the mean of all the 42 yearly rates embraced in the reports ending with that period. It is therefore safe to say that it does not exaggerate the vitality of the population of the State. Nevertheless, it is only for ages over sixty that this makes a safe table for annuity uses, and its comparative fitness and safety for such use at higher ages rests mainly on the fact that all our insured life tables, and even some of those based on annuitant experience are deficient in actual data at very high ages, and have been so graduated as to exaggerate the death rate at such ages.

Mr. McClintock's recent table and Dr. Farr's English Life Healthy Districts table are exceptions and the former, even for females, only falls below Dr. Abbott's table, when computing annuities of 3 per cent. interest, above the age of seventy-seven, and then not more than .4187 of a year's purchase in the extreme case at age eighty-eight. The Healthy Districts falls below above age sixty-one, but also only a fraction of one year's purchase. For males, the latter table only falls slightly below above age seventy-one, and recovers again above age ninety-three. Though Dr. Abbott's table is not the best suited at all ages for annuity uses, as might be expected, for otherwise it would completely negative the common supposition that annuitants evince a strong power of selection in their own favor, it does furnish us a most valuable guide by which to properly graduate other tables for high ages.

Dr. Abbott's table might not be a bad one for general valuation use in the present American fashion, especially if long term endowment insurance policies and annuities purchased by the proceeds of such policies take the place of whole-life insurance contracts. But I have not made a thorough study of it from this point of view. For males, it exceeds the Combined Experience table from age twenty-two to age sixty-two, inclusive.

Mr. Sartelle's computations have been carried out with great precision as respects the interest element, but I doubt the utility of such exactitude, especially when employing mortality data

based on a radix of only 100,000. I am inclined to think that in connection with the uniform notation, which is now so generally favored, it might be well to establish a fixed rule governing the radix, and the number of decimal figures employed, or to be preserved, in preparing commutation columns. Seven significant figures altogether, or six decimal places of figures, are generally enough to furnish monetary values with sufficient accuracy, and all that can be conveniently dealt with when using seven-place logarithms, and in many cases this is the only means of facilitating computation ready at hand.

A SECOND NOTE ON PREMIUM LOADING. WALTER C. WRIGHT.

PAGE 153, VOL. VI.

MR. W. C. WRIGHT: Mr. Craig has successfully explained to himself, in his remarks printed on page 326, Vol. VI., the anomaly pointed out in my note on premium loading, which I called "a gross and injurious, if not a dangerous, violation of equity," because it afforded the opportunity to secure five years' future insurance when ceasing the payment of further premiums at materially less cost than when going on with such payments. He claims to find no reason to increase the charge in the former case. It is possible that my language was too strong to suit the case, but it is nevertheless plain that Mr. Craig leaves the anomaly unexplained on grounds of equity when taking their points of view, why the persistent policyholder should find himself less cheaply served for the next five years than his fellow who has stopped payment. To say that the same opportunity was open to him is only to emphasize the invitation to lapse. The danger of not being able to renew at the expiration of the temporary insurance, in case of lapse, is a good check on lapses as far as it goes, but it does not justify or explain for the persistent premium payer the discrimination in cost of insurance against him. If he goes on and then has to stop, he will pay more than if he stops at once. The scientific value, and equity of the change in method of loading premiums, and settling for expense with all policyholders, which I have explained, lies in the fact that it makes all charges to policyholders equal for equal risks borne and investment gains realized, and leaves with the ingenuity of the actuaries and officers to whom it belongs, the duty of so regulating all sorts of expenses as not to dangerously interfere with preserving the essential requirement of equity above stated. This is the real problem for Mr. Craig and other sceptics to grapple with. The more carefully and dispassionately they study it, I am satisfied the feasibility and advantages of the change will more clearly appear to them.

REMARKS BY THE PRESIDENT IN REFERENCE TO HIS ADDRESS OF MAY 17TH, 1900.

PAGE 310, VOL. VI.

My attention has been called to the fact that the conclusions drawn from the statistics published by the British Medical Association by the authors of the leaflets quoted by me, are not endorsed by the Association itself. A copy of the original report (which I had not previously seen) has been furnished me, and from it I make the following extracts:

"The average age furnished by the total abstainers is some-"what startling, for we find that it is not only far below the "average age attained by the moderate drinkers, but it is even a "year below that reached by the decidedly intemperate. It must, "however, be remembered in interpreting this figure correctly that "the class of total abstainers is somewhat differently constituted "from any of the other classes.

"It will not, I think, be disputed that the total abstinence "movements, which have played so prominent a part in this "country of late years, have made many more converts among "the young than among the middle-aged or elderly. If this is "admitted, it will necessarily follow that the average age of total "abstainers—I mean of living total abstainers—at any time during "the three years covered by this inquiry, was considerably less "than the average age of the rest of the community; so that the "class of abstainers has contained a proportion much greater than "the average of individuals susceptible to early death, or, to put "it in another way, has had a greater average liability to early "death, apart from any question of alcohol, than that of the "other classes.

"These considerations are borne out to some extent by the "tables immediately following.

"It is probable that the average age at death shown by "the other classes is affected to some degree by similar "circumstances. If the alcoholic habit tends to increase, as "generally supposed, from youth up to about 45 or 50 years of "age (cf. Neison, *Journ. Statist. Soc.*, XIV., p. 216), the average age "of the class of the population corresponding to our class B (the "'habitually temperate') will be somewhat lower than that of the "classes corresponding to C, D and E (the 'careless drinkers,' "the 'free drinkers,' and the 'decidedly intemperate'). Our "inquiry has taken no account of the age at which intemperate "habits began in each case. This information we feared we "should fail to obtain."

I have much pleasure in doing this act of justice to our medical friends, and in admitting that these quotations show that they themselves realized some of the weak points in their report.

Abstract from the Minutes of the Fall Meeting of the Actuarial Society of America, held in New York, N. Y., on Thursday and Friday, October 25th and 26th, 1900.

FIRST SESSION.

The "Arena," 41 West 31st Street,
Thursday, October 25th, 1900, 11 A. M.

The meeting was called to order by the President, Mr. Thomas B. Macaulay, who, after the roll call, addressed the Society. (See page 414.)

MEMBERS PRESENT.

Alsop,	Lunger,	Roche,
Brinkerhoff,	McCabe,	Salter,
Craig,	McClintock,	Sartelle,
Fackler,	Macaulay,	Smith,
Gore,	Macdonald,	Stabler,
Hann,	E. P. Marshall,	Starr,
Hildebrand,	Messenger,	Tatlock,
Holcombe,	Miller,	Van Cise,
Huntington,	Nichols,	Welch,
Ireland,	Ogden,	Wells,
Jones,	Paterson,	Woodward,
Lee,	Pierson,	P. T. Wright,
Lewis,	Plumley,	W. C. Wright.
	Reiter,	

ASSOCIATES ATTENDING.

Davenport,	Hall,	Johnston,
Dow,	Henderson,	Kaufman,
Forster,	A. Hunter,	Linzmeyer,
Gaylord,	Hutcheson,	Washburne.

The minutes of the meeting of May 17th and 18th, 1900, were approved as printed.

A summary of the business transacted by the Council was presented and approved.

The Secretary read the names of those admitted to Membership and enrolled as Associates since the last meeting of the Society.

New papers were then presented and read. (See pages 363-413.)

The Society adjourned at 1 P. M.

SECOND SESSION.

THURSDAY, October 25th, 1900, 2 P. M.

The reading of new papers was concluded.

The Society then proceeded to the discussion of papers read at previous meetings. (See pages 426-447.)

On motion, the President appointed Mr. Pierson and Mr. Smith as a Committee to prepare suitable notices of the members lately deceased, namely, Alex. J. Finlaison and Augustus F. Harvey.

The Society adjourned at 5 P. M.

THURSDAY, October 25th, 1900, 7 P. M.

Members and Associates of the Society, to the number of forty-five, dined together at the "Arena." President Macaulay presided.

Speeches were made by Messrs. Holcombe, Jones, Lewis, Pierson, Roche, Sartelle, Starr and Welch.

THIRD SESSION.

FRIDAY, October 26th, 1900, 10.15 A. M.

President Macaulay in the chair.

The discussion of papers was continued.

On motion of Mr. Fackler, seconded by Mr. Sartelle, a Committee consisting of five members, namely, Mr. McClintock, Mr. Miller, Mr. Van Cise, Mr. Weeks and Mr. Wells, was appointed to consider the question of collecting the mortality experience of American life companies, with reference to special points of inquiry, as indicated in the paper read by Mr. McClintock at this meeting (see page 373), and to report the result of their deliberations at the next meeting of the Society.

It was announced that the International Congress of Actuaries had accepted the invitation of the Society to meet in New York in September, 1903.

The Society adjourned at 12.30 P. M.

JOHN TATLOCK, JR.,
Secretary,
32 Nassau Street,
New York.

ALEXANDER JOHN FINLAISON, C. B., F. I. A.

Born, London, March 28th, 1840.
Died, London, September 17th, 1900.
Actuary to the Government of the United Kingdom of Great Britain and Ireland and to the National Debt Commissioners.

IN Actuarial circles, probably Mr. Finlaison was best known and will be longest remembered on account of his connection with the English Government Annuity Experience, which had been compiled and was published in 1883, under his supervision.

He entered the National Debt Office in 1860. In 1875 he was appointed Actuary to succeed his father, Alexander Glen Finlaison, who had followed the grandfather, John Finlaison, in the same office. "His advice and assistance were constantly sought by other Government Departments, and he was held in high esteem by many successive Administrations, more than one Chancellor of the Exchequer having publicly testified in the House of Commons to his abilities."

In the Institute of Actuaries, he became a Fellow by examination in 1864, was elected to the Council in 1875, was Hon. Secretary in 1883 and 1884, and Vice-President in 1887 and 1888. He was honored by election to the Presidency in 1894 and 1895.

He was elected a member of the Actuarial Society of America on October 10th, 1895, by the unanimous vote of the members on the unanimous recommendation of the Council.

At the First International Congress of Actuaries in Brussels, September 2d to 6th, 1895, he was one of the Vice-Presidents and took part in the proceedings, speaking fluently in the French language. The death of his wife a few months before the Second International Congress of Actuaries, in London, May 16th to 20th, 1898, from which grief he never recovered, deprived the members of his genial presence at that time.

Mr. Finlaison was unostentatious in manner, quiet in disposition, refined in habit, cultured in tastes—a gentleman respected and loved by colleagues, friends and all who knew him. A little over sixty years seems an untimely age at which the career of such a man should be terminated.

ALEXANDER JOHN FINLAISON.

AUGUSTUS FORD HARVEY.

Born at Watertown, N. Y., January 19th, 1830.
Died at Kirkwood, Mo., August 28th, 1900.

Augustus Ford Harvey acquired his education in the public schools and in the academy at Erie, Pa., from which he graduated in 1846. His occupations from that time until 1870 were those of teacher, telegrapher, printer, editor, land surveyor, civil engineer, insurance agent, city official and legislator. His first exclusive identification with life insurance was his appointment as actuary of the Insurance Department of Missouri in 1870, which he held with brief interruptions throughout the rest of life: in the intervals he was associated actively with life offices in St. Louis and acted as general consulting actuary.

Few persons possessed his versatility of talent or the power of strong and logical analysis of any subject engaging his interest. In whatever pursuit, his earnestness and sincerity combined with fair and honest estimate of the opinions of others made him always a most valuable counsellor and agreeable companion. Never hasty at conclusions, always frank and free of advice or criticism, without ostentation of his own opinions, his kindly, genial, generous spirit and the affluence of his imperturbable good nature endeared him to the heart and sympathy of every associate. For the lesson and encouragement of all, and especially those looking forward and hoping for the best and highest from their own personal exertions, the story of our neighbor's life should be written with an earnest and faithful pen.

For general, all-around knowledge of insurance in its every aspect of theory and practice, and in the keeping in touch with the business at almost every point, he was surpassed by very few, if any. It was his favorite life work and study, and from it he derived his greatest success and reputation.

The taking away of our friend and fellow member at this time does not come with the shock of the sudden and unexpected, but the bereavement is not the less heavy nor the loss less deeply and sincerely felt. Those who heard the brave words of his touching letter, read at the meeting of October, 1899, were prepared for the sad intelligence when received.

AUGUSTUS FORD HARVEY.

Constitution and By-Laws of the Actuarial Society of America, as Amended October 19th, 1899.

Constitution.

Art. I.—This organization shall be called "The Actuarial Society of America."

Art. II.—*Object.*

The Actuarial Society of America has for its object the promotion of actuarial science by personal intercourse, presentation of appropriate papers, discussion, and such other methods as may be found desirable.

Art. III.—*Membership.*

The membership of the society shall consist of the subscribers and of such other persons, connected with actuarial pursuits, as shall be duly elected or admitted.

Art. IV.—*Officers.*

The officers shall consist of a President, a first and a second Vice-President, a Secretary and a Treasurer.

Art. V.—*Council.*

The officers, with ex-presidents and six other members shall constitute the Council. At the first election of members of Council, after the adoption of this article, two members shall be elected to serve for three years, two for two years and two for one year; and annually thereafter two members shall be elected for three years. No retiring member of the Council shall be eligible for re-election at the same meeting.

Art. VI.—*Duties of Officers and Council.*

The duties of the officers shall be such as usually appertain to their respective offices. The duties of the Council shall be in general to manage the affairs of the Society in the intervals between its meetings, to call meetings and to make arrangements for the same, to decide upon papers offered for reading and discussion, and to pass upon candidates to be presented to the Society for election.

Art. VII.—*Election of Officers and Council.*

The officers shall be elected by ballot at the annual meeting for the term of one year. The President and Vice-Presidents shall

not be eligible for the same office for more than two consecutive years.

ART. VIII.—*Meetings.*

The annual meetings of the Society shall be held on the first Thursday after the fourteenth day of May in each year. Other meetings may be called by the Council from time to time, and shall be called by the President at any time upon the written request of ten members; twenty days' notice of such meetings shall be given by the Secretary.

ART. IX.—*Associates.*

Any one twenty-one years of age, pursuing actuarial studies and looking forward to future membership, may, on the conditions hereinafter stated, be enrolled as an associate, and in such capacity may be present, without participation, at the meetings of the Society, and receive its publications. The conditions are, that he shall be nominated to the Council by at least two members of the Society; that notice of such nomination shall be given by the Secretary to each member of the Society; that, not less than twenty days after such notice has been given, his nomination shall be approved by the Council, with not more than one negative vote, and that he shall subsequently pay such examination fee and pass such preliminary examination as may be prescribed by the Council. When the Council finds that a candidate has passed an equivalent examination required by another recognized society of actuaries it may waive the examination hereby required.

ART. X.—*Admission to Membership.*

Any associate, who has been such for at least one year, and is twenty-five years of age, may apply to the Council for membership. Every member of the Council shall be notified confidentially of such application at least twenty days before the meeting at which it shall be acted upon. If his application be approved by the Council he shall become a member on paying such examination fee and passing such final examination as shall be prescribed by the Council. Otherwise, no one shall be admitted as a member unless by the unanimous recommendation of the Council, followed by a unanimous ballot of the Society.

ART. XI.—*Quorum.*

Four members of the Council shall constitute a quorum. Ten members of the Society at any duly called meeting shall constitute a quorum.

ART. XII. No resolution expressive of opinion shall be entertained at any meeting.

ART. XIII.—*Amendments.*

This Constitution may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of such proposed amendment shall have been sent to each member by the Secretary.

BY-LAWS.

ART. I.—*Order of Business.*

At the meeting of the Society the following order of business shall be observed, unless the members vote otherwise for the time being :

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. The minutes of the last meeting read and passed.
5. Summary of the business transacted by the Council since the last meeting of the Society presented for its information and approval.
6. Candidates for membership considered and voted upon.
7. Reports of Committees and Officers.
8. At annual meeting, election of Officers and Council.
9. Unfinished business ; new business.
10. Reading of papers prepared for presentation to the Society.
11. Discussion of papers presented.
12. Reading of a summary of the business just transacted at the meeting for correction if necessary.
13. The Council meetings shall proceed in this order as far as it may apply to them.

ART. II.—Meetings of the Council shall be called whenever the President or four members of the Council so request, but not without sending notice to each member of the Council ten or more days before the time appointed ; such notice shall state the subjects intended to be brought before the meeting, and should

other matter be passed upon, any member of the Council shall have the right to re-open the question at the next meeting.

ART. III.—*Duties of Officers.*

The President, or, in his absence, the next Vice-President in order, shall preside at meetings of the Society and of the Council. At the Society meetings he shall vote only in case of a tie, but at the Council meetings he may vote in all cases.

The duties of the Secretary shall consist in keeping a full and accurate record of the proceedings at the meetings of the Society and of the Council, sending out calls for the said meetings, and, with the approval of the President and Council, carrying on the correspondence of the Society.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Society or the Council; keep a detailed account of all receipts and expenditures, and present an abstract of the same at the annual meetings, after it has been audited by a Committee of the Council.

ART. IV.—*Dues.*

Each member shall pay ten dollars, and each associate five dollars, upon entrance, and a like amount at each annual meeting thereafter, except in the case of members not residing on this continent, who shall pay five dollars at the times stated.

It shall be the duty of the Treasurer to notify by mail any member or associate whose dues may be six months in arrear, and to accompany such notice by a copy of this article. If such member or associate shall fail to pay his dues within three months from the date of mailing such notice, the Treasurer shall report the delinquent to the Secretary, by whom his name shall be stricken from the rolls, and he shall thereupon cease to be a member or associate of the Society. He may, however, be reinstated by vote of the Council, and upon payment of arrears of dues.

ART. V.—*Amendments.*

These By-Laws may be amended by an affirmative vote of two-thirds of the members present at any meeting held at least three months after notice of the proposed amendment shall have been sent to each member by the Secretary.

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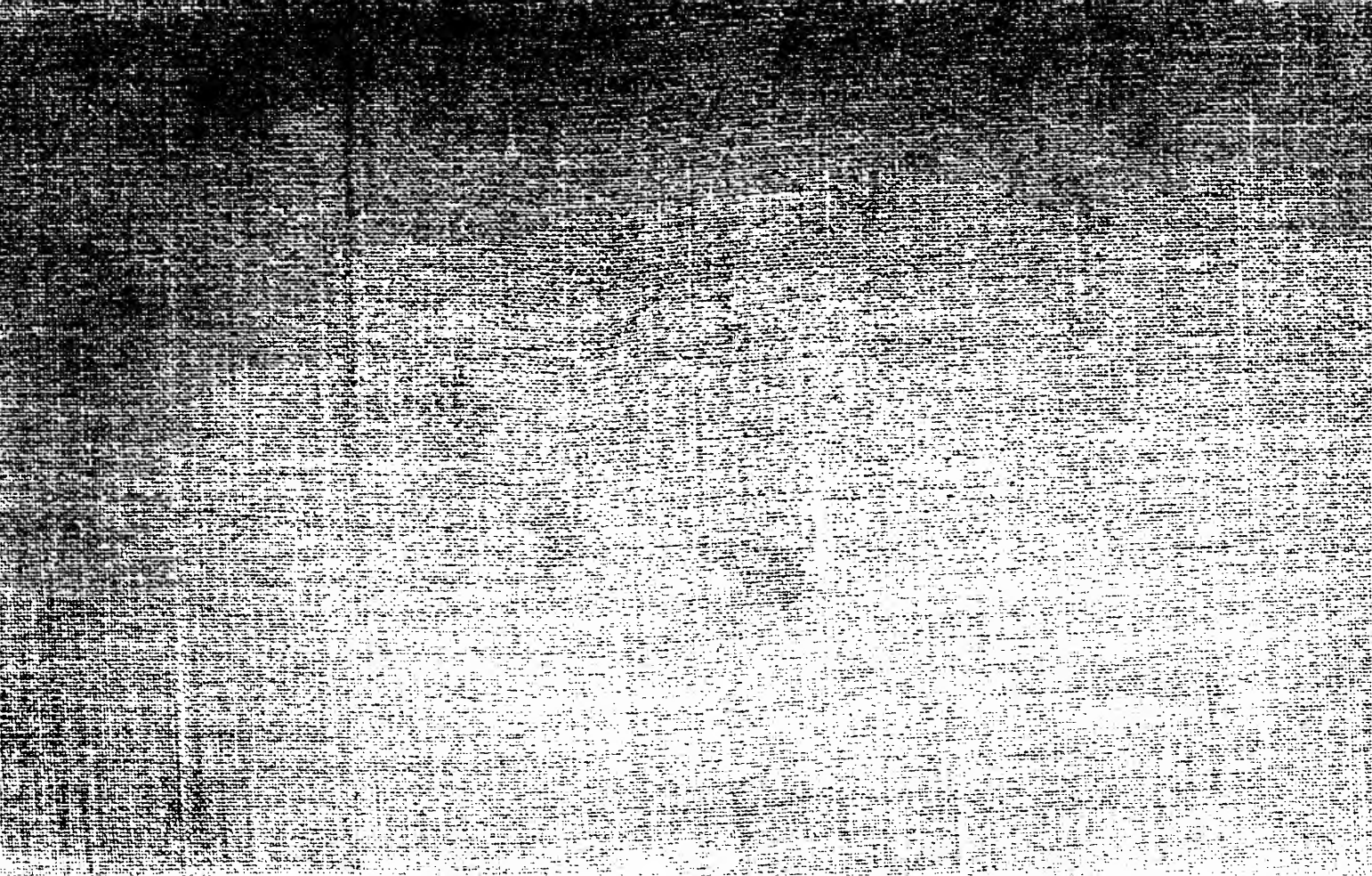
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